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ADDENDUM TO THE PAPER: SPHERE THEOREM BY MEANS OF THE RATIO OF MEAN CURVATURE FUNCTIONS*

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Abstract. It is shown that an immersion of n dimensional compact oriented manifold without boundary into the $n+1$ dimensional Euclidean space, hyperbolic space or open half sphere is a totally umbilic immersion if one of the mean curvature function H_l does not vanish and the ratio H_k/H_l is constant, $1 \leq k, l \leq n, k \neq l$.

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The following theorem was proved in [**1**].

THEOREM 1. *Let N^{n+1} be one of the Euclidean space $\mathbb{R}^{n+1}$, the hyperbolic space $\mathbb{H}^{n+1}$ or the open half sphere $\mathbb{S}_+^{n+1}$ and $\phi : M^n \to N^{n+1}$ be an isometric embedding of a compact oriented n-dimensional manifold without boundary M^n. If the ratio H_k/H_l is constant for some $k, l = 0, 1, 2 \ldots, n,\ \ k > l$ and H_l does not vanish on M^n, then $\phi(M^n)$ is a geodesic hypersphere.*

If we assume that ϕ is an immersion, the proof in [**1**] does not apply directly. For Theorem A in [**1**] is not true in this case as Wente's disproving the Hopf's conjecture [**3**] shows. In this note, however, we prove the theorem above for an *immersion* ϕ by slightly changing the argument of [**1**].

THEOREM 2. *Let N^{n+1} be one of the Euclidean space $\mathbb{R}^{n+1}$, the hyperbolic space $\mathbb{H}^{n+1}$ or the open half sphere $\mathbb{S}_+^{n+1}$ and $\phi : M^n \to N^{n+1}$ be an isometric immersion of a compact oriented n-dimensional manifold without boundary M^n. If the ratio H_k/H_l is constant for some $k, l = 1, 2 \ldots, n,\ \ k > l$ and H_l does not vanish on M^n, then $\phi(M^n)$ is a geodesic hypersphere.*

This theorem is also a generalization of [**2**], where the same theorem was proved when $k = l + 1$. Note that the case $l = 0$ is omitted. As H_0 is defined to be 1, $H_k/H_0 = H_k$. Thus, if H_k/H_0 is constant, the theorem above does not hold for the same reason that the proof of [**1**] does not apply directly. The first-named author would like to thank the referee of [**1**] for suggesting Theorem 2.

The proof is as follows. In the proof of [**1**], we showed that

$$H_k/H_l = H_{k-1}/H_{l-1}.$$

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Proceeding inductively, we have

$$H_{p+1}/H_1 = H_p/H_0 = H_p, \quad (p = k - l);$$

that is,

$$H_{p+1}/H_p = H_1. \tag{1}$$

On the other hand, we also have from Lemma B (2) in [**1**],

$$H_{p+1}/H_p \leq H_p/H_{p-1} \leq \cdots \leq H_1. \tag{2}$$

From (1) and (2), we have

$$H_{p+1}/H_p = H_p/H_{p-1} = \cdots = H_1.$$

From this equality we have

$$H_r = H_1^r, \qquad r = 1, 2, \ldots, p+1.$$

Since these equalities hold only at umbilical points, it follows that every point is an umbilical point; that is, $\phi(M^n)$ is a geodesic hypersphere.

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