Contents

Preface to the second eatton Preface to the first edition List of Symbols		pug
PAR	T I COMBINATORIAL ENUMERATION	
Intro	oduction	
1.1	Generating functions and asymptotics	
1.2	New multivariate methods	
1.3	Outline of the remaining chapters	
Gen	erating functions	
2.1	Power series	
2.2	Rational operations on generating functions	
2.3	Algebraic generating functions	
2.4	D-finite generating functions and diagonals	
2.5	Labeled classes	
Univ	variate asymptotics	
3.1	An explicit formula for rational functions	
3.2	Meromorphic asymptotics	
3.3	Darboux's method	
3.4	Transfer theorems	
3.5	The saddle point method	

	PART II MATHEMATICAL BACKGROUND	87	
4	Fourier–Laplace integrals in one variable		
	4.1 Real integrands	91	

1

2

3

Contents

97

Complex phase

	4.3	Analytic versus smooth functions	105
5	Mult	ivariate Fourier–Laplace integrals	114
	5.1	Overview	114
	5.2	Standard phase	118
	5.3	Real part of phase has a strict minimum	121
	5.4	General nondegenerate phase with finite critical set	125
	5.5	Higher order terms in the expansions	129
6	Laurent series, amoebas, and convex geometry		134
	6.1	Laurent series	135
	6.2	Polynomial amoebas	141
	6.3	Convex cones and exponential bounds	147
	6.4	Singularities, amoeba boundaries, and minimal points	153
	6.5	Additional constructions	162
	PAR'	T III MULTIVARIATE ENUMERATION	167
7	Over	view of analytic methods for multivariate GFs	169
	7.1	Some illustrative examples	171
	7.2	The smooth case	182
	7.3	The general case via stratified Morse theory	196
	7.4	Geometry	206
	7.5	Deformations	214
8	Effec	tive computations and ACSV	221
	8.1	Techniques for polynomial systems	222
	8.2	Computing critical points	226
	8.3	Verifying minimal points	235
	8.4	Further computations for asymptotics	238
9	Smoo	oth point asymptotics	245
	9.1	Finitely minimal points and the surgery method	253
	9.2	The method of residue forms	257
	9.3	Smooth bivariate functions	262
	9.4	Additional formulae for asymptotics	279
10	Mult	iple point asymptotics	291
	10.1	A taxonomy of multiple points	293
	10.2	Main results on integrals	299
	10.3	Main results on coefficient asymptotics	302
	10.4	Classifying multiple points	322

viii

4.2

		Contents	ix
	10.5	Surgery, non-generic directions, and non-arrangement	
		points	327
11	Cone	point asymptotics	339
	11.1	Results on cones and deformations	341
	11.2	Proof of Theorem 11.1	347
	11.3	Evaluating asymptotics	361
	11.4	Examples and consequences	368
12	Com	pinatorial applications	381
	12.1	Some classifications	381
	12.2	Powers, quasi-powers, and Riordan arrays	385
	12.3	Lagrange inversion	390
	12.4	Transfer matrices	393
	12.5	Higher order asymptotics	399
	12.6	Algebraic generating functions	402
	12.7	Additional worked examples	404
	12.8	Limit laws from probability theory	412
13	Chall	enges and extensions	424
	13.1	Contributing singularities and diagonals	424
	13.2	Phase transitions	426
	13.3	Degenerate phase	426
	13.4	Critical points at infinity	429
	13.5	Algebraic GFs	430
	13.6	Asymptotic formulae	432
	13.7	Symmetric functions	433
	13.8	Conclusion	435
Appe	endix A	Integration on manifolds	438
	A.1	Manifolds	438
	A.2	Vector fields and differential forms	443
	A.3	Integration of forms	452
	A.4	Complex manifolds and differential forms in \mathbb{C}^n	456
Appe	endix B	Algebraic topology	462
	B .1	Chain complexes and homology theory	462
	B.2	Tools for homology	468
	B.3	Cohomology	475
	B.4	Topology of complex manifolds	477
Appe	endix C	Residue forms and classical Morse theory	480
	C.1	Intersection classes	481
	C.2	Residue forms and the residue integral theorem	485

Contents

C.3 Classical Morse theory		499
C.4	Description at the level of homology	505
Appendix D	Stratification and stratified Morse theory	513
D.1	Whitney stratified spaces	513
D.2	Critical points and the fundamental lemma	518
D.3	Description of the attachments	523
D.4	Stratified Morse theory for complex manifolds	527
References Author Index		533
		553
Subje	559	