Part 2. Stellar Pulsation in Rotating Stars

Part 2.1. Theoretical Aspects

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The Effects of Rapid Rotation on Pulsation

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Abstract. We discuss the properties of pulsations in rotating stars, which include frequency shifts, the effects on the stability of p- and g-modes, and the properties of low-frequency modes whose frequencies are comparable to or less than the rotation frequency.

1. Basic properties of pulsations in a rotating star

Stellar rotation affects the properties of nonradial pulsations through the Coriolis force and the deformation of the equilibrium structure. The Coriolis force effect is large when the ratio of the rotation frequency Ω to the pulsation frequency (in the co-rotating frame of the star) is large, while the deformation effect is important when the ratio of Ω to the Kepler frequency $\sqrt{GM/R^3}$ is large. The pulsation frequency is shifted and the stability of pulsation is modified. In addition, rotation allows the existence of r-modes and, more generally, inertial modes. This paper gives an overview of these effects.

In this paper, we assume adiabatic pulsations and neglect the Eulerian perturbation of gravitational potential (Cowling approximation) for simplicity. Under these approximations, the linearized basic equations for nonradial pulsations (see e.g., Unno et al., 1989 for detail) are

$$-(\sigma+m\Omega)^2\boldsymbol{\xi}+2i(\sigma+m\Omega)(\boldsymbol{\Omega}\times\boldsymbol{\xi})+\boldsymbol{e}_s r\sin\theta\boldsymbol{\xi}\cdot\nabla\Omega^2=\frac{\rho'}{\rho^2}\nabla P-\frac{1}{\rho}\nabla P', \quad (1)$$

$$\rho'+\nabla\cdot(\rho\boldsymbol{\xi})=0, \quad (2)$$

$$\frac{\delta P}{P}=\Gamma_1\frac{\delta\rho}{\rho}, \quad \text{or} \quad \frac{P'}{P}=\Gamma_1\frac{\rho'}{\rho}+\boldsymbol{\xi}\cdot(\Gamma_1\nabla\ln\rho-\nabla\ln P), \quad (3)$$

where σ is the pulsation frequency observed in an inertial frame, m is the azimuthal order, $\boldsymbol{\Omega}$ is the rotation vector, in the direction of the rotation axis and of magnitude Ω, and $\boldsymbol{e}_s$ is a unit vector pointing in the outward direction from the rotation axis. Eqs. (1) – (3) can be reduced to an equation for the displacement vector $\boldsymbol{\xi}$;

$$-(\sigma+m\Omega)^2\rho\boldsymbol{\xi}+2i(\sigma+m\Omega)\rho\boldsymbol{\Omega}\times\boldsymbol{\xi}+\boldsymbol{C}(\boldsymbol{\xi})=0, \quad (4)$$

where $\boldsymbol{C}$ is a differential operator. Lynden-Bell & Ostriker (1967) have shown that all the operators in Eq. (4) are Hermitian. Taking the inner product with

$\boldsymbol{\xi}^*$ and integrating over the entire volume of the star, we obtain

$$-(\sigma+m\Omega)^2 a+(\sigma+m\Omega)b+c=0, \tag{5}$$

where $a=\int_V \boldsymbol{\xi}^*\cdot\boldsymbol{\xi}\rho d^3\boldsymbol{r}$, $b=2i\int_V \boldsymbol{\xi}^*\cdot(\boldsymbol{\Omega}\times\boldsymbol{\xi})\rho d^3\boldsymbol{r}$, and $c=\int_V \boldsymbol{\xi}^*\cdot\boldsymbol{\mathcal{C}}(\boldsymbol{\xi})d^3\boldsymbol{r}$, which are real since the corresponding operators are Hermitian. Then, we obtain

$$\sigma+m\Omega=\frac{1}{2a}(b\pm\sqrt{b^2+4ac}). \tag{6}$$

If $\Omega=0$, the above equation reduces to $\sigma=\sqrt{c/a}\equiv\sigma_0$. Since c does not depend on m $(-\ell\le m\le\ell)$, the eigenfrequencies of nonradial pulsations in a non-rotating star are $(2\ell+1)$-fold degenerate. In the following, we consider only uniform rotation, for which $\sigma+m\Omega$ corresponds to the frequency ω observed in the co-rotating frame of the star; i.e., $\omega=\sigma+m\Omega$.

