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BIMODULES FOR CUNTZ–KRIEGER ALGEBRAS OF INFINITE MATRICES

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We show that any Cuntz–Krieger algebra $\mathcal{O}_A$ over an infinite 0–1 matrix A may be realised as a Cuntz–Pimsner algebra $\mathcal{O}_X$ for a Hilbert bimodule X over a suitable Abelian C^*-algebra with totally disconnected spectrum. Using Pimsner's six-term exact sequence for the KK-groups we calculate the K-groups of $\mathcal{O}_A$. We also give a description of the corresponding Toeplitz algebra $\mathcal{T}_X$ in terms of generators and relations.

0.

Exel and Laca [3] have recently constructed and described a class of C^*-algebras corresponding to infinite 0–1 matrices, which generalises the classical work of Cuntz and Krieger [2]. Exel and Laca's primary approach is based on crossed products for partial actions. However, they also recognise that the algebras are universal for a suitable set of relations imposed on the generating families of partial isometries. On the other hand, Pimsner [11] defined a large class of C^*-algebras based on Hilbert bimodules. We refer to these as to Cuntz–Pimsner algebras. Pimsner showed, in particular, that his construction encompasses both Cuntz–Krieger algebras (of finite 0–1 matrices) and crossed products by the integers.

It is the purpose of this note to show that the Cuntz–Krieger algebras of infinite matrices constructed by Exel and Laca fall into the general framework of Cuntz–Pimsner algebras. Namely, for any infinite 0–1 matrix A without zero rows we construct a Hilbert bimodule X such that the C^*-algebras $\mathcal{O}_A$ and $\mathcal{O}_X$ are canonically isomorphic. The bimodule X is over an Abelian C^*-algebra with totally disconnected spectrum. As a consequence of this result we may now use general methods developed for Cuntz–Pimsner algebras to investigate Cuntz–Krieger algebras. This in particular applies to the exact sequences for the KK-groups of $\mathcal{O}_X$ found in [11], which in special cases yield the K-groups of Cuntz–Krieger algebras of infinite matrices, calculated earlier by Exel and Laca [4] in a different way. One may also try to get in this way an insight into the structure of ideals. To the author's best knowledge none of the presently available results in this

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direction, for example, [**7, 10**], covers all possible cases resulting from Cuntz–Krieger algebras built on infinite matrices. However, a necessary and sufficient condition for their simplicity has been already found in [**13**]. It does not seem unreasonable to expect that this and some other results that have been obtained for Cuntz–Krieger algebras by other methods may now help develop analogous techniques applicable to a more general class of Cuntz–Pimsner algebras.

Before Exel and Laca came up with their extension of the Cuntz and Krieger theory to infinite matrices, a very satisfactory generalisation of Cuntz–Krieger algebras was given in terms of directed graphs, see, for example, [**9, 1**]. It was shown in [**5**] that if E is a directed graph without sinks or sources then the corresponding algebra $C^*(E)$ may be realised as a Cuntz–Krieger algebra $\mathcal{O}_A$, for a suitable infinite 0–1 matrix A, as well as a Cuntz–Pimsner algebra $\mathcal{O}_X$ for a suitable Hilbert bimodule X. The bimodule is over an Abelian C^*-algebra with discrete spectrum. For locally finite graphs without sinks or sources an analogous result was obtained in [**8**].

1.

In what follows, we always assume that N is an at most countable, nonempty set and $A = \big[A(i,j)\big]_{i,j\in N}$ is a matrix with entries in $\{0,1\}$, no row of which is identically zero. Exel and Laca define in [**3**, Theorem 8.6] a Cuntz–Krieger algebra $\mathcal{O}_A$ corresponding to A as the universal C^*-algebra generated by a family of partial isometries $\{S_i \mid i \in N\}$, subject to the following relations:

