A MOLECULAR WEIGHT GRADIENT BATTERY MECHANISM FOR STELLAR MAGNETIC FIELDS

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SUMMARY

Dolginov (1977) pointed out that a non spherical distribution of molecular weight μ can drive a Biermann type "battery" mechanism in a star and so generate a toroidal magnetic field. We consider uniformly rotating axisymmetric models of this type, containing a 'primeval' poloidal field, in which the currents maintaining the toroidal field flow along poloidal field lines, thus ensuring that the configuration is torque free. When the molecular weight distribution is chosen so as to produce a meridional circulation which exactly cancels the Eddington-Sweet circulation, then battery fields of order 1000 gauss can be maintained against ohmic decay in models appropriate to Ap stars near the main sequence with period of order 5 days.

1. INTRODUCTION

Under stellar conditions Ohm's law takes the form

$$\frac{\underline{j}}{\sigma} = \underline{E} + \frac{\underline{v}\times\underline{B}}{c} + \frac{\nabla p_e}{n_e e} - \frac{\underline{j}\times\underline{B}}{cn_e e}, \tag{1}$$

in a standard notation. We also have

$$\frac{\partial \underline{B}}{\partial t} = - c\nabla \times \underline{E}, \tag{2}$$

and the stellar material satisfies the equation of hydrostatic support

$$- \nabla p + \rho\nabla\phi + \rho\Omega^2\underline{s} = \underline{0}\ , \tag{3}$$

where $\Omega = \Omega(\underline{r})$ is the angular velocity, (s, z) are cylindrical polar coordinates, and we have neglected the Lorentz force. In a chemically homogeneous non-magnetic star, from (1) and (2)

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$$\nabla \times \underline{E} = - \nabla \times \left(\frac{\nabla p_e}{n_e e}\right) \propto \nabla \times \Omega^2 \underline{s} \ ,$$

and so $\nabla \times \underline{E} = \underline{0}$ for conservative rotation fields $(\Omega = \Omega(s))$. In this case the centrifugal force is balanced by a Coulomb electric field, maintained by an extremely small charge separation analogous to the "Pannekoek-Rosseland" field. But if $\Omega = \Omega(s, z)$ so that the centrifugal force is not irrotational, then $\nabla \times \underline{E} \neq \underline{0}$, and a toroidal magnetic field may be generated by Faraday's law (2). This was pointed out by Biermann (1950), who estimated that asymptotic fields of order 10^3 gauss could be built up in the sun, and much larger fields $(B_t \propto \Omega^2)$ were possible in rapid rotators. However Mestel and Roxburgh (1962) demonstrated that in the presence of even a weak poloidal field the requirement that the total field be torque free, together with isorotation, effectively 'short circuits' Biermann's battery.

In a chemically inhomogeneous star, even if it rotates uniformly, the battery term $\nabla p_e/n_e e$ may still have a non-zero curl (e.g. Dolginov, 1977). We pursue this idea by constructing axisymmetric torque-free mixed poloidal-toroidal fields, whose toroidal components are maintained against ohmic decay by the battery field generated by non-spherical μ-distributions that might plausibly arise spontaneously during stellar evolution.

2. THE QUASI-STEADY AXISYMMETRIC THEORY

We assume the presence of a "primeval" poloidal field $\underline{B}_p$. In a torque free steady axisymmetric system the currents $\underline{j}_p$ maintaining the battery field $\underline{B}_t$ must flow parallel to $\underline{B}_p$, and the torque free condition can be written (e.g. Lüst & Schlüter, 1954) as

$$\underline{j}_p = K\underline{B}_p, \ \underline{B}_p.\nabla K = 0. \qquad (4)$$

If meridional circulation is neglected so that in equation (1) $\underline{v} = \Omega s\hat{\underline{t}} \times \underline{B}$, Mestel and Roxburgh (1962) showed that to a very good approximation Ω must be constant along poloidal field lines. We specialize by assuming Ω constant throughout the star. Then, integrating the poloidal component of equation (1) around poloidal field lines, assuming that a steady state has been reached, and using equation (4), we obtain

$$\oint \frac{\underline{j}_p.\underline{dr}}{\sigma} = K \oint \frac{\underline{B}_p.\underline{dr}}{\sigma} = \oint \frac{\nabla p_e}{n_e e} . \underline{dr} \ . \qquad (5)$$