Any displacement in a star can be represented by a superposition of spheroidal displacement $\boldsymbol{\xi}_{\rm s}$ and toroidal displacement $\boldsymbol{\xi}_{\rm t}$ (Aizenman & Smeyers, 1977);

$$\boldsymbol{\xi}_{\rm s}=\left[\zeta(r),h(r)\frac{\partial}{\partial\theta},\frac{h(r)}{\sin\theta}\frac{\partial}{\partial\phi}\right]Y_\ell^m,\quad \boldsymbol{\xi}_{\rm t}=\left[0,\frac{t(r)}{\sin\theta}\frac{\partial}{\partial\phi},-t(r)\frac{\partial}{\partial\theta}\right]Y_\ell^m. \tag{7}$$

For $\boldsymbol{\xi}_{\rm s}$ we obtain $a_{\rm s}=\int_0^R[|\zeta|^2+\ell(\ell+1)|h|^2]\rho r^2dr$, and $b_{\rm s}=2m\Omega\int_0^R(\zeta^*h+\zeta h^*+|h|^2)\rho r^2dr$. If $\sigma_0\gg\Omega$, Eq. (6) leads to

$$\omega_{\rm s}=\sigma_0+\frac{b_s}{2a_s}+O(\Omega^2)=\sigma_0+m\Omega\frac{\int_0^R(\zeta^*h+\zeta h^*+|h|^2)\rho r^2dr}{\int_0^R[|\zeta|^2+\ell(\ell+1)|h|^2]\rho r^2dr}+O(\Omega^2). \tag{8}$$

This shows that rotation lifts the $(2\ell+1)$-fold degeneracy completely.

For $\boldsymbol{\xi}_{\rm t}$ we obtain $a_{\rm t}=\ell(\ell+1)\int_0^R|t(r)|^2\rho r^2dr$, $b_{\rm t}=2m\Omega\int_0^R|t(r)|^2\rho r^2dr$, and $c_{\rm t}=0$. Then we obtain

$$\omega_{\rm t}=\frac{b_t}{a_t}=\frac{2m\Omega}{\ell(\ell+1)}. \tag{9}$$

This is the zero-th order approximation for the frequency of r-mode pulsations, which indicates that all r-modes are retrograde (Papaloizou & Pringle, 1978).

If $\Omega\neq0$, the actual displacement $\boldsymbol{\xi}$ is a mix of $\boldsymbol{\xi}_s$ and $\boldsymbol{\xi}_t$, which may be understood from the horizontal components of the momentum Eq. (1);

$$\xi_\theta=\frac{\partial(P'/\rho)}{\omega^2r\partial\theta}-2i\frac{\Omega_r}{\omega}\xi_\phi-\frac{\rho'}{\omega^2\rho^2r}\frac{\partial P}{\partial\theta},\ \xi_\phi=\frac{1}{\sin\theta}\frac{\partial(P'/\rho)}{\omega^2r\partial\phi}+\frac{2i}{\omega}(\Omega_r\xi_\theta-\Omega_\theta\xi_r), \tag{10}$$

where $\partial P/\partial\theta$ $[\sim O(\Omega^2)]$ arises from the rotational deformation. Apparently, when $\Omega=0$, the horizontal displacement is of pure spheroidal form. The necessity of a toroidal component becomes more apparent if we eliminate ξ_ϕ from the first equation and ξ_θ from the second in Eq. (10);

$$\begin{aligned}\xi_\theta\left[1-4\left(\frac{\Omega_r}{\omega}\right)^2\right]&=\frac{1}{\omega^2r}\left(\frac{\partial}{\partial\theta}-\frac{2i\Omega_r}{\omega}\frac{1}{\sin\theta}\frac{\partial}{\partial\phi}\right)\frac{P'}{\rho}-4\frac{\Omega_r\Omega_\theta}{\omega^2}\xi_r-\frac{\rho'}{\omega^2\rho^2r}\frac{\partial P}{\partial\theta},\\ \xi_\phi\left[1-4\left(\frac{\Omega_r}{\omega}\right)^2\right]&=\frac{1}{\omega^2r}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\phi}+\frac{2i\Omega_r}{\omega}\frac{\partial}{\partial\theta}\right)\frac{P'}{\rho}-\frac{2i\Omega_\theta}{\omega}\xi_r-\frac{2i\Omega_r\rho'}{\omega^3\rho^2r}\frac{\partial P}{\partial\theta}.\end{aligned} \tag{11}$$

The toroidal terms appear in the order of Ω/ω, which affect line profile variations (Aerts & Waelkens, 1993) as well as the frequency shifts in $O(\Omega^2)$.

