A PRELIMINARY ESTIMATION OF THE ACCURACY OF THE INNER PLANET'S CO-ORDINATES

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RÉSUMÉ. — L'auteur étudie successivement les effets d'erreurs dans les développements analytiques des théories des planètes, ou dans les valeurs adoptées pour les masses, pour les éléments et pour le temps des éphémérides, sur la précision des coordonnées des planètes inférieures calculées à partir des développements et des constantes de Newcomb (éléments de Ross pour Mars). La précision est de cinq décimales. La source principale d'erreurs sur les positions des planètes est l'imprécision des éléments. On pourrait améliorer considérablement les éphémérides en introduisant des corrections appropriées.

ABSTRACT. — In the construction of analytical theories of planetary motion, the effect is considered of errors in the adopted values of masses, orbital elements and ephemeris time upon the accuracy of computed co-ordinates of the inner planets in accordance with Newcomb's expansions and constants (with Ross's elements for Mars). These are accurate to five decimal places. The errors in the orbital elements are the principal source of inaccuracy in the positions of the planets. The precision of the ephemerides can be considerably improved by introducing appropriate corrections.

ZUSAMMENFASSUNG. — Der Einfluss von Fehlern bei der Aufstellung analytischer Theorien der Planetenbewegung und in den angenommenen Werten für die Massen, die Bahnelemente und die Ephemeridenzeit auf die Genauigkeit der berechneten Koordinaten der inneren Planeten nach den Ausdrücken und mit den Konstanten von Newcomb (mit den Marselementen von Ross) wird untersucht. Die Genauigkeit beträgt fünf Dezimalen. Die Fehler in den Bahnelementen sind die hauptsächliche Ursache für die Ungenauigkeit der Planetenörter. Die Genauigkeit der Ephemeriden kann beträchtlich gesteigert werden, wenn man geeignete Korrektionen einführt.

Резюме. — Автор последовательно изучает эффекты погрешностей в аналитических разложениях в теориях планет, или в принятых значениях масс и элементов, или еще в эфемеридном времени, на точность координат внутренних планет, вычисленных пользуясь разложениями и постоянными Ньюкомба (для Марса, элементы Росса). Они точны до пяти десятичных знаков. Погрешности в орбитальных элементах являются главной причиной неточности положений планет. Эфемериды могут быть значительно уточнены, применяя надлежащие поправки.

The *National Almanacs*, since 1960, give the ephemerides of the Moon, the outer planets and the day numbers with an additional decimal figure. Apparent places of stars are computed more precisely. However the accuracy of ephemerides of the inner planets has remained unchanged. Besides it is to be noted, that more than a hundred years have elapsed since Newcomb had worked out his theory of motion of the inner planets, and the accumulated observational material remains unused in the computation of ephemerides; whereas the demands for precision of the astronomical ephemerides rise continuously. At the present time astronomical almanacs are used not only in studies of the motion of the bodies of the solar system, and in discussions of different kinds of astronomical observations, but also in computations of the paths of space rockets.

In accordance with the need for higher precision in astronomical ephemerides by both theory and practice, especially in experimental astronomy, it may be of interest to make an at least provisional estimate of the accuracy of the ephemerides of the inner planets, based on Newcomb's theory ([1], [2]), which are at present published in astronomical almanacs.

The principal sources of errors in the determination of planetary co-ordinates are the following :

1. Errors in the analytical theories;
2. Errors in the adopted masses of the planets;
3. Errors in the determination of orbital elements;
4. Errors in the determination of ephemeris time.

We shall discuss the effect of the enumerated sources of errors on the heliocentric position of the planets, resolving the vector of error into three components — along the radius-vector (r), in the normal to the orbital plane (s), and in that direction perpendicular to the radius-vector, which lies in the orbital plane (τ). It is convenient to express the vector of error in astronomical units of length (a. u.).

Errors in the analytical theories of planetary motion arise from unaccounted or incorrectly computed terms in Newcomb's expansions.

