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Magnetic Diffusion in Star Formation

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Abstract. Magnetic diffusion plays a vital role in star formation. We trace its influence from interstellar cloud scales down to star-disk scales. On both scales, we find that magnetic diffusion can be significantly enhanced by the buildup of strong gradients in magnetic field structure. Large scale nonlinear flows can create compressed cloud layers within which ambipolar diffusion occurs rapidly. However, in the flux-freezing limit that may be applicable to photoionized molecular cloud envelopes, supersonic motions can persist for long times if driven by an externally generated magnetic field that corresponds to a subcritical mass-to-flux ratio. In the case of protostellar accretion, rapid magnetic diffusion (through Ohmic dissipation with additional support from ambipolar diffusion) near the protostar causes dramatic magnetic flux loss. By doing so, it also allows the formation of a centrifugal disk, thereby avoiding the magnetic braking catastrophe.

Keywords. MHD, turbulence, waves, stars: formation, ISM: clouds, ISM: magnetic fields

1. Clouds to Cores

Magnetic energy dominates self-gravitational energy in the H I clouds that occupy the interstellar medium (ISM) of our Galaxy (Heiles & Troland 2005). In other words, their mass-to-flux ratio M/Φ is significantly less than the critical value required for gravitational collapse and fragmentation. On the other hand, molecular clouds, the birthplaces of stars, have mass-to-flux ratios that are very close to the critical value (Crutcher 2004). Mass-to-flux ratios in molecular clouds (or cloud fragments) that are significantly greater than the ambient ISM value can be achieved by two distinct but not mutually exclusive mechanisms. One is the accumulation of matter *along* the ambient magnetic field direction in the local spiral arm. However, this places severe constraints on the accumulation length of molecular clouds, and on either the associated formation timescale or the magnitude of streaming motions that can create the clouds (Mestel 1999). The second and rather attractive possibility is that the formation process of the molecular cloud, in a medium pervaded by turbulence or nonlinear flows, will lead to rapid ambipolar diffusion in at least the compressed (filamentary or sheet-like) regions. This scenario of turbulent ambipolar diffusion has been explored in several recent works, e.g., Li & Nakamura (2004), Kudoh & Basu (2008), and Basu *et al.* (2009).

Here, we report the results of Basu & Dapp (2010). Solution of the thin-sheet MHD equations, including ambipolar diffusion, shows that the rapid accumulation of subcritical gas is expected to lead to islands of higher mass-to-flux ratio. These regions undergo enhanced ambipolar diffusion during the compression, and if they are still subcritical, they then undergo ambipolar-diffusion-driven contraction more rapidly than their surroundings due to the elevated density in those regions. The qualitative result is a handful of cores that are formed within elongated ridges. These cores undergo runaway collapse with subsequent near-flux-trapping as soon as gravity overwhelms magnetic and thermal pressure forces within a supercritical region. However, most of the gas in the cloud

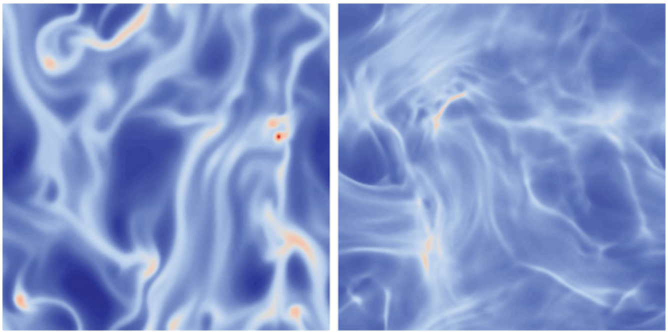

Figure 1. Images of gas column density for initially turbulent models with ambipolar diffusion (left) and flux freezing (right), shown in identical logarithmic color schemes. Both models have initially the same subcritical mass-to-flux ratio and turbulent power spectrum and amplitude, that is allowed to decay. Both models are shown at the same physical time, with the model on the left undergoing runaway collapse in isolated cores and the model on the right in the midst of indefinite supersonic motions. From Basu & Dapp (2010).

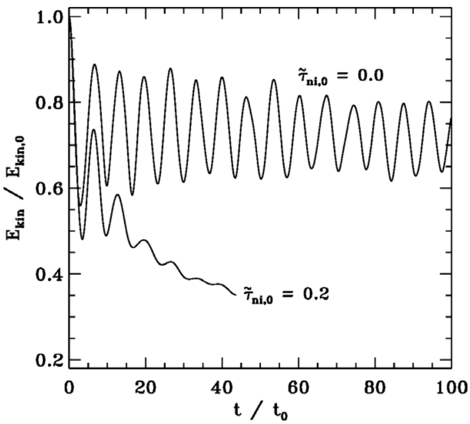

Figure 2. Decay of kinetic energy. The time evolution of total kinetic energy, normalized to its initial value, for a model with flux freezing ($\tilde{\tau}_{\rm ni,0} = 0$) and another with ambipolar diffusion corresponding to a canonical ionization fraction for molecular clouds ($\tilde{\tau}_{\rm ni,0} = 0.2$). From Basu & Dapp (2010).

