OPTIMUM SEARCH STRATEGY FOR RANDOMLY DISTRIBUTED CW TRANSMITTERS

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ABSTRACT. The relative probability of detecting randomly distributed CW transmitters as a function of the fraction of the sky which is searched (in a fixed time interval) is given. It is shown that the probability of detecting such a class of transmitters with a given receiving system is a maximum if the entire sky is searched, provided that the receiving system is sufficiently sensitive to detect the nearest transmitter in the allocated time and that the integration time - bandwidth product in a specified direction is greater than 8.

1. INTRODUCTION

It is important in designing a search program for SETI (Search for Extraterrestrial Intelligence) to understand the factors that affect the probability for detecting signals from distant transmitters. An optimum search strategy cannot be identified at the present time since the statistical properties of the transmitters and the motivations of the senders are not known. Nevertheless, a simple model in which the transmitters are randomly distributed in space and have a power law distribution of intrinsic transmitted power appears to have sufficient generality to provide some guidance in designing a search program. We examine in this note the relative probability of detecting signals from such a class of transmitters in a fixed time interval, as a function of the fraction of sky that is searched. The scenario envisaged is that the observer may choose between scanning slowly and achieving high sensitivity in a few directions, or scanning the entire sky at reduced sensitivity.

2. THE PROBABILITY RELATIONSHIP

Following Drake (1983), we assume that signals from civilizations radiating an equivalent isotropic radiated power (EIRP), P_{eirp}, are originating from randomly spaced locations having a density of ρ transmitters per unit volume. We ignore the fact that the galaxy is a highly flattened disk and consider the spherical volume in the vicinity of the

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sun where stars are distributed more or less at random except for a tendency to cluster. This volume has a maximum radius of approximately 1 kiloparsec. The maximum range of a given search system is related to the minimum detectable flux, S, by the expression

$$R = \left(\frac{P_{eirp}}{4\pi S}\right)^{1/2} \tag{1}$$

The total number of detectable signals within the spherical volume is given by

$$N = \left(\frac{4\pi\rho}{3}\right)\left(\frac{P_{eirp}}{4\pi S}\right)^{3/2} \tag{2}$$

The minimum detectable flux, S, depends on the diameter, D, of the receiving antenna, the system noise temperature, T, and the receiver channel bandwidth, B. Oliver and Billingham (1971) show that the minimun detectable (defined as Signal/Noise = 1) coherent flux is equal to

$$S = \frac{4kTB}{\pi D^2} \; \frac{1+(1+n)^{1/2}}{n} \tag{3}$$

and the maximum range at which a signal can be detected with a given receiving system is given by

$$R = \left(\frac{D}{4}\right)\left[\frac{P_{eirp}}{kTB} \; \frac{n}{(1+(1+n)^{1/2})}\right]^{1/2} . \tag{4}$$

In these expressions

k = Boltzmann's constant
T = system noise temperature
D = receiving antenna diameter
B = receiver channel bandwidth
t = integration time
n = (Bt) = number of independent samples averaged.

Drake (1983) shows that the probability of success of an observation program is proportional to a) the total frequency searched b) the total solid angle searched, and c) the spherical volume defined by the radius R. These lead to the following expression for the probability of success of an observation program under the assumption that the probability of success of a single observation is small (Gulkis, 1984).

$$P = K\rho MCB\lambda^3 G^{1/2}\left[\frac{P_{eirp}}{kTB} \; \frac{B\tau}{M+\sqrt{M^2+MB\tau}}\right]^{3/2} \tag{5}$$

In this expression, M is the number of different directions searched, λ is the wavelength of the search, G is the antenna gain, B is the channel bandwidth, C is the total number of channels, τ is the total observation time, and K is a constant of proportionality. Equation 5 shows that the probability of success is a function of the number of directions (M) which is searched. Figure 1 shows the dependence of the probability on M for a number of different telescopes whose gains vary from 10^2 to 10^8. The advantages of working with a high gain antenna are evident. The value of $B\tau$ used is 10^8.

We note in Figure 1 that for any given telescope, the probability increases as M increases up to the point where the telescope gain becomes nearly equal to the number of directions examined. M cannot exceed the gain since the gain sets the limit on the number of independent areas in the sky which can be examined. The maximum probability is achieved when $M = B\tau/8$. This corresponds to 8 independent samples per beam area. If the antenna gain is less than the value of M which maximizes the probablity, the entire sky should be searched in order to maximize the probability.

