IMPAIRED CAPITAL REALLOCATION AND PRODUCTIVITY

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The level of private sector labour productivity has been particularly weak since the start of the crisis. In this paper we explore whether impairment to capital reallocation has been contributing to this weakness. The recent increase in the dispersion of output, prices and rates of return across firms and sectors is stark, and suggests that resources have had incentives to move. Efficient allocation requires that capital moves to firms and sectors where rates of return are relatively high. And the change in capital levels across sectors has been particularly low, suggesting there has been an unusually slow process of capital reallocation since 2008 compared to previous UK recessions and other banking crises. This result is also apparent within sectors. We use a simple and general model to show that increased price dispersion can be a consequence of frictions to efficient capital allocation. And the size of this dispersion can usefully inform us about the size of the associated output and productivity loss. We then find that – using firm level data – the relationship between rates of return and subsequent capital movements has changed since the financial crisis. Overall, our results suggest that impaired capital reallocation across the UK economy is likely to have been one factor contributing to the recent weakness in productivity growth.

Keywords: capital allocation; productivity; financial crisis

JEL Classifications: D24; E22; O47

1. Introduction

At the end of 2013, private sector output per worker remained around 18 percentage points below the level implied by a simple extrapolation of its pre-crisis trend.[1] The scale of the productivity fall and the continued stagnation has been the defining feature of the UK's recent recession and stands in contrast to previous UK recessions. Some other European countries have also experienced a productivity slowdown since 2008, although the UK has seen one of the largest falls relative to its pre-crisis trend. The US experience, on the other hand, has been very different: US productivity showed a small initial drop but quickly returned to its pre-recession trend (Hughes and Saleheen, 2012). Understanding the factors behind the weakness in productivity following the 2008 recession, including how persistent these factors might be, is important in identifying the policy challenges and designing appropriate solutions.[2]

A growing body of research has put forward various explanations for this weakness. Some of these stress the cyclical or temporary nature of the slowdown while others highlight the structural or the more persistent nature of the productivity weakness. Those in the former camp rely on the fact that firms face adjustment costs when hiring or firing and therefore may choose to hold on to labour in order to retain their skills and experience for when the economy recovers. Blundell, Crawford and Jin (2013) find that flexible wages and increased labour supply are likely to have affected aggregate productivity. In addition, if firms divert resources to activities dedicated to winning work and securing contracts, this may also lead to temporary weakness. Those in the latter camp stress the importance of shocks that had a persistent effect on the UK's productive capacity such as negative shocks to the availability of credit to UK companies, for

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example. Indeed, Oulton and Sebastia-Barriel (2013) find that financial crises tend to reduce the long-run level of productivity.

It may well be that the behaviour of productivity cannot be ascribed to any one particular explanation. But rather a number of factors and explanations may have driven the weakness we have seen.

In this paper, we aim to investigate the extent to which some of the weakness in productivity can be explained by impaired allocation of capital across firms and sectors. We do not identify individual channels through which the allocation may have been impaired. Instead we try to assess the overall incidence of capital misallocation and assess the effect on TFP. We see this as complementary to existing evidence rather than providing a full alternative explanation.

The UK faced a series of shocks at the onset of the financial crisis which are likely to have affected some firms and sectors more than others. For example, the exchange rate depreciation may have had a stronger effect on those firms that produce or consume products with a tradable content. Such uneven shocks are likely to have caused shifts in relative marginal returns. In theory, in a world in which resources are mobile, capital and labour would move to where their respective marginal return is higher until these are equalised. Frictions to the allocation of capital will impede this process.

We present two pieces of evidence for the existence of such frictions. First, we document a sharp increase in the dispersion of output prices across sectors and relate this to an aggregate TFP loss. Drawing on a simple model in which we simulate cross-sectional demand shocks, we argue that the persistent price dispersion represents a signal that capital is not moving to equalise rates of return across sectors. Second, we use rich firm-level data to estimate the relationship between the rates of return on capital and subsequent investment rates. As expected, this relationship is positive before the crisis. However we find an insignificant relationship after the crisis: capital is not being attracted to where the returns are highest. In both exercises we attempt to quantify the potential size of the aggregate TFP loss that results from impaired allocation, and conclude that it is likely to play a role in explaining the extent to which productivity is below its pre-crisis trend.

The paper proceeds as follows. Section 2 highlights the growing literature on the role of capital allocation in driving productivity growth and describes the framework in which we consider impaired capital allocation. Section 3 relates this to dispersion in output prices. Section 4 presents empirical, firm-level, estimates of the relationship between rates of return and subsequent capital growth. A final section discusses our results and concludes.

2. Resource allocation and productivity

Economic theory suggests that more productive companies should have a greater incentive to, and be more able to, attract inputs, be it capital or labour, relative to companies that are less efficient. Over time, less productive companies are forced to become more efficient or go out of business. This process brings about capital and labour reallocation, which will show up in measured aggregate Total Factor Productivity (TFP), and which will foster productivity growth across the economy as a whole.

There is a substantial body of literature demonstrating the importance of resource allocation in driving productivity. Disney *et al*. (2003) find that during the 1980s and 1990s recessions external restructuring (measured as the exit, entry or changing market share of firms) could explain around 50 per cent of UK labour productivity growth within the manufacturing sector. More recently, Barnett *et al.* (2014) find that labour reallocation across firms explained 48 per cent of labour productivity growth for most UK sectors[3] in the 5 years prior to 2007. They also find, however, that since 2007, the contribution of reallocation to aggregate productive growth fell to nearly zero, indicating that the process of efficient resource allocation may have been impaired. Their work focuses on labour reallocation across firms and sectors. This paper builds on that work but focusses specifically on the capital reallocation channel. It contributes to the growing body of literature linking resource misallocation to aggregate productivity (Bartelsman *et al*., 2013; Restuccia and Rogerson, 2013). And it also relates to the body of work, dating back to Schumpeter, that considers the role of resource restructuring in crises (Caballero, 2007).[4]

Misallocation of resources can arise if there are impediments to the movement of factors between heterogeneous firms and sectors (Broadbent, 2012). This can give rise to persistent rates of return differentials across firms, and reduce aggregate TFP and labour productivity growth.[5] Impediments to the efficient allocation of capital can take various forms. In the context of the recent financial crisis, it is plausible that these impediments have intensified – notably in the form of financial market frictions and weak and uncertain demand conditions.

At the same time the shocks associated with the crisis may have increased the necessity for some reallocation of resources. Since only a part of capital reallocation occurs as a result of relatively slow depreciation of less productive capital assets, any restraint on investment by more productive firms can have a significant impact in slowing the process of reallocation.

