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THESIS ABSTRACTS

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ARTHUR FREITAS RAMOS, ***Explicit Computational Paths in Type Theory***, Universidade Federal de Pernambuco, Brazil, 2018. Supervised by Anjolina G. de Oliveira and Ruy J. G. B. de Queiroz. MSC: 03B, 03B15, 03B70. Keywords: computational paths, homotopy type theory, type theory, identity type, equality theory, category theory, term rewrite system, uniqueness of identity proofs.

Abstract

The treatment of an equality as a type in type theory gives rise to an interesting type known as identity type. The idea is that, given terms a, b of a type A, one may form the type $Id_A(a, b)$, whose elements are proofs that a and b are equal elements of type A. A term of this type, $p : Id_A(a, b)$, makes up for the grounds (or proof) that establishes that a is indeed equal to b. Many interesting results have been achieved using the identity type. One of these was the discovery of the Univalent Models in 2005 by Vladimir Voevodsky. A groundbreaking result has arisen from Voevodsky's work: the connection between type theory and homotopy theory. The intuitive connection is simple: a term $a : A$ can be considered as a point of the space A and $p : Id_A(a, b)$ is a homotopy path between points $a, b \in A$. This semantical interpretation has given rise to a whole new area of research known as Homotopy Type Theory. Inspired by the path-based approach of the homotopy interpretation, we propose that a proof of equality can be seen as a sequence of substitutions and rewrites, also known as a computational path. The idea is that a term $p : Id_A(a, b)$ will be a computational path between terms $a, b : A$. With that in mind, our work has three main objectives. The first one is the proposal of computational paths as a new entity of type theory. In this proposal, we point out the fact that computational paths should be seen as the syntax counterpart of the homotopical paths between terms of a type. We also propose a formalization of the identity type using computational paths. The second objective is the proposal of a mathematical structure for a type using computational paths. We show that using categorical semantics it is possible to induce a groupoid structure for a type and also a higher groupoid structure, using computational paths and a rewrite system. We use this groupoid structure to prove that computational paths also refute the uniqueness of identity proofs. The last objective is to formulate and prove the main concepts and building blocks of homotopy type theory, now using terms which represent (explicit) computational paths. We wrap up this last objective with a proof of the isomorphism between the fundamental group of the circle and the group of the integers.

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Marcus Vinícius Midena Ramos, ***Formalization of Context-Free Language Theory***, Universidade Federal de Pernambuco, Brazil, 2016. Supervised by Ruy J. G. B. de Queiroz. MSC: 03B35, 03D05, 03F65, 68Q42, 68Q45, 68T15. Keywords: proof assistants, Coq, formalization, language theory, context-free languages, context-free grammars.

Abstract

Proof assistants are software-based tools that are used in the mechanization of proof construction and validation in mathematics and computer science, and also in certified program development. Different such tools are being increasingly used in order to accelerate and simplify proof checking, and the Coq proof assistant is one of the most well known and used in large-scale projects. Language and automata theory is a well-established area of mathematics, relevant to computer science foundations and information technology. In particular, context-free language theory is of fundamental importance in the analysis, design, and implementation of computer programming languages. This work describes a formalization effort, using the Coq proof assistant, of fundamental results of the classical theory of context-free grammars and languages. These include closure properties (union, concatenation, and Kleene star), grammar simplification (elimination of useless symbols, inaccessible symbols, empty rules, and unit rules), the existence of a Chomsky Normal Form for context-free grammars and the Pumping Lemma for context-free languages. The result is an important set of libraries covering the main results of context-free language theory, with more than 500 lemmas and theorems fully proved and checked. As it turns out, this is a comprehensive formalization of the classical context-free language theory in the Coq proof assistant and includes the formalization of the Pumping Lemma for context-free languages. The perspectives for the further development of this work are diverse and can be grouped in three different areas: inclusion of new devices and results, code extraction, and general enhancements of its libraries.

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Douglas Ulrich, ***Some Applications of Set Theory to Model Theory***, University of Maryland, College Park, USA, 2018. Supervised by Michael C. Laskowski. MSC: 03C55. Keywords: Keisler's order, Borel complexity.

Abstract

We investigate set-theoretic dividing lines in model theory. In particular, we are interested in Keisler's order and Borel complexity.

Keisler's order $\trianglelefteq$ is a pre-order on complete countable theories T, measuring the saturation of ultrapowers of models of T. In Chapter 3, we present a self-contained survey on Keisler's order, cast in terms of full Boolean-valued models and pseudosaturation. The cornerstone of our development is a compactness theorem for full Boolean-valued models. As an application, we show that if T is a complete countable theory and $\mathcal{B}$ is a complete Boolean algebra, then λ^+-saturated $\mathcal{B}$-valued models of T exist. Moreover, if $\mathcal{U}$ is an ultrafilter on T and $\mathbf{M}$ is a λ^+-saturated $\mathbf{B}$-valued model of T, then whether or not $\mathbf{M}/\mathcal{U}$ is λ^+-saturated just depends on $\mathcal{U}$ and T; we say that $\mathcal{U}$ λ^+-saturates T in this case. We show that Keisler's order can be formulated as follows: $T_0 \trianglelefteq T_1$ if and only if for every cardinal λ, for every complete Boolean algebra $\mathcal{B}$ with the λ^+-c.c., and for every ultrafilter $\mathcal{U}$ on $\mathcal{B}$, if $\mathcal{U}$ λ^+-saturates T_1, then $\mathcal{U}$ λ^+-saturates T_0. We also prove that if $\mathcal{B}$ is a complete Boolean algebra with an antichain of size λ, then there is a λ^+-good ultrafilter on $\mathcal{B}$; conversely, if $\mathcal{B}$ is a complete Boolean algebra with the λ-c.c., then then no nonprincipal ultrafilter on $\mathcal{U}$ λ^+-saturates any unsimple theory, and no $\aleph_1$-incomplete ultrafilter on $\mathcal{U}$ λ^+-saturates any nonlow theory. Finally, we