remains subcritical, does not form stars, and maintains a higher velocity dispersion. This mode of star formation is illustrated in the left panel of Fig. 1. The dark (red) dots represent high density regions of cores that are undergoing runaway collapse. This occurs at a time $43.5\,t_0$ in this model, where $t_0 = c_{\rm s}/(2\pi G\Sigma_0)$, $c_{\rm s}$ is the isothermal sound speed, and Σ_0 is the mean column density of the sheet. The collapse time is a factor ≈ 6 shorter than if starting with small-amplitude initial perturbations. The right panel shows the state of evolution at the same physical time and for statistically identical initial conditions but with ambipolar diffusion turned off, i.e., pure flux freezing. In this case, a startling result is that supersonic motions persist indefinitely. This follows an initial phase in which some but not all of the initial kinetic energy is lost due to shocks. A full explanation and theory is given by Basu & Dapp (2010). Fully three-dimensional simulations by Kudoh & Basu (2008) confirm the rapid core formation found in thin-sheet models such as this and those of Li & Nakamura (2004) and Basu *et al.* (2009). The thin-sheet flux-freezing result of long-lived supersonic motions (Basu & Dapp 2010) remains to be tested fully in three dimensions since the result is dependent upon the existence of a low density

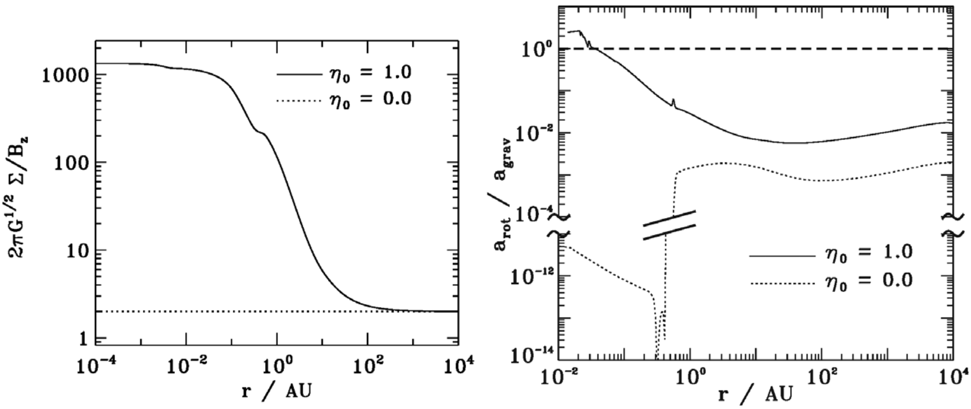

Figure 3. Spatial profiles of mass-to-flux ratio (left) and centrifugal support level (right). Both profiles are shown after the formation of the second core, with extent $\sim R_\odot$. The plot on the right is shown at a time shortly after the introduction of a central sink cell of radius $3\,R_\odot$ that masks the newly-formed second core. From Dapp & Basu (2010).

external medium in which the magnetic field can quickly adjust to a current-free configuration. The latter is built in to the thin-sheet models and may be approximately true in a three-dimensional model of a cold magnetized cloud embedded in a hot tenuous medium.

2. Cores to Star-Disk Systems

Core collapse inevitably begins when the mass-to-flux ratio is a factor ≈ 2 above the critical value (e.g., Basu & Mouschovias 1994). Rapid collapse on a dynamical timescale is able to effectively trap the remaining magnetic flux during the prestellar runaway collapse phase. If this flux trapping continued indefinitely, there would remain a big magnetic flux problem for the final star. A cloud core with twice the critical mass-to-flux ratio would still contain 10^8 times as much magnetic flux per mass as threads the solar surface, and $10^3 - 10^5$ times as much as a magnetic Ap star or a T Tauri star.

Fortunately, a resolution to the magnetic flux problem is facilitated in the post-stellar-core formation epoch, also known as the accretion phase or "$t > 0$", where $t = 0$ is the pivotal moment at which a central protostar is formed. In the spherically symmetric model of Shu (1977), $t = 0$ is pivotal in that an expansion wave subsequently moves outward from the center and envelopes a region of near-free-fall that is dominated by the potential of the central point mass. For the purpose of the magnetic flux problem, $t = 0$ is also pivotal. The subsequent collapse leads to sharp magnetic field gradients in the innermost regions such that the diffusion terms dominate the advection term in the magnetic induction equation, as shown by Li & McKee (1996) and Contopoulos *et al.* (1998). Expressed more loosely, the extreme dragging-in of magnetic field lines in the flux-freezing limit leads to a split-monopole configuration that would in reality be prone to rapid magnetic diffusion due its sharp magnetic field gradient. It is essentially a self-regulation by magnetic diffusion that prevents a split-monopole from forming in the accretion phase.

