PROPER ELEMENTS, FAMILIES, AND BELT BOUNDARIES

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Families of asteroids were first found by Hirayama (1918, 1923, 1928). More recently Brouwer (1951) and Arnold (1969) have extended greatly the number of families known from the cataloged asteroids. The Palomar-Leiden survey (PLS) (van Houten et al., 1970), which mainly covered very faint, uncataloged objects, found several more families.

Except for the work of Hirayama (1918), which used osculating elements, all of the above studies looked for clusterings of the semimajor axis a, the proper eccentricities e', and proper inclinations i'. The calculation of proper elements involves using a theory of secular perturbations to remove the long-period, large-amplitude disturbances of the major planets. The theory used up to now (Brouwer and van Woerkom, 1950; Brouwer and Clemence, 1961) involved a low-order expansion in the eccentricities and inclinations. There is now a theory available (Williams, 1969) that will accurately handle much higher eccentricities and inclinations than before.

I am now applying this new theory to calculate improved proper elements. The job is only partly completed, but several interesting results have emerged. These preliminary results are based upon reductions for asteroids with $a < 2.61$ AU.

The theory may be used to calculate the closest approach between a major planet and an asteroid. When this is done for Mars, it is found that the density of asteroids drops sharply when one crosses into the region where an asteroid can encounter Mars. This effect can be seen in figure 1 where a histogram of the number of asteroids per 0.02 AU interval of the closest distance of approach to Mars is given. A negative distance means that the asteroid can pass that distance within the orbit of Mars and is subject to close encounters and eventual removal from the solar system by planetary collision or ejection. The evolution of planetary crossers was first discussed by Öpik (1963). There is a tail in the distribution for small negative distances. As one approaches the Mars crossing boundary from the negative side, the number of times the secular perturbations caused the orbits to intersect during the age of the solar system becomes fewer. Objects with small negative distances have very long lifetimes. The inner boundary of the asteroid belt in a, e', sin i' space is determined by the Mars crossing boundary. It is an almost inescapable conclusion that the belt

https://doi.org/10.1017/S0252921100089028 Published online by Cambridge University Press

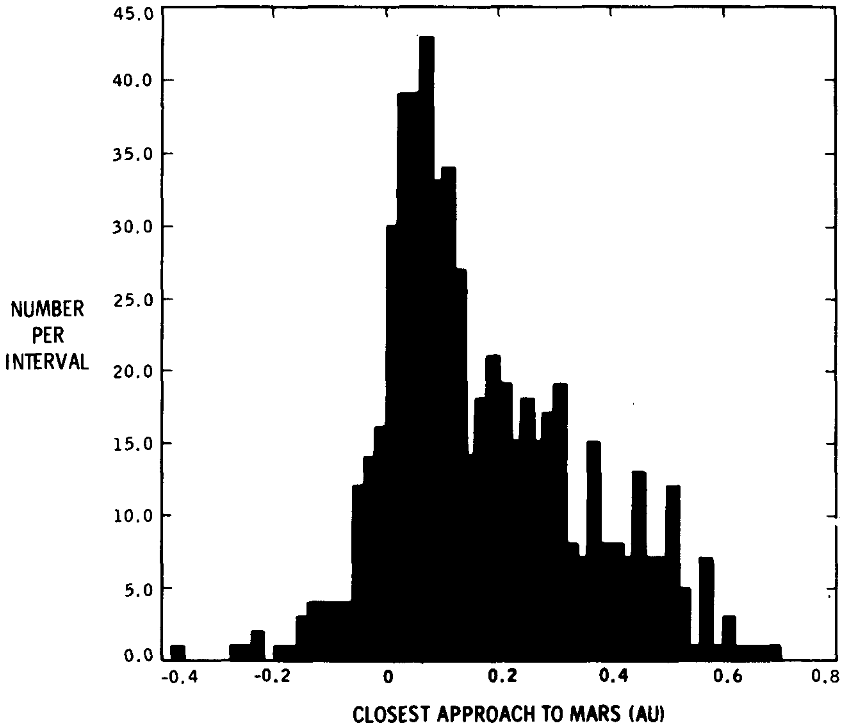

Figure 1.–Number of asteroids per 0.02 AU interval of the distance of closest approach to Mars. Negative distances are Mars crossers. Data are for belt asteroids with $a < 2.61$ AU.

once extended closer to the Sun and that only objects in stable or nearly stable orbits remain.

From the old theory it was known that there are strong resonances of the node rate $\dot{\Omega}$ and longitude of perihelion rate $\dot{\tilde{\omega}}$ at 1.94 and 2.03 AU, respectively, for $e' = i' = 0$. In the new theory these resonances form surfaces in a, e', sin i' space. It is found that there are very few asteroids in the vicinity of these resonance surfaces. In the vicinity of a resonance, the oscillations of e' and i' become very large and the Mars crossing boundary is pushed deeper into the belt. The resonances are not completely understood and it is not clear whether the displacement of the Mars crossing boundary is sufficient to explain all of the depopulation in the neighborhood of resonance surfaces.