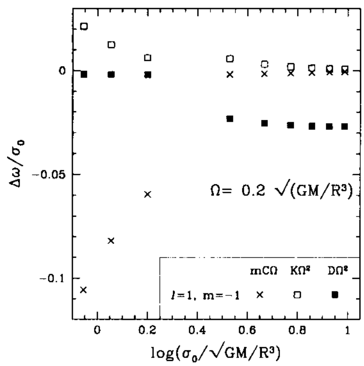

Figure 1. Each contribution to the frequency shifts given in Eq. (12).

2. Frequency shifts

A change in the displacement vector of $O(\Omega/\omega)$ produces frequency shifts of $O(\Omega^2)$. A shift of the same order also comes from the rotational deformation of the equilibrium structure. Therefore, equations to obtain the frequency shift in $O(\Omega^2)$ are very complicated even for uniformly rotating stars (see e.g., Chlebowski, 1978; Saio, 1981; Martens & Smeyers, 1982; Christensen-Dalsgaard & Thompson, 1999). The frequencies in the co-rotating frame of spheroidal modes in uniformly rotating stars may be written as

$$\omega = \sigma_0 + mC_{nl}\Omega + (K + D)\Omega^2 + O(\Omega^3), \tag{12}$$

where $K\Omega^2$ represents the 2nd order Coriolis force effects being proportional to Ω^2/ω^2, and $D\Omega^2$ the deformation effects proportional to $\Omega^2/(GM/R^3)$.

Figure 1 shows the contribution of each term in Eq. (12) for an $n = 3$ polytrope with $\Omega = 0.2\sqrt{GM/R^3}$. Since the p-modes are trapped in the envelope where rotational deformation is large, the contribution from the deformation to p-modes is larger than to g-modes trapped in the core. Dziembowski & Goode (1992) found that the deformation effect dominates the frequency shift of $O(\Omega^2)$, even for the solar 5-min oscillations.

The effects of differential rotation up to $O(\Omega^2)$ were studied by Gough & Thompson (1990) and Dziembowski & Goode (1992). Even $O(\Omega^3)$ effects were formulated by Soufi et al. (1998), but the formulae have not been used yet.

3. The effect on stability

The κ-mechanism driving works optimally for a pulsation mode whose period Π is comparable to the thermal time-scale at the driving zone; i.e., when

$$\Pi \sim \tau_{\rm th} \equiv c_{\rm v}T\Delta m/L \tag{13}$$

(Cox 1974), where c_v and T are the specific heat at constant volume and the temperature at the driving zone, L is luminosity, and Δm is the mass lying above the driving zone. The stellar rotation affects the Π–$\tau_{\rm th}$ matching in two ways: it modifies the period and the thermal time-scale.

The rotational deformation reduces $\tau_{\rm th}$ (Lee 1998), which tends to stabilize p-modes that are excited by the Z-bump κ-mechanism (Lee & Baraffe 1995; Lee 1998; Saio et al. 2000). Since the excitation zone for unstable g-modes lies deeper in the star (because of longer periods) than that for unstable p-modes, the rotational deformation hardly affects the stability of g-modes (Lee 2001).

On the other hand, the rotational period change is larger for g-modes (see Fig. 1), which shifts the range of g-modes that are excited by the κ-mechanism, but it cannot stabilize all the g-modes (Ushomirsky & Bildsten 1998; Lee 2001).

4. Low-frequency modes

For low-frequency modes ($2\Omega/\omega \gtrsim 1$) the effect of deformation is not important (see Fig. 1) because the pulsation energy is confined to the deep interior, and Coriolis force terms are much larger because $(\Omega/\omega)^2 \gg \Omega^2/(GM/R^3)$.

4.1. Local analysis

A usual local analysis under the Boussinesq approximation assuming $P', \boldsymbol{\xi} \propto e^{i\boldsymbol{k}\cdot\boldsymbol{r}}$ leads to (e.g., Unno et al., 1989, Chapter 6)

$$\omega^2 = \frac{k_h^2 N^2 + 4(\boldsymbol{k}\cdot\boldsymbol{\Omega})^2}{k^2}, \tag{14}$$

where N is the Brunt-Väisälä frequency. The term $4(\boldsymbol{k}\cdot\boldsymbol{\Omega})^2$ comes from the Coriolis force. Eq. (14) indicates two types of oscillations: g-modes modified by rotation with $k_h^2N^2 > 4(\boldsymbol{k}\cdot\boldsymbol{\Omega})^2$; and inertial modes, for which the inertial force is more important than gravity; i.e., $k_h^2N^2 < 4(\boldsymbol{k}\cdot\boldsymbol{\Omega})^2$.