The 2008 financial crisis was associated with weak demand conditions and a large increase in uncertainty, both of which can distort the market signals that drive incentives to reallocate and depress investment.[6] Weak demand can mean that current resources are underutilised, leading firms (including those with high productivity) to delay expansion plans. Uncertainty can lead firms to delay investment decisions because capital choices are (at least partially) irreversible (Dixit and Pindyck, 1994; Caballero and Pindyck, 1996). Indeed, it is the costs associated with adjusting capital that help explain the 'lumpy' nature of firm investment: firms' capital stocks do not tend to evolve in a continuous and linear way but include discrete steps (Doms and Dunne, 1998; Nickell, 1978). Delaying investment decisions until more information is available creates an option value that can be shown to be increasing in the level of uncertainty (Abel *et al*., 1996; Abel and Eberly, 1996). Bloom *et al*. (2007) show that higher uncertainty reduces firms' responsiveness to demand shocks.

Financial market frictions, in particular, are commonly cited as affecting the allocation of capital.[7] They do this by increasing the cost of capital or producing capital constraints for some firms in ways that are unrelated to fundamental characteristics and thereby distorting investment decisions. Gilchrist *et al*. (2013) relate financial frictions, measured as an increased dispersion in borrowing costs, to capital misallocation for a subset of US manufacturing firms over the period 1985–2010.[8] The 2008 financial crisis was associated with a sharp fall in liquidity and an increase in borrowing costs (both in the banking sector and on capital markets). Lenders may also have become more risk averse, such that some firms (e.g small firms) or some projects (e.g. those involving assets that cannot be collateralised) found it harder to finance new investments. In addition, accommodative monetary policy, forbearance by banks and tax forbearance by HMRC may have contributed to preventing unproductive firms from exiting the market, thereby slowing one part of the reallocative process (Arrowsmith *et al*., 2013).

Hshieh and Klenow (2009) develop a framework with many industries, heterogeneous firms and monopolistic competition, to show that firm-level distortions to capital choices lead firms to make inefficient input choices. This in turn prevents the rates of return to capital from equalising across firms and leads to lower aggregate TFP.[9] In such a framework, in response to relative demand shocks for example, distortions will prevent capital from fully adjusting across firms, which will lead to persistent dispersion in prices and rates of return across firms.

The framework underlying our analysis draws on the spirit of Hsieh and Klenow (2009). We consider the financial crisis as having generated incentives for capital and labour to re-allocate because there were large relative shocks – including the large exchange rate depreciation in 2007 and commodity price shocks. At the same time, uncertainty and financial market frictions potentially created higher barriers to capital reallocation. These uneven shocks may have reduced prices for some products and increased them for others, resulting in changes in relative profitability. In the longer run we would expect resources to move towards the more profitable firms. If, however, there are distortions to capital reallocation, we would expect to see this manifested in a persistently higher dispersion of prices, and indeed marginal products of capital, across firms and sectors.

We consider evidence relating to dispersion in prices in section 3 and across rates of return in section 4. In both cases we attempt to measure the scale of the loss of TFP resulting from allocation frictions.

3. Price dispersion and allocation

Figures 1 and 2 provide some evidence that UK firms have faced large relative shocks since the 2008 crisis. Figure 1 plots the mean (black) and two standard deviations around the mean (the red and dashed red lines) of the distribution of deviations in output from long-run industry trends across 17 industrial sectors. Figure 2 plots the comparable dispersion in output prices.[10] In both cases dispersion increased markedly since the crisis suggesting that resources may have faced incentives to move. If that was the case we would expect a move in resources towards firms and sectors where returns were highest, which in time may act to reduce the dispersion in price. The persistence of the dispersion in prices suggests that relatively little reallocation of resources has taken place to date.

Hsieh and Klenow (2009) show that frictions to labour and capital choices lead to persistent dispersions in prices and output. In turn, under some assumptions, these dispersions can be mapped into an estimate of the aggregate loss to TFP. However, this approach relies on a particular functional form for firms' production

Figure 1. Dispersion in gross value added across sectors

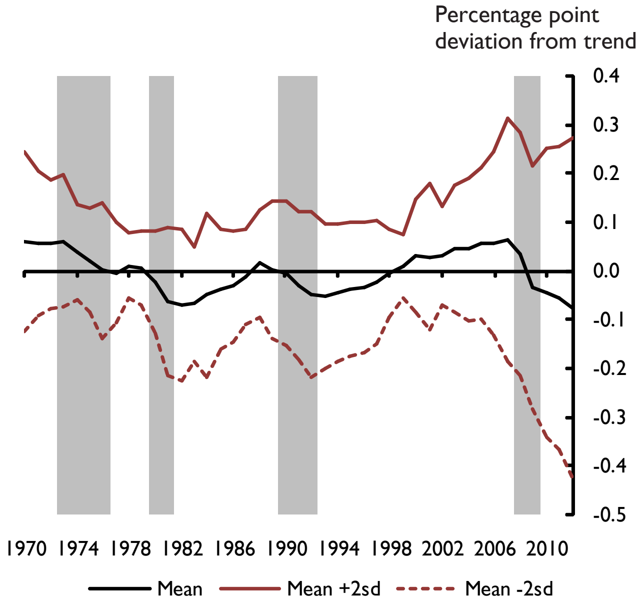
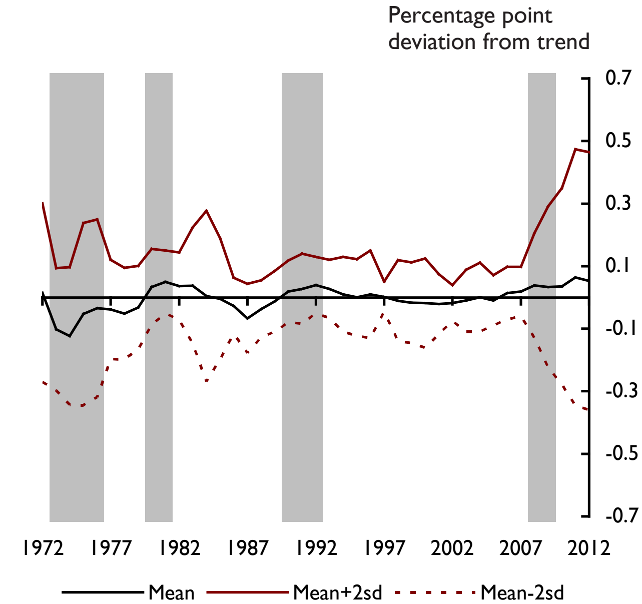


Figure 2. Dispersion in output price deflators across sectors



Source: ONS, EU KLEMS and Bank calculations.
Notes: Mean and standard deviations calculated from the distribution of deviations in productivity (or output price deflators) from their log trend in 17 SIC industrial sectors.

functions, and a specific way of aggregating over firm and sector level production functions.

In what follows, we employ a model used by Broadbent (2013) in a policy speech. This model uses a general static perfect competition framework that does not require a specific production function to map dispersion of sectoral prices following a demand shock into labour productivity changes. And although this relies on a perfect competition assumption, the implications for productivity are similar if one relaxes this assumption.