Figure 2 shows a plot of proper sin i' against a for the cataloged asteroids that have been studied. Circles are used to denote Mars crossers. The high-inclination region at about 1.9 AU is the Hungaria region, and the one centered at 2.37 AU is the Phocaea region. The resonance that starts at 1.94 AU at $e' = i' = 0$ runs up between the Hungaria and Phocaea regions, whereas

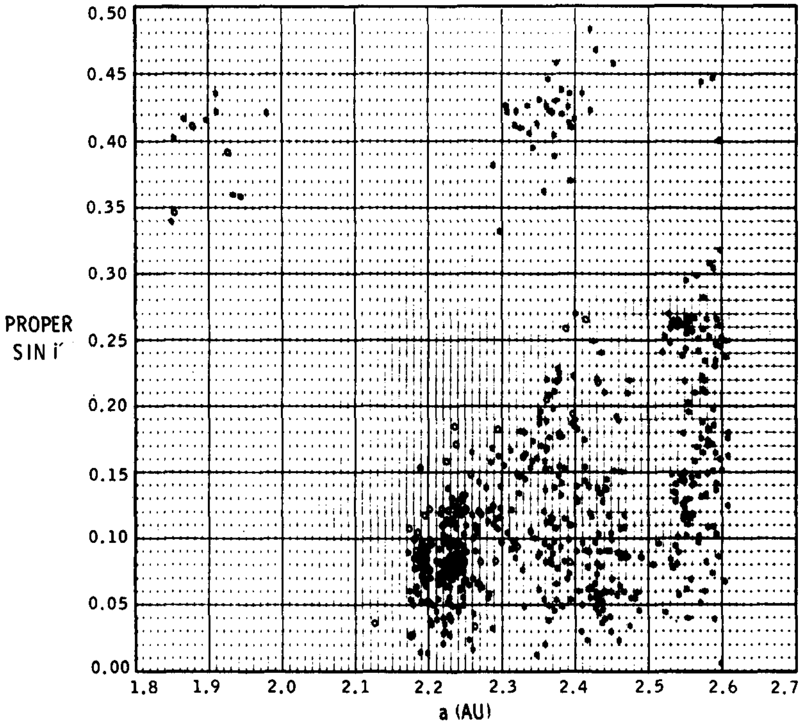

Figure 2.–Asteroid sin i' plotted against semimajor axes. Open circles are for objects that can encounter Mars during the age of the solar system. The two high-inclination regions are isolated from the main belt by secular resonances.

the one that starts at 2.03 AU runs between the Phocaea region and the main belt. A third strong resonance caps the belt off at about 30° inclination. The gap at 2.5 AU is the Kirkwood gap at the 3:1 commensurability with Jupiter. The Hungaria and Phocaea regions are not large families but segments of the belt isolated by resonances.[1] The former region contains one normal-sized family and the latter contains two.

Numerous families can be recognized in the data. Arnold's (1969) work is strongly confirmed. Many of the families are found to contain a relatively large object, and this large object is always at the edge of the family in a, e', sin i' space. These families with large objects would seem to be debris from a major cratering event on the large asteroid. The families without large objects presumably result from total disruption of the parent asteroid. The distinction between families with and without a large object is somewhat artificial. The two classes really merge. Of course, total disruption is the limiting case of

[1]See, however, p. 363 of PLS.

cratering. The above opinions obviously rely on the collision theory of the origin of asteroid families.

ACKNOWLEDGMENT

This work was one phase of research carried out at the Jet Propulsion Laboratory, California Institute of Technology, under Contract no. NAS 7-100, sponsored by the National Aeronautics and Space Administration.

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DISCUSSION

MARSDEN: Does the 9:2 resonance with Jupiter (or 2:3 resonance with Mars) really have a decisive influence on the motions of the Hungaria asteroids?

WILLIAMS: These are only approximate commensurabilities and probably unimportant as far as the existence of these objects is concerned. Their mean motions spread over a large range, 1270 to 1410 arcsec/day, whereas the commensurabilities in question lie at 1346 and 1258 arcsec/day. The Jovian commensurability is of seventh order and should be very small, whereas Mars has such a small mass that it is hard for it to have a significant influence. Actually the Hungaria asteroids lie in a small island of stability between two of the secular resonances and the Mars crossing boundary. I think they demonstrate that the belt was once much larger but that Mars has swept out, by collisions and close approaches, the regions that are now empty.

BRATENAHL: The histogram of N versus a is remarkable in showing how sharply Mars defines the inner boundary of the asteroid belt. Do you have any estimate of the lifetime of Eros, and is that limited by an impact on Mars or on which planet?

WILLIAMS: The Mars crossers typically have lifetimes of 10^8 to 10^9 yr and may impact any of the terrestrial planets. If Eros could evolve into an Earth crosser through secular perturbations, then its lifetime might be an order of magnitude smaller.

ANONYMOUS: Can an explanation be given of the mechanism by which observational selection can give rise to an apparent jetstream?

WILLIAMS: I will give an example of a selection effect for the Flora family due to secular perturbations. These perturbations cause a bias in the eccentricities that has an approximately sinusoidal dependence on the longitude of perihelion and an amplitude of 0.05. This causes the average perihelion distance of 1.9 AU to have peak variations of ±0.1 AU. The objects with perihelia of 1.8 AU will be 0.7 mag brighter than those objects with perihelia of 2.0 AU, the two extremes being 180° apart in the sky. Because an asteroid is discovered in the vicinity of its perihelion, more small objects will be seen in the direction of the closer perihelia. Using the factor of 2.5 per mag for the differential number density from the PLS gives a factor of $2.5^{0.7} = 1.9$ in the ratio of the number of objects discovered at the two extremes. To be cataloged, an object must be seen at a minimum of three different oppositions. Because the discoveries at the different oppositions are usually independent of one another, the peak factor among cataloged objects will be $1.9^3 = 6.8$. Averaging the sinusoidal bias over a 180° range of longitude of perihelia, in the direction of the minimum perihelia, results in about four times as many cataloged objects as in the opposite half of the sky. Such a concentration would be considered to be evidence of a jetstream. There are also seasonal selection effects due to weather and altitude of the ecliptic.

The above is only an order of magnitude calculation, but it illustrates the severity of the selection effects among the cataloged asteroids fainter than mean opposition photographic magnitude 15.0.