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These errors may be detected by a comparison of Newcomb's expansions, representing an approximate solution of differential equations, with an accurate solution of these equations, which can be readily obtained within a given number of decimals by numerical integration.

In order that the co-ordinates computed directly from Newcomb's expansions might be comparable with those obtained by numerical integration, the initial conditions of motion and the parameters of the differential equations must be consistent with the constants adopted by Newcomb in his analytical theories of the motion of the planets. Besides, care must be taken that the accumulation of errors in the process of numerical integration should not affect the result within the stated limits of accuracy.

To make the initial conditions of motion agree, we used the formulae of Eckert-Brouwer in the form

$$(1) \qquad \Delta \bar{r} = \bar{C} \times \bar{r} + \dot{\bar{r}}\, C_4 + \left(\bar{r} - \frac{3}{2} t\, \dot{\bar{r}}\right) C_5 + (H\bar{r} + K\dot{\bar{r}})\, C_6,$$

where

$$H = \frac{r + p - 2a}{p}, \qquad K = \frac{(r + p)\, r\dot{r}}{pa^2 n^2}.$$

The integration of the equations of motion for the period from 1961 to 1965 has been performed by Cowell's modificated method of quadratures ([3], [4]), taking account of mutual perturbations, and perturbations from Jupiter, Saturn and Uranus. An estimate of the accumulation of errors in the numerical integration shows that these errors are practically zero.

Thus, the discrepancies between the integrated values of the co-ordinates and Newcomb's are the result of a deficiency in the analytical theories of motion of the inner planets. The maximum discrepancies between integrated values of the co-ordinates and Newcomb's in the interval 1961-1965 are shown in the first line of table IV.

It may be seen from this table that the defect of the theory is especially great in the case of Mars.

A similar estimate of accuracy of the new theory of Mars by Clemence ([5]-[7]) yields the following results :

$$10^8\,|\Delta r| < 22, \qquad 10^8\,|\Delta \tau| < 26, \qquad 10^8\,|\Delta s| < 10.$$

Now let us consider the influence of errors in the values of masses adopted by Newcomb. First these errors will cause an error in the semi-major axis and secondly they will affect the perturbations in longitude and in the radius-vector. Perturbations in latitude are very small, owing to the small mutual inclinations of the orbits, and are beyond the limits of accuracy.

Periodic perturbations in longitude and in the radius-vector by planet i are of the form

$$\Delta r_i = m_i \sum_k a_{ik} \cos D_{ik},$$

$$\Delta l_i = m_i \sum_k b_{ik} \cos D'_{ik},$$

where m_i is the mass of the perturbing planet. Therefore we may write

$$(2) \quad \begin{cases} \Delta r \leqq \sum_i A_i \left| \frac{\Delta m_i}{m_i} \right|, \\ \Delta \tau \leqq a(1+e) \sum_i B_i \left| \frac{\Delta m_i}{m_i} \right|, \end{cases}$$

where

$$A_i = m_i \sum_k |a_{ik}|, \qquad B_i = m_i \sum_k |b_{ik}| \sin 1''.$$

On the basis of Newcomb's expansions we easily find the values of A_i and B_i. They are shown in table I.

TABLE I.

Planets	Mercury.		Venus.		Earth.		Mars.	
	$10^6 A_i$.	$10^6 B_i$.	$10^6 A_i$.	$10^6 B_i$.	$10^6 A_i$.	$10^6 B_i$.	$10^6 A_i$	$10^6 B_i$.
1. Mercury......	–	–	0.8	3.8	0.1	0.3	0.0	0.0
2. Venus........	17.7	127.0	–	–	34.3	85.3	12.5	74.7
3. Earth........	2.3	15.0	45.2	176.5	–	–	185.8	423.7
4. Mars.........	0.0	0.0	1.6	10.7	10.0	33.0	–	–
5. Jupiter.......	8.1	36.8	9.0	30.5	34.5	75.6	569.8	724.8
6. Saturn.......	0.0	3.9	0.4	2.4	1.9	4.8	20.0	27.6
7. Uranus.......	–	–	–	–	–	–	0.5	1.0

To estimate the errors in the values of planetary masses adopted by Newcomb, the most reliable values of masses obtained by different authors from independent determinations are summarized in table II.