Specifically, all firms, indexed i, use inputs K_i and L_i to produce $y_i = f_i(K_i,L_i)$. Firms operate in a perfectly competitive market, where the price of a firm's output, denoted p_i, is equal to marginal cost.[11] The question that we want to ask is what the loss in output and productivity is when firms face uneven demand shocks and labour can freely adjust, but capital cannot. More specifically, suppose we increase a firm's employment by ΔL_i, holding fixed K_i. Then the (base-weighted) value of its output will change by

$$p_i^0 \Delta y_i = p_i^0 \int_{L_i^0}^{L_i^0+\Delta L_i} f'_{iL}(K,L)dL \tag{1}$$

where a zero superscript indicates the starting value and f'_{iL} the marginal product of labour. We want to think about what happens to the change in aggregate output $\Delta Y = \Sigma_i p_i^0 \Delta y_i$ when shifts in relative demand are met by changes in labour alone. In doing so we assume there is a fixed supply of labour in aggregate (call it L) and that the labour market clears.

The intuition behind this model is relatively straightforward. A sector with a positive demand shock will grow by increasing its labour input. The marginal cost of production will increase. Since capital is fixed, the marginal cost, and therefore the price, will be higher than if capital could fully adjust. Output will be lower and so will aggregate labour productivity.

More generally, the resulting percentage change in output is proportional to the cross-sectoral covariance between inflation and size-weighted employment growth (see appendix for details on how to derive equation 2) as in the expression below:

$$\frac{\Delta Y}{Y} \approx \frac{w^0 L}{Y} \Sigma_i \left[1 + \tfrac{1}{2}\left(\frac{\Delta w}{w^0} - \frac{\Delta p_i}{p_i^0} \right) \right] \frac{\Delta L_i}{L}$$

$$= -\tfrac{1}{2}\alpha \,\mathrm{cov}_i \left(\frac{\Delta p_i}{p_i^0}, \lambda_i \frac{\Delta L_i}{L_i^0} \right) \tag{2}$$

where α is the share of wages in national income (wL/Y)

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and λ_i is employment in sector i relative to the average, namely $\frac{L_i}{L/N}$.

The share of wages in GDP is roughly two-thirds. So this relationship says the loss in productivity is (to a first-order approximation) one third the cross-sectoral covariance between inflation and size-weighted employment growth. Further approximating the relationship between price and employment growth from (2), and using σ as the elasticity of substitution between capital and labour in sector i, and α_i as the share of labour income in that sector, one can re-express this in terms of prices alone:

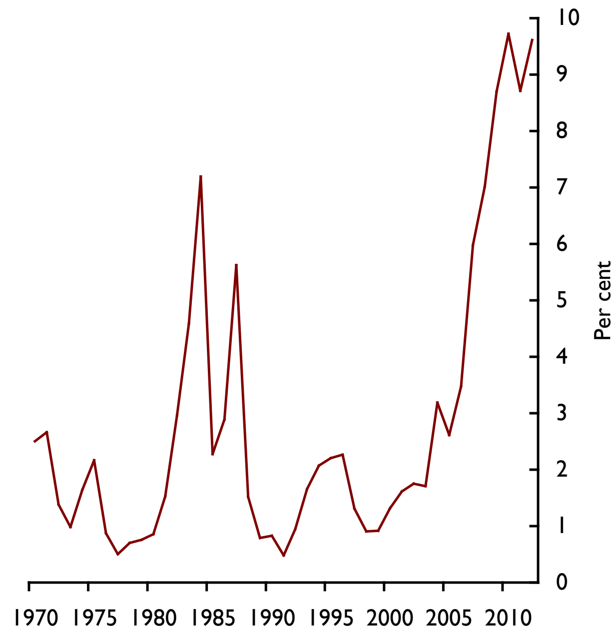
$$\begin{aligned}\frac{\Delta Y}{Y} &\approx -\tfrac{1}{2}\alpha\,\mathrm{cov}_i\left(\frac{\lambda_i\sigma_i}{1-\alpha_i}\frac{\Delta p_i}{p_i^0},\frac{\Delta p_i}{p_i^0}\right) \\ &= -\tfrac{1}{2}\alpha\mu\,\mathrm{var}_i\left(\sqrt{\frac{\mu_i}{\mu}}\frac{\Delta p_i}{p_i^0}\right)\end{aligned} \tag{3}$$

where $\mu_i = \frac{\lambda_i\sigma_i}{1-\alpha_i}$ and $\mu = \frac{\sigma}{1-\alpha}$ is the same quantity for the economy as a whole. Intuitively, equation (3) shows that relative demand shocks won't affect aggregate productivity if prices remain unchanged. Prices remain unchanged as long as productive resources, and in our case capital, move seamlessly in response to demand shocks. When there are frictions to the movement of capital across firms or sectors, relative marginal costs and therefore relative prices will change. In reality, there will be some frictions to capital mobility and relative prices will fluctuate in the short term as the economy reacts to shocks. However, large and persistent increases in price dispersion would be indicative of capital misallocation and more binding constraints on capital movement.

Figure 3 plots the cross-sectoral variance derived in equation (3). The scale of the variance since 2008 suggests that slow reallocation might have knocked 3–4 per cent off aggregate labour productivity compared with the pre-crisis period.[12]

Aggregate labour productivity can be thought of as a combination of capital per worker and Total Factor Productivity (TFP), the latter embodying a range of factors such as technology. There is a large degree of uncertainty around aggregate estimates of the UK capital stock. However, latest estimates suggest that the change in the capital to labour ratio since the crisis can only account for a small part of the shortfall in productivity relative to its pre-crisis trend. Therefore, it is likely that much of the fall in measured labour productivity is accounted for by a fall in TFP.[13] As such, we make the inference that the loss in labour productivity identified in this model will largely reflect a loss in measured aggregate TFP due to the misallocation of capital across sectors.

Figure 3. Significant increase in price dispersion in the UK[(a)]



Source: ONS, EU KLEMS and Bank calculations.
Note: (a) Size-weighted variance of real prices, relative to pre-crisis trends, across 29 sectors calculated using equation (3).

These results are very general. All they require is that the labour demand curves are well defined,[14] but they do not need restrictions on production functions: these don't need to be the same in all sectors, or to exhibit constant returns to scale (CRS).[15] In the extreme case that there *are* constant returns, and if capital too is fully mobile, then relative prices don't change in response to demand shocks (relative supply curves are flat) and productivity is invariant to shifts in relative demand.[16]

We have presented a model in which competitive markets mean that prices are equal to marginal costs. However, the broad point that higher price dispersion is indicative of capital misallocation requires only that a demand shock creates or increases differences in marginal costs across firms that feed through into prices if capital does not move freely to where returns are highest.