To obtain a dispersion relation for r-modes, the latitudinal gradient of $\boldsymbol{\Omega}$ must be taken into account. Letting Ω_r and Ω_θ be the components of $\boldsymbol{\Omega}$ in the r and θ directions and keeping $\partial\Omega_r/\partial\theta$, we obtain

$$k_\theta\omega[k^2\omega^2 - k_h^2N^2 - 4(\boldsymbol{k}\cdot\boldsymbol{\Omega})^2] - 2\frac{\partial\Omega_r}{r\partial\theta}[(N^2-\omega^2)k_\theta k_\phi - 2i\omega k_r\boldsymbol{k}\cdot\boldsymbol{\Omega})] = 0. \tag{15}$$

In deriving Eq. (15) we neglected $\partial\Omega_\theta/\partial\theta$ because it always appears with $\boldsymbol{k}\cdot\boldsymbol{\Omega}$ and the latter is larger. Eq. (15) is reduced to Eq. (14) if $\partial\Omega_r/\partial\theta$ is set to be zero. A dispersion relation for r-modes is obtained from Eq. (15) by assuming $N \gg \omega, \Omega$ as

$$\omega \simeq \omega_0 = -2\frac{k_\phi}{k_h^2}\frac{1}{r}\frac{\partial\Omega_r}{\partial\theta} = \frac{2m\Omega}{\ell(\ell+1)}. \tag{16}$$

The last relation is obtained by substituting $k_\phi = m/(r\sin\theta)$, $k_h^2 = \ell(\ell+1)/r^2$, and $\Omega_r = \Omega\cos\theta$. The frequency ω_0 is the same as $\omega_{\rm t}$ given in Eq. (9) for r-modes. This exercise clearly shows that the latitudinal dependence of Ω_r is essential for r-modes. Furthermore, using Eq. (15) we may estimate the next order term of the r-mode frequency as

$$\frac{\omega-\omega_0}{\omega_0} \simeq \frac{1}{k_h^2 N^2}[k_r^2\omega_0^2 - 4(\boldsymbol{k}\cdot\boldsymbol{\Omega})^2] = O\left[\frac{k_r^2 R^2}{\ell(\ell+1)}\frac{H_p}{R}\frac{\Omega^2}{GM/R^3}\right], \tag{17}$$

where H_p is the pressure scale height. This was estimated first by Papaloizou & Pringle (1978) and obtained numerically by Provost et al. (1981) and Saio (1982).

We note that the properties of r-modes in an isentropic star ($N = 0$) are considerably different from those discussed above. In an isentropic star, r-modes are allowed only for $\ell = |m|$ and the displacements should be proportional to r^ℓ (Provost et al., 1981). Apparently, Eq. (17) is not appropriate for $N = 0$, but the relation for ω_0 in Eq. (16) is recovered if we assume $k_h \gg k_\theta$ and $k_r = 0$ in Eq. (15). The former assumption may be consistent with the condition $\ell = |m|$. The latter assumption, however, contradicts the concept of local analysis and the required r^ℓ dependence. We also note that $\omega - \omega_0 = \omega_0 O(\Omega^2/(GM/R^3))$ in this case (Yoshida & Lee, 2000), which arises from the rotational deformation. Such r-modes got much attention recently because they are unstable due to the emission of gravitational radiation and possibly regulate the rotation speed in young neutron stars (e.g., Andersson, 1998; Lindblom et al., 1998; Yoshida & Lee, 2000).

4.2. Traditional approximation

For low-frequency modes in rotating stars it is convenient to use the so called 'Traditional Approximation', in which Ω_θ is neglected. This approximation is justified because for low-frequency modes $\xi_r \ll \xi_\theta$ and ρ' can be neglected except for the buoyancy term (Boussinesq or anelastic approximation). Under the traditional approximation, Eq. (11) reduces to

$$\boldsymbol{\xi}_h \equiv \begin{pmatrix} \xi_\theta \\ \xi_\phi \end{pmatrix} = \frac{1}{r\omega^2}\frac{1}{1-\nu^2\cos^2\theta}\begin{pmatrix} \dfrac{\partial}{\partial\theta} - \dfrac{i\nu\cos\theta}{\sin\theta}\dfrac{\partial}{\partial\phi} \\ \dfrac{1}{\sin\theta}\dfrac{\partial}{\partial\phi} + i\nu\cos\theta\dfrac{\partial}{\partial\theta} \end{pmatrix}\frac{P'}{\rho}, \tag{18}$$

