## **BOOK REVIEWS**

 $\Delta(>0) \equiv 1 \pmod{8}$  for which the  $Q_i$  are powers of 2 (§7.1). An important key exchange system of Buchmann and Williams in cryptology uses the infrastructure of real quadratic fields and large values of  $h_{\Delta}$  (§8.2).

We briefly survey further features of this volume. There are eighty pages of tables on fundamental units, class numbers and the structure of the class group for D < 2000. There are also sections on the significance of the GRH and the philosophy of computer-assisted proofs as well as appendices on analytic material and the conventional theory of quadratic forms. Throughout one becomes aware of the advantages that the power of modern computing provides in terms of insights that can lead to proofs of results which might otherwise have been unsuspected or in some cases yield counterexamples to what had seemed reasonable conjectures. On the other hand many plausible conjectures and questions are stated which remain to be settled and might serve to motivate interested readers.

With its many footnotes giving items of mathematical, historical and personal interest (some anecdotal), its full sets of references and examples and its wealth of curiosities and numerical facts, this volume is bound to be stimulating for expert and student alike. Certainly it is a browser's delight. Nevertheless, from an early stage (for example, Theorem 1.3.2 on page 16) it touches on the frontiers of knowledge and in many places it is a tour along such frontiers, so that a serious reader should not expect to proceed quickly. It will certainly become an invaluable handbook on "quadratics".

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RAMM, A. G. and KATSEVICH, A. I. The Radon transform and local tomography (CRC Press, Boca Raton-New York-London-Tokyo, 1996), xviii + 485 pp., 0 8493 9492 9, \$79.95.

The Radon transform of a function  $f : \mathbb{R}^2 \to \mathbb{R}$  gives the line integrals of f along straight lines in the plane. Thus the Radon transform  $\hat{f} : \{\text{lines in } \mathbb{R}^2\} \to \mathbb{R}$  is defined as

$$\hat{f}(\ell) = \int_{\ell} f(x) d\ell$$

and there are obvious extensions to integrals of functions on  $\mathbb{R}^n$  over translates of subspaces of a given dimension. A natural problem to ask is: how can one recover f knowing  $\hat{f}$  or, in other words, what is the inverse of the transform? An inversion formula was published by Johann Radon in 1917, but this attracted little interest for many years.

It was in the 1970s with the development of X-ray and computer technology that the Radon transform and its inversion became enormously important. Given a section of a head or body, the local density of X-ray absorption at the point x, given by f(x) say, varies considerably between different types of bone and tissue. The attenuation of X-rays along a line represents the integral of f along the line and this can be measured using X-ray photography. Thus a large number of X-ray photographs taken in different directions enable the Radon transform of f to be estimated. Inversion of the transform gives f, providing a "map" of the head or body section that enables tumours, blood clots, etc. to be detected. The development of a practical scanner to implement this idea led to the Nobel prize in medicine being awarded to Cormack and Hounsfield in 1979 and to the birth of the science of tomography.

Since then an enormous amount of theoretical and practical mathematics of tomography has been developed, with functional analysis and Fourier transforms playing an important role. Many problems arise: for example practical data is discrete rather than continuous, it is more convenient to work with point source rather than parallel beam X-rays, the inversion problem is ill-posed given a finite set of X-ray photographs, and the data is often incomplete – for example the data may be known for only a limited cone of directions.

In the first few chapters this book presents standard tomography theory, including results on the range and uniqueness of the Radon transform, inversion formulae and reconstruction

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algorithms. The book then addresses less familiar material, for example the relation between singularities of the Radon transform and the function, and this is extended to the situation of noisy data. (This is of practical importance since discontinuities of the function correspond to boundaries of bones, etc.). Inversion of the transform where the data is incomplete is discussed, as is the important "cone-beam" problem of reconstructing a function on  $\mathbb{R}^n$  from line integrals through the points on a  $C^1$ -curve outside the support of f.

A substantial part of the book is devoted to local tomography and a variant, pseudolocal tomography. The problem is that, although one often wishes to find the unknown function on a small region, the Radon transform inversion formula involves integrals along lines cutting all parts of the supporting domain – it is unfortunate if, to map a section of the head, one needs to take the feet into consideration! A technique for getting around this difficulty is to replace f by Bf for a pseudodifferential operator B chosen so that the calculation of Bf is local in that it can be found from integrals along just those lines passing through the region where f needs to be found. Then many of the features of f such as its discontinuities can be recovered from Bf in this region. Local tomography and its implementation are discussed for the basic Radon transform and variants such as limited angle cases.

This valuable addition to the literature will benefit those involved in theoretical and practical tomography, giving access both to the basics of the subject and to more sophisticated but important techniques. Much of the work is recent with a substantial amount the authors' own research. However the presentation and material are sophisticated in places and a fairly advanced knowledge of mathematical analysis is needed to follow much of the book. Mathematicians will enjoy the book, but I am less certain about the "engineers, physicists and radiologists who deal with processing of tomographic data" mentioned in the introduction!

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