Thus the condition for the battery to operate is that the right hand integral in (5) be non zero. Suppose that the star consists wholly of hydrogen and helium, with molecular weight μ, so that

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$n_e = \frac{(\mu+2)}{5\mu}\frac{\rho}{m_H}$. Thus in (5)

$$K \oint \frac{\underline{B}_p \cdot dr}{\sigma} = \frac{m_H}{e} \oint \frac{\nabla((\mu+2)p)}{\left(\frac{\mu+2}{\mu}\right)\rho} \cdot \underline{dr} \; . \qquad (6)$$

Note that if μ, p and ρ are equatorially symmetric, then the battery integral is only non zero if $\underline{B}_p$ has a component of even parity, for example if $\underline{B}_p$ is a quadrupole or decentred dipole field.

Introducing a 'stream function' P for $\underline{B}_p$, so that $\underline{B}_p = -\frac{\nabla P \times \hat{\underline{t}}}{r \sin\theta}$, where P is constant along lines of $\underline{B}_p$, then, using equation (4),

$$\underline{j}_p = \frac{c}{4\pi}\nabla \times \underline{B}_t = \frac{c}{4\pi}\nabla(r \sin\theta \, B_t) \times \frac{\hat{\underline{t}}}{r \sin\theta} = -K\frac{\nabla P \times \hat{\underline{t}}}{r \sin\theta} \; . \qquad (7)$$

Thus $r \sin\theta \, B_t = f(P)$, and so

$$\frac{df}{dP} = -\frac{4\pi}{c} K(P) . \qquad (8)$$

From (6) we can determine $K(P)$ by explicit integration along field lines, and then determine f and so $\underline{B}_t$ from (8), given that $f(0) = 0$. We note now that K will be greatly reduced on field lines which pass through low conductivity regions (equation (6)), and that with $\sigma \propto T^{3/2}$ the value of K on field lines which pass through the stellar surface will be sensitive to the value adopted for the temperature (T_c, say) of the surrounding regions. In the next section we present results for several values of this 'coronal temperature'.

3. ILLUSTRATIVE CALCULATIONS

Firstly we choose our μ distribution by the condition that the associated meridional circulation, $\underline{v}_\mu$ (Mestel 1953), exactly opposes the Eddington-Sweet circulation, $\underline{v}_\Omega$, driven by the stellar rotation throughout the radiative region of the star

$$\text{i.e.} \quad \underline{v}_\Omega + \underline{v}_\mu = \underline{0}.$$

The figures we present refer to a "Cowling model" of mass $2M_\odot$ and radius $1.6\ R_\odot$ with period approximately 10 days, so that the perturbation parameter $\lambda_\Omega = \frac{\Omega^2 R^3}{GM} \simeq 10^{-3}$. As the theory is linear the toroidal field scales as λ_Ω. The poloidal field $\underline{B}_p$ is that of the slowest dipole decay eigenmode, decentred by fractional amount d/R along the rotation axis. The salient results for the model are presented in Table I for 3 values of d/R and 3 choices of T_c.

$|B_t, (P_{max})|$ is the value of the toroidal field in gauss at the neutral line of the poloidal field, and $|B_{t, max, surface}|$ is the value where the first interior poloidal field line grazes the stellar surface.

Table I

d/R	T_c (^oK)	$\|B_t(P_{max})\|$	$\|B_{t,max,surface}\|$
0.05	10^4	175	0.3
	10^5	184	2.8
	10^6	192	8.5
0.10	10^4	316	0.6
	10^5	327	6.5
	10^6	348	14.0
0.20	10^4	419	0.9
	10^5	438	9.2
	10^6	473	25.0

Table I demonstrates the sensitivity of the surface field to the value of T_c, but the relative insensitivity of the interior field. Basically this arises from the small value of K (equation (6)) on field lines passing outside of the start when T_c is low. This results in f(P) remaining small until P corresponds to a field line totally within the star - see equation (7).