4. Firm level evidence of lack of capital reallocation

In a neoclassical model of firm factor demand the optimal

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capital stock is chosen to maximise the value of the firm. In a static setting this requires that the marginal product of capital be equated with the user cost of capital in each period. However, firms face adjustment costs when changing the capital stock, such that they may not respond instantly to a demand shock. The presence of adjustment costs also means that firms face a dynamic decision in which investment today is based in part on expected future demand conditions. Commonly, investment in any given year is thought of as advancing the firm towards an optimal capital choice. For a discussion of models of firm investment and empirical counterparts see Bloom *et al*. (2007).

The recession was characterised by a series of large shocks. For example, as highlighted in section 3, we expect firms that experience a positive (negative) demand shock to see an increase (fall) in the marginal product of capital while firms respond to this shock. Initially, this will be seen as an increase in the dispersion in the marginal product of capital across firms. But as capital moves towards firms that produce a higher return on capital, dispersion in returns should decrease. Any frictions to the process of allocation (including those that arise in normal times, and any additional frictions caused by the financial crisis) will impair this allocation of resources, such that the dispersion in rates of return across firms will be persistent.

In this section we are interested in estimating an empirical relationship between rates of return to capital and subsequent investment. We test whether the expected positive relationship has changed since the 2008 financial crisis, more specifically, whether firms have become less responsive to investment incentives.

Although we cannot directly measure the marginal product of capital or observe the demand shocks that firms face, we can calculate the average rate of return to capital at the firm level each year in a rich firm-level dataset. The following three subsections describe the firm level data, our empirical approach and then the results.

Table 1. Descriptive statistics (per cent)

	Mean	Standard deviation	1% percentile	99% percentile
Change in capital stock, ΔK_{ijt}	3.7	7.6	–7.4	37.9
Rate of return ROR_{ijt-1}	17.2	27.8	–28.1	119.7

Notes: Number of observations (firm-year) is 85591; *i* stands for firm, *j* for industry and *t* for time. ΔK_{ijt} is measured as capital expenditure divided by the previous year's capital stock.

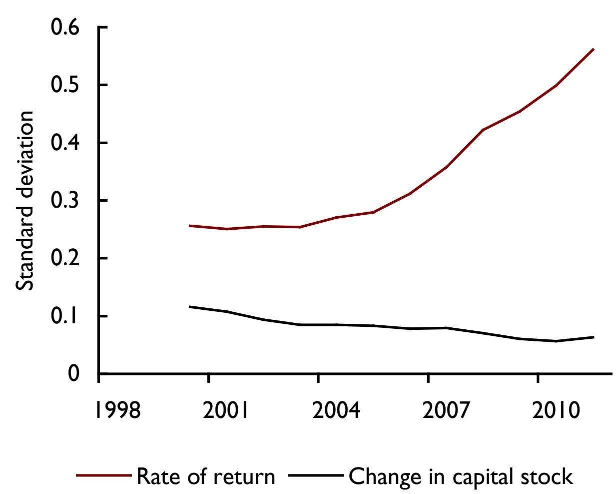
4.1 Data description

We use a data set of around 8,000 UK firms per year over the period 2000–2011. This sample uses data from the ONS Annual Business Survey (ABS) and its predecessor.[17] The ABS is an annual survey of around 60,000 businesses from most sectors of the UK economy[18] and covers around two-thirds of the economy in terms of Gross Value Added (GVA). As well as GVA, the data include information on employment, wages and capital expenditure. For each firm, real GVA is calculated using deflators at the 2 digit Standard Industry Classification (SIC) level.

Following Gilhooly (2009),[19] for each reporting unit, a perpetual inventory method is used to create a measure of three different capital assets namely plant and machinery, buildings and vehicles. We use annual depreciation rates for each asset, as set out in ONS (2007), of 6 per cent, 2 per cent and 20 per cent respectively to calculate an overall net measure of firm-level capital. Rates of return are calculated as Gross Operating Surplus – that part of Gross Value Added that is not used to remunerate labour but retained by firms as a form of profit – divided by the net capital stock. For further details about the dataset please see Barnett *et al*. (2014).

Our sample size is significantly smaller than the ABS full sample mainly because there are many firms for which we do not observe capital expenditure data. In addition, since we use lags in our empirical specification (described

Figure 4. Standard deviation of firm rates of returns has increased since the crisis



Source: ONS (ABS).

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next), the number of firms for which data is available in consecutive years is even smaller. Table 1 summarises the key variables.

Figure 4 plots the standard deviation (second column of table 1) for the rates of return and the change in the capital stock over time. This shows an increase in the variation in rates of return across firms since 2007 (red line) and a flat variance of changes in capital stock (black line).

4.2 Empirical specification

As described in the previous section, our aim is to test the hypothesis that firms have become less responsive to investment incentives since the crisis. To do this, we estimate the reduced form model below:

$$\begin{aligned}\Delta K_{ijt} &= \mu + \Sigma_s(\alpha_s + \beta_s crisis)ROR_{ijt-s} \\ &+\Sigma_1(\alpha_1 + \beta_1 crisis)Demand_{kt+1} \\ &+\beta_5 crisis + \gamma_i + \gamma_j + \varepsilon_{ijt}\end{aligned} \tag{4}$$

where ΔK_{ijt} is the change in the capital stock measured as investment (capital expenditure) divided by the previous years' capital stock for firm i, in industry j, in period t. ROR_{ijt-s} is the rate of return on capital of firm i, in industry j, in period t–s. We include the first and second lag in recognition that there may be a delay to firms' capital adjustment. This is captured by the s subscript. *Crisis* is a dummy variable that takes the value of 0 up to and including 2007 and 1 after.

One possible concern is that the rate of return does not adequately capture investment incentives. To address this we present results of a specification that also includes an independent measure of output demand,[20] as one would expect firms that face higher demand (now and in the future) to face a larger incentive to increase investment. The variable *Demand* is constructed following Bailey *et al*. (2001), Bartelsman *et al*. (1994) and Shea (1993), and is a downstream demand indicator specific to the two-digit industry, k, that each firm belongs to. This sectoral indicator is a weighted average of changes in real GVA of the downstream industries with weights equal to their share of purchases of output from that specific upstream industry. Furthermore, we also apply a Shea-exogeneity test in order to rule out the endogenous effect that the upstream industry's demand may have, via intermediate goods prices, on the downstream industry's activity.[21] More specifically, we calculate this using the input-output matrices and two-digit real output indices from the ONS and exclude from our indicator those downstream industries whose purchases of the upstream industry are larger than 5 per cent of their expenditure on intermediate inputs.

Our specification also uses γ_j as industry fixed effects, defined at the two-digit industry level. These should capture systematic differences in investment rates across firms in different industries. The firm fixed effects, γ_i, should capture systematic firm differences that affect investment choices, such as differences in production technology and time invariant differences in the cost of investment. ε_{ijt} is an i.i.d error, assumed to be normally distributed.