TABLE II.

Author.	Year.	$1/m$.	Prob. error.	Method of determination.
Mercury.				
1. Clemence [8]............	1949	6 400 000	±250 000	From the motion of the Earth's perihelion
2. Rabe [9]................	1950	6 120 000	± 43 000	From the motion of Eros (1926-1945)

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TABLE II (*continued*).

Author.	Year.	1/*m*.	Prob. error.	Method of determination.
		Mercury.		
3. Brouwer ([10], [11])	1950	6 480 000	±350 000	From secular perturbations of Mercury and Earth
4. Duncombe [12]	1956	5 970 000	±450 000	From the motion of Venus (1750-1949)
5. Makover, Bochan [13] ...	1960	5 980 000	±170 000	From the motion of comet Encke-Backlund (1898-1911)
Weighted mean		6 127 000	± 40 000	
Arithmetic mean		6 190 000	± 70 000	
		Venus.		
1. Fotheringham [14]	1926	406 358	± 723	From the motion of inner planets
2. Spencer Jones [15]	1926	404 700	± 800	From the motion of the Earth (1836-1923)
3. Fotheringham [16]	1935	408 000	± 1 900	From perturbations of the obliquity of the ecliptic
4. Morgan, Scott [17]	1939	407 000	± 500	From the motion of the Earth (1900-1937)
5. Clemence [18]	1943	409 300	± 1 400	From the motion of Mercury (1765-1937)
6. Clemence [8]	1949	408 150	± 200	From the motion of the perihelia of Earth and Mercury
7. Brouwer ([10, [11])	1950	408 000	± 800	From secular perturbations of Mercury and Earth
8. Rabe [9]	1950	408 645	± 208	From the motion of Eros (1926-1945)
Weighted mean		408 120	± 130	
Arithmetic mean		407 570	± 350	
		Earth + Moon.		
1. Noteboom [19]	1921	328 370	± 69	From the motion of Eros (1893-1914)
2. Bosch [20]	1927	327 950	± 200	From the weighted mean values of parallaxes
3. Witt [21]	1933	328 390	± 103	From the motion of Eros (1893-1931)
4. Brouwer [11]	1950	328 427	± 67	From the relation of De Sitter with $p_0 = 8''.7984$
5. Rabe ([9], [22])	1954	328 446	± 43	From the motion of Eros (1926-1945)

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TABLE II (*continued*).

Author.	Year.	1/m.		Prob. error.	Method of determination.
		Saturn.			
2. Gaillot [35]	1913	3 499.9	±	1.18	From the motion of Jupiter (1750-1907)
3. Bosch [20]	1927	3 496	±	3	Weighted mean from satellites
4. Hertz [36]	1953	3 497.64	±	0.27	From the motion of Jupiter (1884-1948)
5. Jeffreys [37]	1954	3 494.8	±	2.0	From the motion of satellites (1924-1937)
6. Clemence [38]	1960	3 499.7	±	0.4	From the motion of Jupiter
Weighted mean		3 498.85	±	0.20	
Arithmetic mean		3 498.38	±	1.12	
		Uranus.			
1. Hill [29]	1898	23 239	±	89	From the motion of Saturn (1751-1888)
2. Bosch [20]	1927	22 530	±	50	From the motion of satellites
3. De Sitter [26]	1938	22 750	±	200	Weighted mean
4. Clemence [8]	1949	22 800	±	300	Weighted mean
5. Harris [11]	1950	22 934	±	6	From the motion of satellites
Weighted mean		22 929	±	6	
Arithmetic mean		22 851	±	78	

Rounded-off probable values of the masses of major planets, finally adopted by us, as well as the relative errors of Newcomb's values of these masses are shown in table III.