where $\nu \equiv 2\Omega/\omega$. The horizontal displacement $\boldsymbol{\xi}_h$ couples with the radial component of the momentum equation

$$-\rho\omega^2\xi_r = -\frac{\partial P'}{\partial r} - \rho'\frac{\partial\psi}{\partial r} \tag{19}$$

through the mass conservation equation

$$\rho' + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\rho\xi_r) + \frac{\rho}{r}\nabla_\perp\cdot\boldsymbol{\xi}_h = 0, \quad \text{where} \quad \nabla_\perp \equiv \frac{\partial}{\partial\theta} + \frac{1}{\sin\theta}\frac{\partial}{\partial\phi}. \tag{20}$$

The angular dependence of eigenfunctions can be separated if we assume

$$P',\ \rho',\ \xi_r \propto e^{im\phi}\Theta_\lambda^m(\mu;\nu), \quad \text{with} \quad \mathcal{L}_\nu\Theta_\lambda^m(\mu;\nu) = -\lambda\Theta_\lambda^m(\mu;\nu) \tag{21}$$

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where $\mu \equiv \cos\theta$, and λ is the eigenvalue of the operator $\mathcal{L}_\nu$

$$\mathcal{L}_\nu = \frac{d}{d\mu}\left[\frac{1-\mu^2}{1-\nu^2\mu^2}\left(\frac{d}{d\mu} - \frac{m\nu\mu}{1-\mu^2}\right)\right] - \frac{1}{1-\nu^2\mu^2}\left(\frac{m^2}{1-\mu^2} - m\nu\mu\frac{d}{d\mu}\right). \quad (22)$$

Then we have, for the horizontal divergence of $\boldsymbol{\xi}_h$,

$$\nabla_\perp \cdot \boldsymbol{\xi}_h = -\lambda P'/(\rho r \omega^2). \quad (23)$$

We may consider λ as a measure of horizontal divergence of the displacements. The eigenvalue λ depends on $\nu = 2\Omega/\omega$ (see Fig. 2, in Bildsten et al., 1996, and Figs. 1 and 2 in Lee & Saio, 1997). For a given ν there are an infinite number of values for λ just as an infinite number of values are possible for ℓ when $\Omega = 0$.

For quasi-spheroidal modes λ approaches to $\ell(\ell+1)$ as $\Omega/\omega \to 0$ and $e^{im\phi}\Theta_\lambda^m$ approaches to Y_ℓ^m. For the prograde ($m < 0$) sectoral modes for which $\ell = |m|$ at $\Omega = 0$, $\lambda \to m^2$ as $\nu \to \infty$, while for the other modes λ increases with $|\nu|$ ($\lambda \propto \nu^2$ for $|\nu| \gtrsim 1$) except for a narrow range of $0 < \nu \ll 1$, for which λ's for prograde modes decrease slightly as ν increases (Bildsten et al., 1996).

The amplitude of pulsation at the surface tends to concentrate toward the equator as $|\nu|$ increases. The effect is stronger for modes with larger λ and the prograde sectoral modes are the least affected. How the amplitude distributes on the surface is important for line-profile variations and the visibility of light variations (e.g., Lee & Saio, 1990a; Townsend, 1997; Lee & Saio, 1997; Ushomirsky & Bildsten, 1998; Clement, 1998).

The horizontal divergence is zero for purely toroidal displacements. Therefore, r-modes should correspond to very small $|\lambda|$'s. λ can be negative if $|2\Omega/\omega| > 1$. Actually, λ crosses zero from negative to positive as $|\nu|$ increases. This occurs only for retrograde oscillations, consistent with the fact that all the r-modes are retrograde.

Under the traditional and Cowling approximations, the equations for nonradial adiabatic pulsations are reduced to

$$\begin{aligned} r\frac{dy_1}{dr} &= \left(\frac{V}{\Gamma_1} - 3\right)y_1 + \left(\frac{\lambda}{c_1\overline{\omega}^2} - \frac{V}{\Gamma_1}\right)y_2, \\ r\frac{dy_2}{dr} &= (c_1\overline{\omega}^2 + rA)y_1 + (1 - U - rA)y_2, \end{aligned} \quad (24)$$

where $y_1 = \xi_r/r$ and $y_2 = P'/(\rho g r)$ (with g being the local gravitational acceleration), and $\overline{\omega}^2 = \omega^2/(GM/R^3)$. Equations (24) are the same as those for $\Omega = 0$ except for λ instead of $\ell(\ell+1)$ (e.g., Berthomieu et al., 1978; Dziembowski & Kosovichev, 1987; Lee & Saio, 1987).