This empirical specification should be seen as a reduced form model, in which the change in the capital stock is expressed as a function of factors that affect the choice of the optimal capital stock, as these factors may signal an incentive to increase investment. We have not specified the precise relationship between investment and the optimal capital stock (which will be a function of the expected costs and benefits of capital investment) or the precise functional form of any adjustment process. The coefficients on *ROR* and *Demand* will be some function of the structural parameters relating firms' expectation generating process to the adjustment technology. As such, this approach will not allow us to identify the underlying structural parameters of the firms' problem. However, that is not our goal here. Instead, our interest is in considering whether the relationship between the signal to invest and investment has changed since the recession, controlling for other factors that affect firms' decisions. The idea is to test whether or not there appear to be barriers or constraints to the efficient movement of capital across firms.

We interpret the α coefficients as the average or 'normal' process of adjustment of capital to investment incentives. We interpret the β coefficient as representing distortions to the adjustment of firms' capital stocks since 2008. This may encompass various sources of adjustment friction that arose as the result of (or at least at the same time as) the financial crisis. For example, it may capture the effect of higher uncertainty, which has been shown to make firms less responsive to changes in demand (Bloom *et al*., 2007), or the effects of credit constraints.[22]

The validity of this approach requires two sets of assumptions. First, we assume that firms have not changed the way they calculate their optimal capital stock and therefore that rates of return and demand conditions still provide as much information with regard to firms' incentives to invest as they did before the crisis.

In fact we find that the correlation between investment incentives today and in the future (i.e. the correlation between ROR_t and ROR_{t+1}) did not change after the crisis.

Second, we assume that ROR and *Demand* can be treated as exogenous after controlling for various factors. One further possible concern is that we do not directly include a measure of the user cost of capital (which is a function of price of capital goods relative to output), how this is expected to change, the firm's required rate of return or the rate of depreciation of capital (see Jorgensen, 1963).

We do not observe firm-specific interest rates or investment prices and therefore cannot include these in our specification. The industry fixed effects will capture systematic (time-invariant) differences across firms in different industries. In addition, the firm fixed effects will capture systematic differences in the cost of investment faced by firms. While there may be firm-year specific shocks to the user cost of capital, we argue that these are unlikely to be correlated with rates of return.

4.3 Results

Table 2 presents the results from estimating equation (4). We find that the positive relationship between rates of return and subsequent capital growth at the firm level has broken down since this financial crisis. The coefficients on lagged rates of return interacting with the recession dummy are negative and significant, suggesting a reduction in the sensitivity of capital growth to rates of return after 2007. The magnitude of this reduction is such that it broadly offsets the positive relationship between these two variables that existed prior to the crisis. Indeed the null hypothesis that the association between capital growth and rates of return is zero after 2007 cannot be rejected in any of the regression specifications (the sum of the coefficients on ROR and the coefficients on the interaction of ROR with the recession dummy is statistically not different from zero). The coefficients on lagged rates of return themselves are small. In our third specification for example, they imply that a 1 percentage point increase in the level of rates of return for a particular firm before the crisis increased the level of the capital stock for that firm by around 0.04 per cent (after two years). Although small, we find these coefficients to be statistically significant across a range of specifications. The key point from our results here is that this statistically significant relationship between rates of return and changes in the capital stock appears to disappear after the crisis.

Table 2. Results

Dependent variable: change in capital stock, ΔK_{ijt} (percentage point)	(1)	(2)	(3)
ROR_{ijt-1}	0.029***	0.034***	0.032***
	(0.003)	(0.003)	(0.004)
ROR_{ijt-1} *crisis	–0.029***	–0.029***	–0.032***
	(0.003)	(0.003)	(0.005)
ROR_{ijt-2}		0.010***	0.008**
		(0.002)	(0.003)
ROR_{ijt-2} *crisis		–0.011***	–0.008*
		(0.002)	(0.004)
$Demand_{kt}$			0.099***
			(0.027)
$Demand_{kt}$ *crisis			–0.066**
			(0.031)
Demand_{kt+1}			0.080***
			(0.018)
Demand_{kt+1} *crisis			0.008
			(0.051)
Crisis dummy	–0.017***	–0.015***	–0.014***
	(0.002)	(0.001)	(0.002)
Observations (firm-periods)	85591	85591	60941
Overall R-squared	0.05	0.06	0.05

Notes: Robust standard errors clustered at the firm level are displayed in round brackets. All regressions include industry and firm fixed effects. *significant at 10 per cent; **significant at 5 per cent; ***significant at 1 per cent.

We also find that the relationship between the demand indicator and the change in the capital stock has become weaker since the crisis: the effect of the demand indicator on capital falls by a third after the recession. If we think that uncertainty may be one of the reasons behind this weaker effect, our result is in line with Guiso and Parigi (1999) who find that expected demand conditions have a small effect on current investment for firms that perceive greater uncertainty about this future demand.

4.4 The effect on productivity

We use the estimated pre-crisis relationship between firm-level capital growth and rates of return (table 2) to construct a back-of-the-envelope counterfactual measure of firms' capital stock. The idea here is to estimate what aggregate labour productivity would have been if capital had moved in response to changes in rates of return. In this counterfactual experiment, firms with higher relative rates of return would have invested more, increasing the size of their capital stock. Based on this estimate of the capital stock, and holding the level of each firm's Total Factor Productivity (TFP) and employment constant at 2008 levels, we estimate what the potential level of output could have been using a simple growth accounting identity shown below.

$$Y^{*}_{ijt} = L^{\alpha}_{ijt} K^{*1-\alpha}_{ijt}$$

For this exercise, we assume that the labour share, α, is 2/3. * indicates the estimated counterfactual measures. The difference between the observed and counterfactual capital levels gives us an indication of the degree of capital misallocation. And the difference between observed and counterfactual output levels gives us an indication of the degree of the associated output loss, which we can use to derive an aggregate TFP loss.

However, there are a number of limitations to this type of counterfactual exercise. First, it ignores any beneficial spillover effects to individual firm level TFP through new investment. Second, we do not capture firm formation and bankruptcies that might have taken place had capital been allocated differently, channels that we think are important for the capital allocation process. Third, the estimation strategy only considers within-firm effects since we use a fixed effects panel specification. There may be between-firm effects and within-sector dynamics that are important to capture. Fourth, any counterfactual exercise is highly endogenous, as future rates of return will be affected by changes in the capital stock in the current period. These effects are hard to capture accurately in this simple exercise. Last, companies may respond to incentives differently in recessions compared to normal times. And our data set does not include any previous recessions. Indeed, as one might expect given these caveats, this counterfactual experiment suggests that the aggregate TFP loss for this sample of firms is relatively small and a bit less than 1 percentage point. However, we think that this may be a distinct lower bound when aggregating to the rest of the UK economy, since we are unable to capture any of the effects described above. More importantly, given the uncertainties with this counterfactual exercise, the key finding to highlight is that the relationship between rates of return and subsequent investment growth appears to have materially weakened since the crisis. This is suggestive that this channel may be one contributing factor to the impaired allocation of capital across firms.