Equation 5 is somewhat misleading in that the probability for success can only go to zero if the number density of transmitters approaches zero (assuming the system search parameters are all finite). However if a given receiving system does not have sufficient sensitivity to detect the nearest transmitter, then the probability of success must be zero regardless of the number density. To overcome this difficulty, we introduce a term for the maximum flux density, Smax, which is produced by the nearest transmitter. If the sensitivity that can be achieved with a given system is greater than Smax, then the probability of success is non-zero; otherwise it is zero. We incorporate this term into the equations by noting that there is a minimum radius (Rm) inside of which there are no transmitters. If R is less than Rm, then the probability of success for a given search system is zero. On the other hand, if R is greater than Rm, then the probability of success is non-zero and is proportional to the number of transmitters which lie between the spherical volumes defined by Rm and R. We can take as an estimate for Rm, the average distance between transmitters given by $0.55\ \rho^{1/3}$. Incorporating these terms into the detection equations leads to the following equation for the probability for success. The last term in equation 5a represents the volume around the observer in which there are no transmitters.

$$P = K\rho MCBG^{-1} \left[\left(\frac{\lambda}{4\pi}\right)^3 G^{3/2} \left[\frac{P_{eirp}}{kTB} \frac{B\tau}{M+\sqrt{M^2+MB\tau}} \right]^{3/2} - 0.55^3\rho \right] \qquad (5a)$$

Figure 2 shows the dependence of the probability on M including the term for sensitivity. It is seen in this figure that there is family of curves, characterized by minimum radius. The maximum probability is now determined by the strength of the transmitters. If the nearest (assuming all transmitters have same intrinsic power) transmitter is sufficiently powerful to be observed even when the entire sky is searched, then the entire sky should be searched. If sensitivity is the limiting factor, then the amount of sky searched should be reduced to the point where the strongest transmitter will be detected.

In the derivation of Equation 5, we assumed that each transmitter had the same intrinsic power. It is easy to generalize the equation for the case of a continuous distribution of powers by rewritting Equation 5 as a differential probability of success.

$$dP = KK_1MCB\lambda^3G^{1/2}\left[\frac{1}{kTB}\,\frac{B\tau}{M+\sqrt{M^2+MB\tau}}\right]^{3/2} P^{3/2}\,\rho(P)dP \tag{6}$$

If the distribution of powers is a power law of the form

$$\rho(P) = K_1P^{-\alpha} \tag{7}$$

Equation 6 may be integrated between the limits Pu and PL to yield the following expression for the probability of success (provided that $\alpha \neq 5/2$)

$$P = KK_1MC\lambda^3BG^{1/2}\left[\frac{1}{kTB}\;\frac{B\tau}{M+\sqrt{M^2+MB\tau}}\right]^{3/2}\frac{P_u{}^{5/2-\alpha}-P_L{}^{5/2-\alpha}}{5/2-\alpha} \tag{8}$$

It is seen from this equation that the spectral index, α, determines whether the strong sources or the weak sources will dominate the probability of success. If the spectral index is less than 5/2, then the strong sources, although less numerous will dominate. If the spectral index is greater than 5/2, then the weaker and more numerous sources will dominate. If the spectral index is exactly 5/2, then all transmitters will contribute uniformly to the probability of success.

DISCUSSION

We have shown in this note that the probability of detection a CW signal from randomly distributed transmitters increases faster by increasing the fraction of sky that is searched than by increasing the sensitivity in a given direction. This result is subject to two constraints. First, the system must have sufficient sensitivity to detect the nearest transmitter; second, Bt must be greater than or equal to 8. For smaller values of Bt, the fraction of sky searched should be adjusted so as to yield Bt=8. The reason why it is more advantageous to search a larger solid angle at reduced sensitivity than to search a smaller solid angle at increased sensitivity can be understood through the following argument. The number of detectable transmitters increases as the minimum detectable flux raised to the − 3/2 power (Eq. 1 and 2). Since the minimum detectable flux decreases

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as the inverse square root of time when n is large, the number of detectable transmitters in a given direction increases as the observation time raised to the 3/4 power. On the other hand, the number of detectable transmitters is also proportional to the solid angle searched. Since the solid angle searched can be made to increase in proportion to the time by searching in different directions, the number of detectable transmitters can be made to increase in direct proportion to the time (rather than to the 3/4 power). Hence, the probability of success increases faster by scanning the entire sky than by concentrating in the search to a few directions. The ratio of the number of transmitters which could be detected with a given telescope system in a given time if the entire sky is searched to those that could be detected in the same time if a single direction is observed is proportional to the observation time raised to the 1/4 power.