TABLE III.

Planets.	Probable values of 1/m.			Newcomb's values of 1/m.	Relative errors. $10^2 \left\| \frac{\Delta m}{m} \right\|$.
Mercury	6 150 000	±	50 000	6 000 000	2.500 ± 0.833
Venus	407 850	±	250	408 000	0.037 ± 0.061
Earth + Moon	328 500	±	40	329 390	0.270 ± 0.013
Mars	3 089 000	±	3 000	3 093 500	0.145 ± 0.097
Jupiter	1 047.40	±	0.02	1 047.355	0.004 ± 0.002
Saturn	3 498.6	±	0.5	3 501.6	0.086 ± 0.014
Uranus	22 900	±	50	22 869	0.136 ± 0.219

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Weighted mean...............		3 498.85	± 0.20	
Arithmetic mean..............		3 498.38	± 1.12	
		Uranus.		
1. Hill [29]...............	1898	23 239	± 89	From the motion of Saturn (1751-1888)
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4. Clemence [8]...........	1949	22 800	± 300	Weighted mean
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According to relation (2) and the numerical values listed in tables I and III, we may easily find the maximum errors in the position of inner planets resulting from errors in the values of the masses.

They are represented in the second line of table IV. Large errors in the position of Mars are due mainly to an error in the Earth's mass.

The effect of errors in the masses upon the semi-major axes, according to Kepler's third law, does not exceed some units of the ninth decimal place, i. e. it amounts practically to zero.

The following relations were used to estimate the effect of errors in the elements of the planetary orbits upon the accuracy of heliocentric positions of the planets [39] :

$$(3)\quad \begin{cases} |\Delta r| \leq a\sqrt{(e\Delta L - e\Delta\varpi)^2 + (\Delta e)^2} + \frac{2}{3}a(1+e)\left|\frac{\Delta n}{n}\right|, \\ |\Delta\tau| \leq a(1+e)\left[\Delta L + 2\sqrt{(e\Delta L - e\Delta\varpi)^2 + (1+e)(\Delta e)^2}\right], \\ |\Delta s| \leq a(1+e)\sqrt{(\Delta i)^2 + (\sin i\,\Delta\Omega)^2}, \qquad \Delta L = \Delta L_0 + T\Delta n. \end{cases}$$

Corrections to the orbital elements in formulae (3) can be found only from a discussion of series of planetary observations.

During 1940-1950 discussions of observations of the Sun (1750-1944), Mercury (1762-1937) and Venus (1750-1949) were made by Morgan [40], Clemence [18] and Duncombe [12] respectively as a result of which corrections were obtained to Newcomb's values of the orbital elements. We shall make use of the new theory of the motion of Mars by Clemence [6] for an estimate of the accuracy of the orbital elements of this planet.

Thus we get the following table of corrections to Newcomb's values of the elements with their probable errors.

Mercury.

$$\begin{aligned} \Delta L_0 &= +0''.23 \pm 0''.08 \\ \Delta n &= +0.08 \pm 0.14 \\ \Delta e &= -0.40 \pm 0.04 \\ e\Delta\varpi &= +0.19 \pm 0.05 + 0''.21\,T \\ \sin i\Delta\Omega &= 0.00 \pm 0.06 + 0.19\,T \\ \Delta i &= +0.06 \pm 0.07 \end{aligned}$$

Venus.

$$\begin{aligned} \Delta L_0 &= +\ 0''.10 \pm 0''.06 \\ \Delta n &= +\ 0.53 \pm 0.18 \\ \Delta e &= -\ 0.12 \pm 0.03 + 0''.01\,T \\ e\Delta\varpi &= +\ 0.01 \pm 0.04 - 0.04\,T \\ \sin i\Delta\Omega &= +\ 0.21 \pm 0.03 + 0.02\,T \\ \Delta i &= +\ 0.08 \pm 0.03 - 0.02\,T \end{aligned}$$