5. Discussion

Overall, we observe large and persistent output and price dispersions at the sectoral level. Under certain assumptions, using a relatively simple and tractable model, it can be shown that such dispersions could arise in an economy affected by relative demand shocks and in which capital is unable to move. We show how the degree of price dispersion can be mapped into a reduction in labour productivity. The latest aggregate estimates of capital per worker in the UK, albeit highly uncertain, suggest that changes in the capital to labour ratio since the crisis can only account for a small amount of the shortfall in labour productivity relative to its pre-crisis trend. Therefore, we infer that the implied reduction in labour productivity is representative of a reduction in measured aggregate Total Factor Productivity (TFP). But in order to explore this mechanism further, we use firm-level data to test the hypothesis that firms have become less responsive to investment incentives since the crisis. We find that the positive relationship between the average rate of return to capital and subsequent investment has broken down since 2008, which is suggestive of increased frictions to the allocation of capital. Although there is a large degree of uncertainty around our estimates, our results suggest that frictions to the allocation of capital are likely to be one of the factors that can help to explain the persistent weakness of UK productivity.

A key outstanding question is whether the UK is experiencing a 'normal' response to a financial crisis. Namely, is it always the case that during recessions and financial crises capital adjustment is slower than in normal times, but that it does eventually take place with a lag? Or is it the case that recessions accompanied by financial crises affect TFP growth for prolonged periods of time? Although previous work has found that the level of TFP is permanently reduced following financial crisis (Oulton and Sebastia-Barriel, 2013) the jury is still out. We cannot consider previous recessions in the UK using the micro data, which is available only back to 1998, but we can attempt to consider how unusual the current UK experience is by considering sectoral level data across a range of countries.

5.1 Historic and international experiences

Figures 5 to 8 compare the dispersion in rates of return to capital, and in capital stocks across industrial sectors since 2007, to two previous UK recessions, the current US experience and the experience in a range of other financial crises. In each case, we standardise each industry's rate of return and capital stock using its pre-crises mean and own standard deviation.[23] In this way we attempt to control for any upward trends in the capital stock overtime (which could cause the sectoral dispersion of capital to increase artificially) and any differences in rates of return to capital that occur even in normal times. This could occur, for example, if sectors differ in the average riskiness of projects or

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Figure 5. Standard deviation of rates of return to capital across sectors

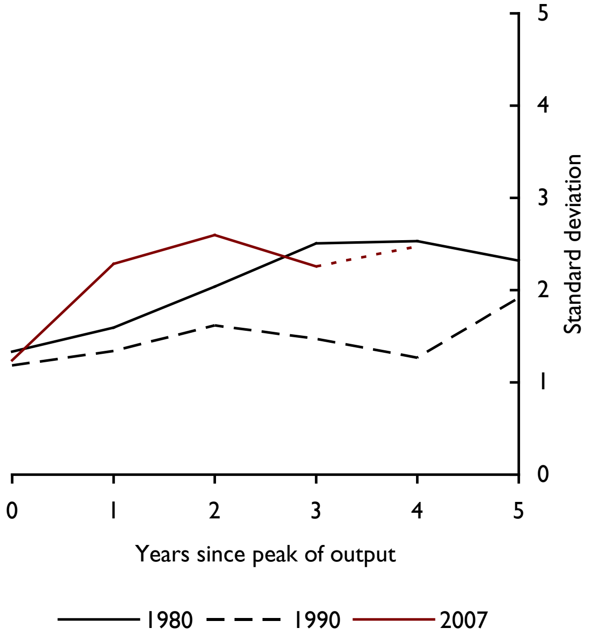
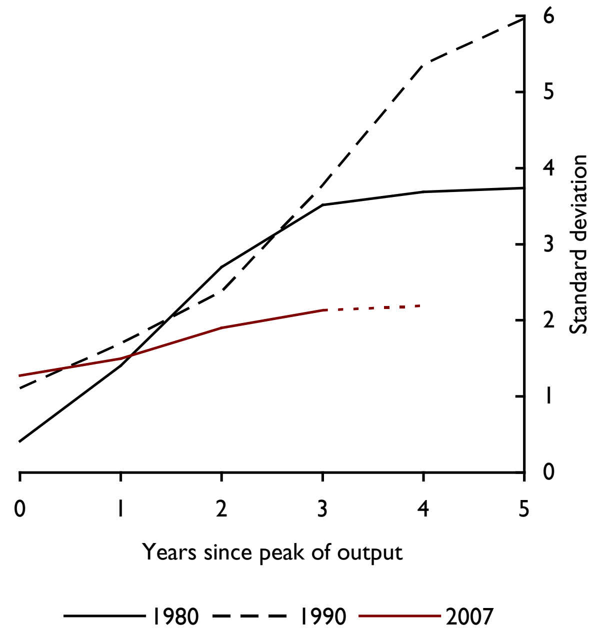


Figure 6. Standard deviation in capital stock levels across sectors



Source: ONS, EU KLEMS and Bank calculations.
Notes: Rates of return are calculated as Gross Operating Surplus (GOS) divided by estimates of the capital stock. These are standardised by their pre-recession means and standard deviations to ensure comparability over time. The lines represent the standard deviation across 12 SIC industrial sectors. The red dashed line is based on a projection of the capital stock data for 2011.

in the degree of competition. We plot the standard deviation of these standardised variables across sectors in figures 5 and 6.

The increase in the dispersion of rates of returns to capital across UK industrial sectors since 2007 is broadly in line with the experience in previous UK recessions (figure 5), and in other banking crises (figure 7). In contrast, the dispersion of the capital stock across industrial sectors has increased by less than in previous recessions (figure 5) and less than is commonly seen following banking crises (figure 7). This suggests that the allocation of capital may have been impaired following the 2008 crisis to a greater degree than would have been expected based on previous experiences.

The idea that frictions to capital allocation may have been important in the UK following the financial crisis also accords with data on firm births and deaths which have been low compared to the early 1990s recession.

5.2 Next steps

There are important questions left to be explored in further work. In this paper we do not distinguish between the sources of impaired capital allocation. However, understanding the source of the frictions, including how persistent these are, is important for considering the appropriate policy response. We highlight uncertainty and credit market imperfections as two important candidates. And there is evidence that both have been defining features of the UK experience. Since 2008 there has been uncertainty over demand conditions, fiscal policy and conditions in the Eurozone. Furthermore, indicators of uncertainty were particularly high in 2008 and 2011 and have only recently returned to more normal levels.

These factors have clearly been persistent, but we think it unlikely that either uncertainty or credit market frictions represent permanent changes. As such, we would expect to see a process of capital reallocation contributing positively to TFP as the economy recovers.