Another classical problem of star formation is due to angular momentum. Even a rotation rate of $\sim 10^{-14}$ rad s^{-1} as observed in molecular clouds (Goodman *et al.* 1993), while not dynamically important on cloud scales, contains enough angular momentum to prevent nearly all the matter from falling in to a central region of stellar dimensions. However, collapse in the prestellar phase is never sufficient to raise the level of centrifugal

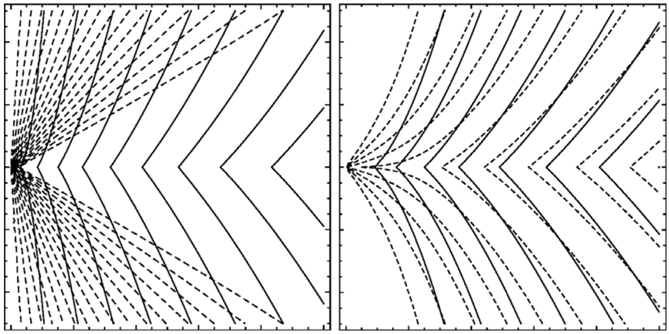

Figure 4. Magnetic field lines. The box on the left has dimensions 10 AU on each side, while the box on the right has dimensions 100 AU on each side. The dashed lines represent the flux-freezing model ($\eta_0 = 0$) while the solid lines show the same field lines for $\eta_0 = 1$. The second core has just formed and is on the left axis midplane.

support relative to gravity, a result decisively shown by the simulations of Norman *et al.* (1980) and explained analytically as a property of self-similar collapse profiles by Basu & Mouschovias (1995). Magnetic braking further weakens the level of centrifugal support, primarily in the core formation epoch, because the subsequent runaway collapse of a prestellar core is generally too rapid for magnetic braking to remain active (Basu & Mouschovias 1994). Here again, $t = 0$ provides a pivot point, after which a centrifugal barrier *does* exist for infalling mass shells, as they fall inward under the gravitational domination of a central protostar. While this implies that a disk will be formed (only) in the $t > 0$ phase, Allen *et al.* (2003) in fact found that magnetic braking gets revitalized in this phase under the assumption of flux freezing. For $t > 0$, the extreme flaring of the magnetic field due to a monopole-like configuration is able to couple near-stellar regions to regions of far greater moment-of-inertia, leading to very efficient magnetic braking. Also, even if a centrifugal disk begins to form during the $t > 0$ phase, the radial velocity is slowed down enough that magnetic braking has time to act and prevent the ultimate formation of the disk.

We have solved the MHD equations for a rotating thin sheet, including the effect of Ohmic dissipation, in axisymmetric geometry. Our results published in Dapp & Basu (2010) show that significant magnetic flux loss occurs within the first core (radius $\sim$ few AU), and effectively shuts down magnetic braking. Ohmic dissipation is added according to the prescription of Nakano *et al.* (2002), similar to the implementation of Machida *et al.* (2007), but accounting for spatial gradients in the resistivity. A dimensionless scaling parameter η_0 characterizes the resistivity, in which the uncertainties hinge largely on the grain size distribution. A canonical value is $\eta_0 = 1$. Additional significant magnetic diffusion can occur at high densities due to ambipolar diffusion, as studied by Kunz & Mouschovias (2010), but is not included in our present study. Ohmic dissipation and its facilitation of disk formation has also been modeled by Krasnopolsky *et al.* (2010) but only on larger scales (>10 AU).

The eventual transformation of a first core to a second core of radius $\sim R_\odot$, due to H_2 dissociation, then occurs with near angular momentum conservation. This is due to rapid Ohmic dissipation having rendered magnetic braking ineffective in this region. The result is that a centrifugal disk does indeed form in the near-environment of the newly-formed second core with mass only $\sim 10^{-3}\, M_\odot$. The rapid flux loss (as seen by a dramatically increased mass-to-flux ratio) is shown in the left panel of Fig. 3. The right panel illustrates the achievement of centrifugal balance in the late stages of the resistive model ($\eta_0 = 1$), and the *magnetic braking catastrophe* in the flux-freezing model ($\eta_0 = 0$). In the latter case, centrifugal support is decimated in the innermost regions by magnetic braking, and a disk cannot form.

Fig. 4 shows the dramatic difference in magnetic field line structure above and below the sheet, calculated using the current-free approximation from a scalar magnetic potential. The split monopole of the $\eta_0 = 0$ model is replaced by a much more relaxed field line structure. The extreme flaring of field lines in the $\eta_0 = 0$ model is a fundamental cause of the magnetic braking catastrophe. More details are in Dapp & Basu (2010). Similar results on magnetic field structure, using a simplified model of Ohmic dissipation, can be found in Galli *et al.* (2009).

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