Resonant absorption of Alfven Waves and the Associated Phenomenon of Magnetic Reconnection.

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ABSTRACT. The mathematical analysis of the Alfven Wave equation in inhomogeneous magnetic fields which explain the resonance absorption of Alfven surface waves near a resonant layer can also be used to show that magnetic reconnection process can arise near the zero frequency resonant layer driven by very low frequency Alfven surface waves. The associated phenomena of resonant absorption and magnetic reconnection in inhomogeneous plasmas can explain the recent obsrevations of intense magnetic activity in the long period geomagnetic micropulsations range, at magnetospheric cusp latitudes, during the time of occurence of flux transfer events.

The Alfven wave equation governing the dynamics of the ideal MHD system in the presence of inhomogeneous magnetic fields with variation in the direction perpendicular to the plane of the magnetic field, shows a singular behaviour at the point where the local Alfven speed becomes equal to the phase speed of the wave. Extensive analysis of this equation shows that due to this singularity the system can absorb energy when it is driven externally if there are well defined discontinuities in the Alfven speed profile then the energy absorption is due to the resonant interaction between the shear Alfven and surface Alfven Waves. In the latter case there is a collisonless damping of the surface waves. When Alfven speed profile passes through a zero value, the singularity occurs at the zero-frequency and the resonant surface is now the neutral surface. The aim of this note is to show that the analysis of

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the Alfven wave equation carried out to understand the resonant phenomena of Alfven waves can be used to show that very low frequency surface waves can drive magnetic reconnection across the neutral layer.

Considering an incompressible medium with density ρ and the magnetic field

$$B_0(x) = [o, B_{0y}(x), B_{0z}(x)] , \qquad (1)$$

the linearized ideal MHD equations give the Alfven wave equation

$$\nabla \cdot \left\{ \rho_0 \left[\frac{\partial^2}{\partial t^2} - \frac{(B_0 \cdot k)^2}{4\pi\rho_0} \right] \nabla u_{xk} \right\} = 0 \qquad (2)$$

where $u_{xk}(x,k,t)$ is the Fourier transform in y and z of the perturbed velocity component $v_x(x,y,z,t)$. The wave number $k=(o,k_y,k_z)$.

Recently it was shown[1] that for a given set of initial conditions Eqn.(2) has a well behaved harmonic solution in the closed form valid only for short intervals of time. This solution is not valid for arbitrary large times as the calculation of current densities J(x) contain terms of order t and therefore become very large as $t \rightarrow \infty$. To be more specific consider a linear variation of the initial magnetic field:

$$B_0(x) = \frac{B_{0y} x}{a} \hat{y} \quad , \qquad a \lesssim x \lesssim a \; ,$$

$$= \text{constant} \quad , \quad |x| > a \; . \qquad (3)$$

The current density component $J_z(x)$ is given as

$$J_z(x) = -\frac{1}{4\pi}\left[\frac{B_{0y} x}{a} \frac{\partial^2 \xi_{xk}}{\partial x^2} + \frac{2B_{0y}}{a} \frac{\partial \xi_{xk}}{\partial x} - k^2 \frac{B_{0y} x}{a} \xi_{xk} \right], \qquad (4)$$

where the displacement vector $\frac{\partial \xi_{xk}}{\partial t} = u_{xk}$, calculated from the solution of (2) given in Ref.1, is given as

$$\frac{\partial \xi_{xk}}{\partial x} = A(x,k,o) \frac{\cos[\omega_A(x)t]}{[\omega_A(x)]^2} + B(x,k,o) \frac{\sin[\omega_A(x)t]}{\omega_A(x)} , \qquad (5)$$

where A and B are constants determined by the initial conditions and $\omega_A(x)$ is the Alfven wave frequency. From (4) and (5) we see that $\partial^2\xi/\partial x^2$ will contain a term of order t which makes $J_z(x)$ assume large values for increas-ing values of t. Considering the values of $J_z(x)$ near theneutral layer we take $\frac{\partial}{\partial x}$ >>k and for simplicity choosing A(z,k,o)=0, Eqn.(4) gives

$$J_z(o) = B(x,k,o)\,\frac{B_{o4}}{a}\,t\ . \qquad (6)$$

which increases with time t.