Figure 7. Standard deviation of rates of return to capital across sectors following banking crises

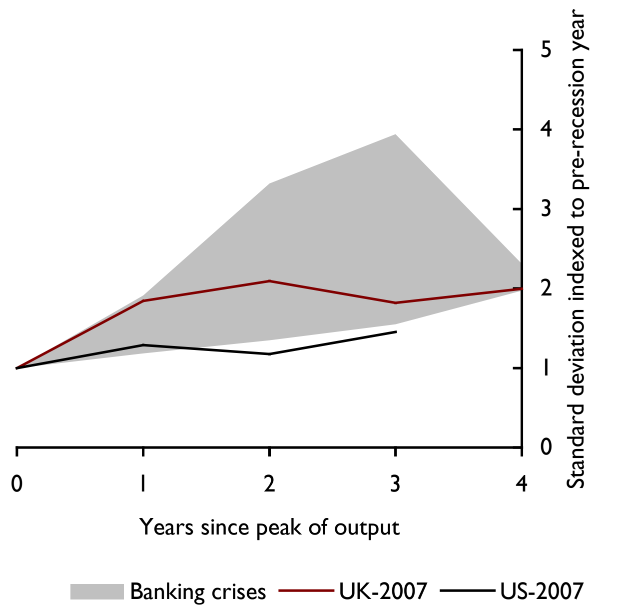
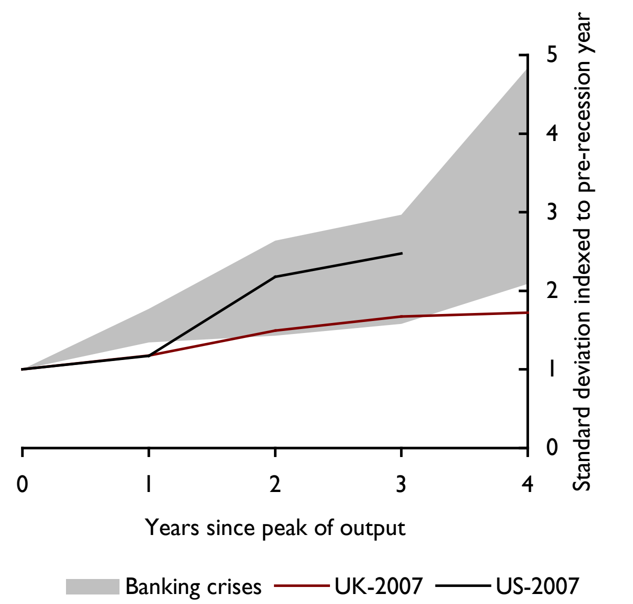


Figure 8. Standard deviation of capital stock levels across sectors following banking crises



Source: ONS, EU KLEMS and Bank calculations.
Notes: The methodology is the same as that described in figures 3 and 4, however to ensure comparability across countries the standard deviations are indexed to 1 at the start of each crisis. Countries in the swathe include: Finland (1991 and 2007); Germany, Italy, Netherlands (2007); Norway (1988 and 2007).

NOTES

1 This weakness is also apparent in hours and TFP space. Private sector output per hour and TFP are around 20 per cent and 14 per cent respectively below the level implied by an extrapolation of a linear pre-crisis trend.

2 Please see the Bank of England November 2013 *Inflation Report* for further details.

3 This study uses Private Non-Financial Corporations (PNFCs) and excludes the agriculture, energy, real estate and the public industries due to their small number of observations and volatile behaviour during the period covered in the sample.

4 Three recent empirical papers consider the role of resource allocation in crises. Sandleris and Wright (2011) and Oberfield (2012) consider the 2001 Argentine crisis and the 1982 Chilean crisis respectively and both find important roles for reduced allocation efficiency in lowering productivity. Ziebarth (2012) finds a similar result for two specific industries (manufactured ice and cement) following the US during the Great Depression.

5 We expect there to be some degree of dispersion in productivity levels across firms and sectors even in normal times. See Bernard and Jones (1996) or Bernard *et al.* (2002) for a discussion of how productivity levels vary across sectors and states both in the UK and US.

6 If the crisis also led to a reduction in product market competition (by leading to fewer new firms entering the market or fewer new products being introduced by current firms) this too would dampen investment incentives.

7 Empirical work includes Erosa and Hidalgo Cabrillana (2008) and Midrigan and Yi Xu (2014). Much of the empirical work is in a context that explains cross country income differences. See Banerjee and Duflo (2005) for a review.

8 Across the whole period they estimate a loss of between 1.5 per cent and 3.5 per cent of measured TFP, and show how TFP losses can be increasing in the dispersion of firm-level borrowing costs.

9 Hsieh and Klenow (2009) use plant-level data to provide evidence that distortions to capital allocation in manufacturing industries in India and China reduce whole economy TFP by 40 per cent and 30 per cent respectively when compared with the US.

10 There's an important technical point here. If relative prices aren't mean-reverting, at least relative to a fixed trend, you'd expect to see increasing divergence over time and the pattern in figure 2 would be an artefact of the way we'd constructed the data. But according to statistical tests, this is not the case: nearly all these relative prices are, indeed, 'trend stationary' prior to 2008.

11 The results and discussion that follows do not rely on the fact that price equals marginal cost but on the fact that following a demand shock marginal costs across firms will differ unless resources get reallocated where their return is highest. The intuition and results follow even if price does not equal marginal cost, although, in this case, our results would overstate the effect of capital immobility. On the other hand any degree of

price stickiness would mean we are underestimating the effect of capital immobility.

12 This is the effect on the assumption that there are only demand shocks in the world, whether at the aggregate or sub-aggregate level. In practice, the dispersion of prices could reflect other things, including cross-sectoral shifts in supply. However, it would be odd to rule out the possibility of supply shocks at an aggregate level, as an explanation for weaker productivity, only to reintroduce them at a sectoral level. That's why the appropriate 'null' hypothesis is a world without such supply shocks.

13 Please see page 29 of the Bank of England's February 2013 *Inflation Report*: http://www.bankofengland.co.uk/publications/Documents/inflationreport/2013/ir13feb3.pdf.

14 In other words, that increases in labour demand lead to proportionate positive changes in wages.

15 We do, however, assume constant production functions through time and are therefore simulating pure (cross-sectoral) demand shocks; if there were also direct shocks to sector level TFP, these would also affect relative prices, as well as having a more direct impact on aggregate TFP.

16 To see this note that CRS means we can write marginal products in any sector i solely in terms of the capital–labour ratio (call that $k_i \equiv \frac{K_i}{L_i}$): defining the function g as $g_i(x) \equiv f_i(x,1)$, we have $y_i = L_i g_i(k_i)$, $f'_{iK} = g'_i$ and $f'_{iL} = g_i - k_i g'_i$. So, under full factor mobility, and for given factor prices r and w, the two first-order conditions for optimal employment of labour are (1) $p_i f'_{iL} = p_i(g_i - k_i g'_i)$ and (2) capital $r = g'_i$ are enough to solve for the two variables k_i and p_i: the scale of production L_i doesn't matter for prices.

17 The ABS has been in place since 2008. It replaced the Annual Business Inquiry (ABI), which ran from 1998–2007. Although the ABI includes information on the number of employees the ABS does not. As a result, post 2007, we use firm-level employment data from the ONS Business Register and Employment Survey (BRES).

