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COMMENT ON A NOTE BY J. MARICA AND J. SCHÖNHEIM

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In [2] it is shown that an $n \times n$ partial latin square with $n-1$ cells occupied on the main diagonal can be completed to a latin square. We can use the technique in [2] to prove the following result.

> An $n \times n$ partial latin square with $n-1$ cells occupied with $n-1$ distinct symbols can be completed to a latin square if the occupied cells are in different rows *or* different columns.

Let P be an $n \times n$ partial latin square based on $0, 1, 2, \ldots, n-1$ satisfying the above conditions, and let $(x_0, y_0), (x_1, y_1), \ldots, (x_{n-2}, y_{n-2})$ be the occupied cells where $y_0, y_1, \ldots, y_{n-2}$ are distinct. In Z_n let $x_{n-1} = x_0 + x_1 + \cdots + x_{n-2}$. Then $(-x_0)+(-x_1)+\cdots+(-x_{n-2})+x_{n-1}=0$ and so by a result due to M. Hall [1] there exists a permutation $\begin{pmatrix} a_0 & a_1 & \ldots & a_{n-1} \\ c_0 & c_1 & \ldots & c_{n-1} \end{pmatrix}$ of the elements of Z_n such that

$$c_i - a_i = -x_i \quad (i = 0, 1, 2, \ldots, n-2) \quad \text{and} \quad c_{n-1} - a_{n-1} = x_{n-1}.$$

This gives

$$c_i + x_i = a_i \quad (i = 0, 1, 2, \ldots, n-2) \quad \text{and} \quad c_{n-1} - x_{n-1} = a_{n-1}.$$

Now define a latin square (b_{ij}) by $b_{ij} = i + c_j$; $i, j = 0, 1, 2, \ldots, n-1$. Then in (b_{ij}) the cells $(x_0, c_0), (x_1, c_1), \ldots, (x_{n-2}, c_{n-2})$ are occupied by distinct symbols. Since $c_0, c_1, \ldots, c_{n-2}$ are distinct, a suitable permutation of the columns of (b_{ij}) places distinct symbols in the cells $(x_0, y_0), (x_1, y_1), \ldots, (x_{n-2}, y_{n-2})$. Hence a permutation on the elements $0, 1, 2, \ldots, n-1$ places the required symbols in these cells giving a completion of P.

References

1. M. Hall, Jr., *A combinatorial problem on abelian groups*, Proc. Amer. Math. Soc. **3** (1952), 584–587.

2. J. Marica and J. Schönheim, *Incomplete diagonals of latin squares*, Canad. Math. Bull. **12** (1969), p. 235.

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