Although this result was derived for the case of coherent radiation, it applies more generally to detection situations in which the minimum detectable flux decreases more slowly than the observation time raised tothe -2/3 power. For example, the minimum detectable flux varies as the inverse square root of time for incoherent radiation. Hence the result holds for incoherent as well as coherent radiation. The result breaks down whenever the minimum detectable flux decreases faster than the observation time raised to the -2/3 power. One example where this occurs is in the detection of strong coherent signals. In this case, the flux varies inversely with time.

Aside from mechanical considerations, a large telescope can always be used to produce a higher probability of success than a smaller telescope. Also, it is possible that a small telescope which surveys the entire sky will have a higher probability of success than a larger telescope that concentrates a search in a few directions.

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FIGURES

Figure 1. Relative probability of success for a number of different antennas whose gains vary from 10^2 to 10^8 as a function of the number of directions in the sky which are searched. The search time - channel bandwidth product is taken to 10^8.

Figure 2. Relative probability of success as a function of the number directions in the sky which are searched. Curves assume search parameters are as follows: 100 m radio telescope, 25 K system temperature, 10^{13} watts effective isotropic power, bandwidth 10 Hz, search time 10^6 seconds. Various curves are for different densities of transmitters, with densities expressed as the mean distance between transmitters in light years.

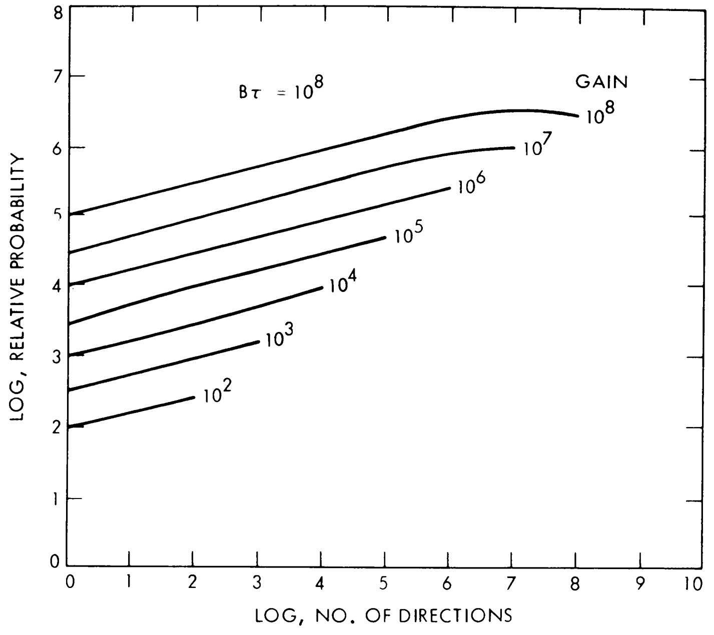

FIGURE 1

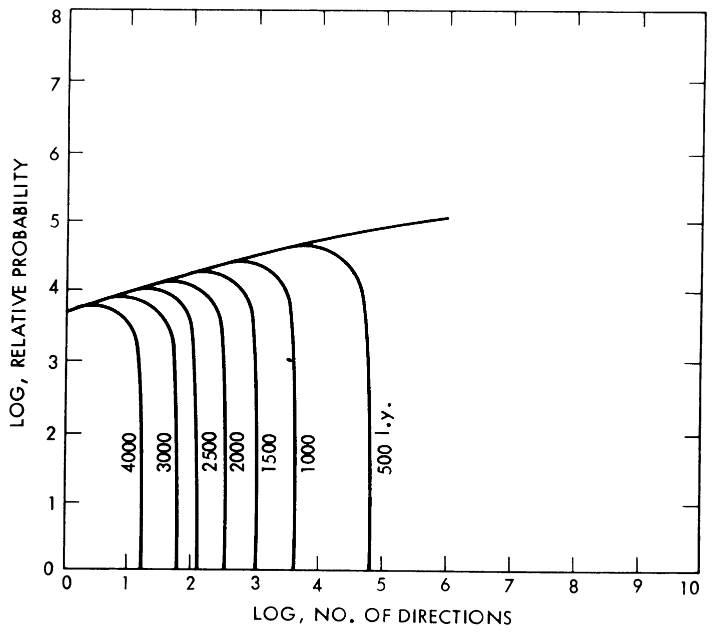

FIGURE 2

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