Assuming that $y_1, y_2 \propto e^{ik_r \ln r}$ with $|k_r| \gg |1 - U - rA|, |V/\Gamma_1 - 3|$, we obtain from Eq. (24) the dispersion relation

$$k_r^2 = \frac{r^2}{\omega^2 c_s^2}(\omega^2 - \frac{\lambda c_s^2}{r^2})(\omega^2 - N^2), \quad (25)$$

where c_s is the adiabatic sound speed. If $k_r^2 > 0$, the oscillation propagates in radial direction. For a positive λ there are two types of radial propagations, as in the non-rotating case. The condition for the propagation of low-frequency oscillations, $\omega^2 < \min\left(\lambda c_s^2/r^2, N^2\right)$ is appropriate here.

To avoid the running-wave leakage at the surface, the frequencies of g-modes must be larger than the critical frequency $\sqrt{\lambda}c_s/R$. Since λ changes with ν, the critical frequency is modified if $\Omega \neq 0$. Townsend (2000a,b) investigated this effect in the context of the influence of rotation for g-mode pulsations in SPB stars. He concluded that the stability of g-modes appropriate for SPB stars is not affected by the above effect because the frequencies are sufficiently high.

When $|\nu| \gtrsim 1$, λ is proportional to ν^2 for quasi-spheroidal modes except for prograde sectoral modes (Bildsten et al. 1996). If we write $\lambda = \Lambda\nu^2$, the condition for the radial propagation and an asymptotic dispersion relation of low-frequency modes may be written as

$$\omega^2 < \min\left(\Lambda^{\frac{1}{2}}\frac{2\Omega c_s}{r}, N^2\right), \quad k_r^2\omega^4 \simeq 4\Lambda\Omega^2 N^2, \tag{26}$$

which correspond to gravito-inertial waves.

Equation (25) indicates that when $\lambda < 0$, oscillatory modes exist in the convection zones. These modes are inertial modes corresponding to rotationally stabilized convective (g^-) modes. Such a mode in the convective core can couple with an envelope g-mode to become a mixed-character mode. These coupled modes have been obtained even without using the traditional approximation (Lee & Saio, 1986). A similar coupled mode was obtained in tidally forced oscillations by Savonije & Papaloizou (1997) with a two-dimensional code.

4.3. Odd properties of low-frequency modes

Negative energy modes?

Lee & Saio (1990b) found that the pulsation (kinetic plus potential) energy in the co-rotating frame is negative if $\lambda(2 + d\ln|\lambda|/d\ln|\nu|) < 0$ in the traditional approximation. Negative energy occurs for some prograde modes with negative λ (Lee & Saio, 1997). If a negative-energy mode couples with a g-mode ($E > 0$), the combined mode can be overstable because energy can flow from the negative energy part to the positive energy part without changing the total energy. This gives an explanation for the overstable modes obtained by Lee & Saio (1986), in which the traditional approximation was not used but the eigenfunctions were expanded by only two spherical harmonics.

Singular modes?

Recently, using the anelastic approximation, Dintrans & Rieutord (2000) have found that some low-frequency modes in a rotating star have a singular velocity distribution in the adiabatic limit. In a non-adiabatic analysis these modes have strong velocity shear zones, which cause a strong damping. They argued that these modes may play an important role to transport angular-momentum in a star with a tidal interaction. Certainly, further investigation of these nearly singular modes is needed.

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Discussion

M. Rieutord : One should keep in mind that r-modes are only an infinitely small fraction of inertial modes which are the modes whose restoring force is the Coriolis force.

H. Saio : That is true, but I would like to emphasize that the toroidal displacements are dominant in r-modes in contrast to the other inertial modes.

M. Rieutord : You seem to find unstable gravito-inertial modes, i.e., modes in which gravity and Coriolis forces are both important. We never found such an instability in our models. Could your result be due to the finite resolution of your computation?

H. Saio : It is difficult to find unstable modes because the frequencies are proportional to the square-root of the superadiabatic temperature gradient in the convective core. Therefore, if you assume adiabatic convection, you would not find one. It is possible that the properties of these modes depend considerably on how many spherical harmonics are included to expand eigenfunctions. But we still find unstable modes even when we use 10 spherical harmonics.