The growth of solutions like (4) and (6) can be understood from the results obtained by the normal mode analysis[2] of the Eqn.(2), which predicts only singular modes and an entirely continuum spectrum. No dispersion relation exists. Thus we can conclude that current density increases at every point in space in the region in which magnetic field varies continuously including the neutral point (which forms the end point of the continuum spectrum) as the disturbance cannot be carried away by the Alfven waves due to lack of dispersion. It is interesting to point out here that Dungey [3], in his historical paper on magnetic reconnection explains the accumulation of energy near a neutral point but not elsewhere in the following way: "If there is a small disturbance in a region of non-zero magnetic field the current density will not become large in this region, but the disturbance will spread in the form of Alfven waves, if, however, there is a neutral point anywhere a small disturbance can cause the current density to become large in the neighbourhood of the neutral point". Here we see that the current density increases in the neighbhourhood of every point in space not just at the neutral point as due to lack of dispersion the Alfven wave cannot propagate away from any point in space in the region in which the magnetic field varies continuously. The normal mode analysis also shows that the singularity of the Eqn.(2) at $\omega=\omega_A(x)$ implies a process of absorption of energy by the plasma when it is driven externally in steady state at frequency ω. The Laplace transform method [2] to understand the initial value problem of the Eqn.(2), however, shows that if magnetic field profile has finite number

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of discontinuities, then the energy is absorbed from the Alfven surface waves propagating along the discontinuities. At the neutral surface the energy is derived from the zero frequency surface waves. The surface waves, which form the only collective modes of colisionless system are damped away. This can be noted from the large time asymptotic solution given in Ref.2.

It is seen that the physical processes near the resonant and neutral points do not show any different behaviour for the ideal MHD system with magnetic fields of type (1). To further understand the dissipation of the accumulated energy near these prints, we take finite resistivity into account. The Alfven wave equation is now a fourth order differential equation with no singular point. The singularity is removed by the finite resistivity term and a resistive boundary layer forms around the singular point. The time behaviour of this eqn. shows that [4] for $t<t_h$ the current density shows a linear increase as in the ideal MHD case but for $t>t_h$ the solution becomes oscillatory, where

$$t_h=\left(\frac{24}{\omega_A[x_0]^2\eta}\right)^{1/3}\times\left[\frac{2B_0'(x_0)}{B}-\frac{\rho'(x_0)}{\rho}\right]^{-2/3}. \qquad (7)$$

For linear variation $t_h=2^{-1/3}\tau_A^{2/3}\tau_R^{1/3}$, $\tau_A=a/v_A$ and $\tau_R=4\pi a^2/\eta$. The analysis of the fourth order differential equation holds good at the neutral point as this equation remains unchanged by shifting x to x_0+x.

Considering the mechanism of dissipation of the accumulated energy in the resistive layer it was shown[5] that near the Alfven resonant point the surface wave resonantly interacts with the shear Alfven wave modified by the finite conductivity effects. The energy of the surface wave transformed to the damped Alfven wave thus gets dissipated by the collisional effects.

The surface wave energy in the neighbourhood of the neutral point, however, cannot propagate away as the damped Alfven wave. The nonoscillating type of surface purterbations (as the surface wave frequency is nearly zero) can give rise to tearing mode instability and the reconnection of the magnetic lines across the neutral layer.

We suggest that the associated phenomena of resonant absorption and magnetic reconnection can explain the recent observations [6] of intense magnetic activity in the long period geomagnetic micropulsations range, at cusp latitudes, during the time of occurrence of flux transfer events. The k-H instability at the magnetopause generates a spectrum of Alfven surface waves. The higher frequency surface waves are associated with the excitation of micropulsation activity [7]but the low frequency surface waves can give rise to the magnetic reconnection process, which is associated with the flux transfer events at the magnetopause.

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