Earth.

$$\begin{aligned} \Delta L_0 &= +0''.03 \pm 0''.05 \\ \Delta n &= -0.11 \pm 0.16 \\ \Delta e &= -0.12 \pm 0.01 \\ e\Delta\varpi &= -0.07 \pm 0.01 - 0''.08\,T \end{aligned}$$

Mars.

$$\begin{aligned} \Delta L_0 &= -\ 5''.50 \pm 0''.11 \\ \Delta n &= +13.22 \pm 0.14 \\ \Delta e &= +\ 0.18 \pm 0.06 \\ e\Delta\varpi &= +\ 0.36 \pm 0.04 \\ \sin i\Delta\Omega &= -\ 0.10 \pm 0.08 \\ \Delta i &= +\ 0.05 \pm 0.07 \end{aligned}$$

TABLE IV.

Sources of error.	Mercury. Δr.	Δz.	Δs.	Venus. Δr.	Δz.	Δs.	Earth. Δr.	Δz.	Δs.	Mars. Δr.	Δz.	Δs.
Systematic errors (in 0.0000 0001 a. u.).												
Analyt. theory..........	41	37	6	33	84	20	27	24	13	264	275	88
Planet. masses..........	2	5	0	15	43	0	4	10	0	55	205	0
Orbital elem............	91	294	32	50	248	89	83	188	0	160	295	90
Ephemeris time.........	0	77	0	0	47	0	0	41	0	0	35	0
TOTAL ERROR............	134	413	38	98	422	109	114	263	13	479	810	178
Fortuitous errors (in 0.0000 0001 a. u.).												
Planet. masses..........	±1	±4	0	±1	±3	0	±3	±7	0	±3	±12	0
Orbital elem...........	±13	±50	±21	±18	±57	±15	±10	±58	0	±46	±180	±89
TOTAL ERROR............	±13	±50	±21	±18	±57	±15	±11	±59	0	±46	±180	±89

The time T is reckoned in Julian centuries from 1900 Jan. 0 Greenwich mean noon. The correction Δn applies to the mean motion in a Julian century. Corrections to the mean longitude and mean motion of the Earth are taken according to Clemence [18]; they agree well with the correction to the mean longitude for the epoch 1940 found by Kulikov [41] from a discussion of observations of the Sun (1925-1953).

An estimate of errors in the planetary co-ordinates for the epoch 1960, resulting from an inaccuracy of the elements, has been made on the basis of the above data and relations (3) (¹). These estimates are shown in the third line of table IV, the probable errors of the estimates being found by means of formulae [39] :

$$
(4) \quad \left\{
\begin{aligned}
|\varepsilon_r| &\leq a\sqrt{e^2\varepsilon_L^2 + e^2\varepsilon_\varpi^2 + \varepsilon_e^2 + \frac{4(1+e)^2}{9n^2}\varepsilon_n^2},\\
|\varepsilon_\tau| &\leq a(1+e)\sqrt{(1+2e)^2\varepsilon_L^2 + 4e^2\varepsilon_\varpi^2 + 4(1+e)^2\varepsilon_e^2},\\
|\varepsilon_s| &\leq a(1+e)\sqrt{\varepsilon_i^2 + \sin^2 i\,\varepsilon_\Omega^2}, \qquad \varepsilon_L^2 = \varepsilon_{L_0}^2 + T^2\varepsilon_n^2,
\end{aligned}
\right.
$$

where ε_j is the probable error of element j.

The fourth line of table IV contains the maximum errors in the position of the planets, resulting from an error in ephemeris time. At present, ephemeris time is known only approximately, and we may assume that the error does not exceed 2 s.

The total effect of all the above sources of error is shown in the fifth line of table IV.