We studied this molecular weight distribution feeling that it might be an approximation to what happens as a rotating star evolves from the zero age main sequence (e.g. Mestel, 1981). More probably this asymptotic state will be approached slowly, and so we also investigated a model in which the molecular weight gradient was only present in the inner 65% by radius. Again the molecular weight distribution was determined by the condition that the total circulation, $\underline{v}_\Omega + \underline{v}_\mu$, be zero in this region. The resulting values of $|\underline{B}_t|$ were about 75% of those given in Table I. This agrees with the qualitative argument that most of the battery effect arises from high conductivity regions where $\nabla\mu \neq \underline{0}$. Provided that most field lines continue to pass through such a region, as they do here since the neutral line is at a fractional radius of about 0.45, the battery effect is not much reduced.

Clearly other sources of chemical inhomogeneity can be considered, such as the surface helium anomaly spots of Dolginov (1977). Trial calculations for plausible parameters again suggest that toroidal fields of around 1000 gauss are attainable, possibly with significantly larger surface fields than those of Table I. The extension of the theory outlined above to an oblique rotator model is difficult in detail, but estimates suggest that the asymptotic battery field is of a comparable order of magnitude to that found in the case of aligned rotational and poloidal magnetic axes.

4. CONCLUSIONS

It has been shown that the toroidal component of a torque-free, mixed poloidal -toroidal field can be maintained against decay in a uniformly rotating star with plausible non-spherical μ-distributions. The figures presented are for a modest value of λ_Ω , and larger fields are possible for shorter periods. Analogous processes may be occurring in the surface regions of chemically peculiar stars (Dolginov, 1977). Toroidal fields are necessary for the dynamical stability of poloidal fields (and v.v.). The potential observability of surface toroidal fields (Barker et al, 1981), and their possible importance for element diffusion (Alecian & Vauclair, 1981) are additional motivation for studying their generation and maintenance.

REFERENCES

Alecian, G., Vauclair, S.: 1981, Astron.Astrophys. 101, p.16.
Barker, P.L., Landstreet, J.D., Marlborough, J.M., Thompson, I., and Maza, J.: 1981, Astrophys. J. 250, p.300.
Biermann, L.: 1950, Zs. f. Naturforsch. 5a, p.65.
Dolginov, A.Z.: 1977, Astron.Astrophys. 54, p.17.
Lust, R., and Schlüter, A.: 1954, Zs. f. Astrophys. 34, p.263.
Mestel, L.: 1953, Mon. Not. R. astr. Soc. 113, p.716.
Mestel, L.: 1981, In "Fundamental Problems in the Theory of Stellar Evolution", IAU Symposium 93.
Mestel, L., and Roxburgh, I.W.: 1962, Astrophys. J. 136, p.615.

DISCUSSION

KUPERUS: About 16 years ago I made a similar calculation on battery generation of toroidal magnetic fields in early-type stars. I found using the best available stellar models present at that time a peak value of a few hundred Gauss around the A type stars, which actually is the dominant spectral type of magnetic stars. The influence of ionization of H and He as well as the rotation was taken into account. It was published in *Bulletin of the Astronomical Institutes of the Netherlands* 1966 (P.S. The journal merged into *Astronomy and Astrophysics*).

MOSS: Did these models take account of the torque-free condition? Did they use a molecular weight gradient to drive the battery?

KUPERUS: No.

SPICER: How does the thermal electric field, which is proportional to the electron temperature gradient, compare with the electric field due to the electron pressure gradient?

IONSON: I agree with Spicer that you should include the thermoelectric field, $E_{thermo} = 0.7\nabla_{||}T_e/e$, which can be comparable to the electron pressure electric field, $E_{\nabla_{||}P_e} = \nabla_{||}P_e/me$.

MOSS: This term does not arise in the form of Ohm's law resulting from the mean free path approximation, which was used in the analysis presented. In any case it is a pure gradient term, and so can only give rise to a Coulomb electric field. As shown, this would not contribute to the battery mechanism.

ROXBURGH: Did you take a molecular weight distribution that cancelled the rotational circulation at all points in the star — or just in the central regions?

MOSS: Firstly we took a molecular weight gradient such that $v_\Omega + v_\mu = 0$ everywhere. This gave the fields in Table I. Another calculation with non-zero molecular weight gradients and $v_\Omega + v_\mu = 0$ in the inner 60% or so by radius gave fields about 0.75 of those of Table I.