18 We use only Private Non-Financial Corporations (PNFCs), excluding the agriculture, energy, real estate and the public industries which seem to display volatile behaviour during the period covered in our sample.

19 Harris (2005) applies a different methodology for constructing firm level capital stock. He constructs capital stock at the level of the plant rather than reporting unit. In future work we aim to look at the sensitivity of our capital stock estimates relative to Harris (2005) methodology.

20 One could also use the market values for equity as a forward looking element but as most of the firms in our dataset are not quoted on the stock exchange it would be a nontrivial exercise to estimate this. We leave this for future work.

21 In the main results reported below we use the exogeneity-adjusted demand indicator. However, we found that the exogeneity assumption does not lead to materially different results.

22 It is also possible that we are capturing a non-linear relationship between rates of return and investment. For example, it may always be the case that adjustment is slower in response to large shocks during recession. As we do not have firm-level data that covers previous recessions, in section 5 we present sector level evidence that capital was slow to adjust in the UK compared with previous UK and other countries' financial crises.

23 Specifically, for each industry, j, we calculate the standardised variable, y, as: $y_{jt} = \frac{x_{jt} - \bar{x}_{j,pre-crisis}}{\sigma_{j,pre-crisis}}$ where xjt is either the industry rate of return on capital or the capital stock at time t and $\bar{x}_{j,pre-crisis}$ and $\sigma_{j,pre-crisis}$ are the industry mean and standard deviation respectively in the eight years prior to the recession.

24 The results and discussion that follow do not rely on the fact that price equals marginal cost but on the fact that following a demand shock marginal costs across firms will differ unless resources get reallocated where their return is highest.

25 Since the size of the aggregate labour force has not changed the loss in productivity is equal to the loss in output.

26 See for example Timmer *et al.* (2007)

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Appendix: Perfect Competition Model

Firms operate in a perfectly competitive market, where the price of a firm's output, denoted p_i, is equal to marginal cost.[24] The question that we want to ask is what the loss in output and productivity is when firms face uneven demand shocks, labour can freely adjust but capital cannot. More specifically, suppose we increase a firm's employment by ΔL_i, holding fixed K_i. Then the (base-weighted) value of its output will change by

$$p_i^0 \Delta y_i = p_i^0 \int_{L_i^0}^{L_i^0+\Delta L_i} f'_{iL}(K,L)dL \tag{1}$$

where a zero superscript indicates the starting value and f'_{iL} the marginal product of labour. We want to think about what happens to the change in aggregate output $\Delta Y = \Sigma_i p_i^0 \Delta y_i$ when shifts in relative demand are met by changes in labour alone. In doing so we assume there is a fixed supply of labour in aggregate (call it L) and that the labour market clears.

The first condition means that $\Sigma_i \Delta L_i = 0$. The second means there is a common wage w, across all sectors, and that firms are on their labour demand curve

$$w = p_i f'_{iL} \tag{2}$$

To work out the effect of a reallocation of labour on aggregate output note that the first-order approximation to the integral in (1) is

$$p_i^0 \int_{L_i^0}^{L_i^0+\Delta L_i} f'_{iL}(K,l)dl \approx p_i^0 [f'^0_{iL} + \tfrac{1}{2}\Delta f'_{iL}]\Delta L_i \tag{3}$$

If we now substitute (2) into (3) we get the equation below

$$p_i^0 \int_{L_i^0}^{L_i^0+\Delta L_i} f'_{iL}(K,l)dl \approx w^0 \left[1+\tfrac{1}{2}\left(\tfrac{\Delta w}{w^0}-\tfrac{\Delta p_i}{p_i^0}\right)\right]\Delta L_i \tag{4}$$

Aggregating (1) over all firms in this economy and substituting from (4), the proportionate change in productivity[25] is

$$\tfrac{\Delta Y}{Y} = \sum_i \tfrac{p_i^0 \Delta y_i}{Y} \approx \tfrac{w^0 L}{Y} \sum_i \left[1+\tfrac{1}{2}\left(\tfrac{\Delta w}{w^0}-\tfrac{\Delta p_i}{p_i^0}\right)\right]\tfrac{\Delta L_i}{L} \tag{5}$$

Note that, of the three terms in square brackets, the first two aggregate to zero because the wage (and its change) are common to all sectors, so can be taken out of the summation, and we have restricted $\Sigma_i \Delta L_i = 0$. Therefore we can re-write (5) as follows:

$$\frac{\Delta Y}{Y} \approx \frac{w^0}{Y}\left[1+\tfrac{1}{2}\frac{\Delta w}{w^0}\right]\sum_i \Delta L_i - \tfrac{1}{2}\frac{w^0 L}{Y}\sum_i \frac{\Delta p_i}{p_i^0} * \frac{\Delta L_i}{L} = -\tfrac{1}{2}\frac{w^0 L}{Y}\sum_i \frac{\Delta p_i}{p_i^0} * \frac{\Delta L_i}{L} \tag{6}$$

$$\frac{\Delta Y}{Y} \approx \frac{w^0 L}{Y}\sum_i \left[1+\tfrac{1}{2}\left(\frac{\Delta w}{w^0}-\frac{\Delta p_i}{p_i^0}\right)\right]\frac{\Delta L_i}{L} = -\tfrac{1}{2}\alpha \operatorname{cov}_i\left(\frac{\Delta p_i}{p_i^0}, \lambda_i \frac{\Delta L_i}{L_i^0}\right)$$

where α is the share of wages in national income (wL/Y) and λ_i is employment in sector i relative to the average, namely $\frac{L_i}{L/N}$.

The share of wages in GDP is roughly two-thirds. So this relationship says the loss in productivity is (to a first-order approximation) one third the cross-sectoral covariance between inflation and size-weighted employment growth.

Further approximating the relationship between price and employment growth from (2), and using σ as the elasticity of substitution between capital and labour in sector i, and α_i as the share of labour income in that sector, one can re-express this in terms of prices alone:

$$\frac{\Delta Y}{Y} \approx \tfrac{1}{2}\alpha \operatorname{cov}_i\left(\frac{\lambda_i \sigma_i}{1-\alpha_i}\frac{\Delta p_i}{p_i^0}, \frac{\Delta p_i}{p_i^0}\right) = -\tfrac{1}{2}\alpha\mu \operatorname{var}_i\left(\sqrt{\frac{\mu_i}{\mu}}\frac{\Delta p_i}{p_i^0}\right) \tag{7}$$

where $\mu_i = \dfrac{\lambda_i \sigma_i}{1-\alpha_i}$ and $\mu = \dfrac{\sigma}{1-\alpha}$ is the same quantity for the economy as a whole. Empirical estimates suggest that whole-economy σ is around a half and α two-thirds.[26] On that basis $\alpha\mu$ is one and the loss of productivity will be around one half the cross-sectoral variance of (μ-weighted) inflation.