Thus the provisional estimation of accuracy of planetary positions computed strictly in accordance with Newcomb's theory on the one hand, and with Newcomb's theory with the corrections of the elements on the other, may be presented as follows :

Mercury.

Theory and elements of Newcomb.	Theory of Newcomb. Elements of Clemence.
$10^8\,\lvert\Delta r\rvert < 134 \pm 13$	$10^8\,\lvert\Delta r\rvert < 43 \pm 13$
$10^8\,\lvert\Delta \tau\rvert < 413 \pm 50$	$10^8\,\lvert\Delta \tau\rvert < 119 \pm 50$
$10^8\,\lvert\Delta s\rvert < 38 \pm 21$	$10^8\,\lvert\Delta s\rvert < 6 \pm 21$
$10^8\,\lvert\Delta \bar{r}\rvert < 436 \pm 56$	$10^8\,\lvert\Delta \bar{r}\rvert < 127 \pm 56$

Venus.

Theory and elements of Newcomb.	Theory of Newcomb. Elements of Duncombe.
$10^8\,\lvert\Delta r\rvert < 98 \pm 18$	$10^8\,\lvert\Delta r\rvert < 48 \pm 18$
$10^8\,\lvert\Delta \tau\rvert < 422 \pm 57$	$10^8\,\lvert\Delta \tau\rvert < 174 \pm 57$
$10^8\,\lvert\Delta s\rvert < 109 \pm 15$	$10^8\,\lvert\Delta s\rvert < 20 \pm 15$
$10^8\,\lvert\Delta \bar{r}\rvert < 447 \pm 62$	$10^8\,\lvert\Delta \bar{r}\rvert < 182 \pm 62$

(¹) The estimations for Mars are obtained by direct comparison of the co-ordinates from Clemence's [7] theory with those from Newcomb [1].

Earth.

Theory and elements of Newcomb.	Theory of Newcomb. Elements of Morgan, Clemence, Duncombe.
$10^8 \mid \Delta r \mid < 114 \pm 11$	$10^8 \mid \Delta r \mid < 31 \pm 11$
$10^8 \mid \Delta \tau \mid < 263 \pm 59$	$10^8 \mid \Delta \tau \mid < 75 \pm 59$
$10^8 \mid \Delta s \mid < 13 \pm 0$	$10^8 \mid \Delta s \mid < 13 \pm 0$
$10^8 \mid \Delta \bar{r} \mid < 286 \pm 60$	$10^8 \mid \Delta \bar{r} \mid < 82 \pm 60$

Mars.

Theory of Newcomb. Elements of Ross.	Theory and elements of Clemence.
$10^8 \mid \Delta r \mid < 479 \pm 46$	$10^8 \mid \Delta r \mid < 77 \pm 46$
$10^8 \mid \Delta \tau \mid < 810 \pm 180$	$10^8 \mid \Delta \tau \mid < 266 \pm 180$
$10^8 \mid \Delta s \mid < 178 \pm 89$	$10^8 \mid \Delta s \mid < 10 \pm 89$
$10^8 \mid \Delta \bar{r} \mid < 957 \pm 206$	$10^8 \mid \Delta \bar{r} \mid < 277 \pm 206$

It may be seen from these data, that the heliocentric planetary co-ordinates computed in accordance with Newcomb's expansions and constants (with Ross's elements for Mars) are accurate to only five decimal places.

However we must bear in mind, that this estimate of accuracy of planetary co-ordinates is considerably overstated. It may be seen from table IV, that the main source of errors in the planetary positions is the inaccuracy of the orbital elements. Introducing appropriate corrections into the elements we can essentially increase the precision of the ephemerides of Mercury, Venus and Earth. As to Mars, the new theory of its motion by Clemence ([5] to [7]) must be used to improve the accuracy of its ephemeris.

However the most efficient solution of this problem of a higher accuracy of ephemerides would be a complete revision of the theories of motion of the inner planets.

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