(EL1) $S_i^*S_i$ and $S_j^*S_j$ commute for any $i,j \in N$,

(EL2) $S_i^*S_j = 0$ for any $i \neq j$ in N,

(EL3) $(S_i^*S_i)S_j = A(i,j)S_j$ for any $i,j \in N$,

(EL4) For any finite subsets K, L of N such that

$$A(K,L,j) \overset{\mathrm{def}}{=} \prod_{k\in K} A(k,j) \prod_{l\in L}\big(I - A(l,j)\big)$$

is nonzero for only finitely many $j \in N$, we have

$$\prod_{k\in K} S_k^*S_k \prod_{l\in L} (I - S_l^*S_l) = \sum_{j\in N} A(K,L,j)S_jS_j^*.$$

Above, I is the identity of the multiplier algebra of $\mathcal{O}_A$. This definition generalises that of Cuntz and Krieger [**2**] for finite matrices. In what follows for finite subsets K, L of N we denote

$$P_{K,L} = \prod_{k\in K} S_k^*S_k \prod_{l\in L} (I - S_l^*S_l),$$

and if $A(K,L,j)$ is nonzero for only finitely many $j \in N$ then we write $(K,L) \in C_A$.

2.

We denote by D the C^*-subalgebra of $\mathcal{O}_A$ generated by the domain projections $S_i^*S_i$ with $i \in N$, of all the generators of $\mathcal{O}_A$. It follows from (EL1) that D is a separable Abelian C^*-algebra with totally disconnected spectrum. D may or may not have a unit. We denote by D_0 its dense $*$-subalgebra generated algebraically by $\{S_i^*S_i \mid i \in N\}$. If $0 \neq a \in D_0$ then there exist finite collections K_ν, L_ν of finite subsets of N such that

$$a = \sum_\nu \lambda_\nu P_{K_\nu,L_\nu}, \tag{1}$$

where the λ_ν are nonzero complex numbers, $K_\nu \neq \emptyset$, and P_{K_ν,L_ν} are nonzero, mutually orthogonal projections. We denote by D_{00} the ideal of D_0 generated by all projections $P_{K,L}$ with $(K,L) \in C_A$. Any element of D_{00} may be written in the form (1) with $(K_\nu, L_\nu) \in C_A$ for all ν. We define

$$X = \overline{\operatorname{span}}\{S_i a \mid i \in N, a \in D\}.$$

X has a natural structure of a D-bimodule, with both actions given by the multiplication in $\mathcal{O}_A$. This is obvious for the right action. For the left action, this follows from the fact that aS_ib is either a scalar multiple of S_ib or 0 for any $i \in N$ and $a, b \in D$, by virtue of (EL3). Furthermore, X carries a natural structure of a right Hilbert D-bimodule [**12**], with the D-valued inner product given by $\langle x, y\rangle_D = x^*y$. This inner product is well-defined by virtue of (EL2). It follows from the definitions that X is full, that is, the closure of $\langle X, X\rangle_D$ is the entire algebra D. The homomorphism $\phi : D \to \mathcal{L}(X_D)$, determined by the left action of D on X, is injective. Indeed, suppose that $0 \neq a \in D_0$ and write $a = \sum_\nu \lambda_\nu P_{K_\nu,L_\nu}$, as in (1). Let μ be such that $|\lambda_\mu| = \|a\|$. It follows from (EL4) that there exists $i \in N$ such that $P_{K_\mu,L_\nu}S_i = S_i$ and, hence, $\|aS_i\| = |\lambda_\mu S_i| = \|a\|$. Consequently, ϕ is isometric on a dense subalgebra, and thus injective. We denote by $\mathcal{K}(X_D)$ the space $\overline{\operatorname{span}}\{\theta_{x,y} \mid x, y \in X\}$, with $\theta_{x,y} : X \to X$ defined as $\theta_{x,y}(z) = x\langle y, z\rangle_D$ for $z \in X$. Later we shall need the following.

Lemma 3. *If X is a Hilbert D-bimodule as in Section 2 above, then D_{00} is a dense $*$-subalgebra of $\phi^{-1}\big(\mathcal{K}(X_D)\big)$.*

Proof: To this end we show that for any $a \in D$

$$\big\|\phi(a) - \mathcal{K}(X_D)\big\| = \|a - D_{00}\|. \tag{2}$$

If $(K,L) \in C_A$, then $P_{K,L} = \sum_{j\in N} A(K,L,j)S_jS_j^*$ by (EL4) and, hence, $\phi(P_{K,L}) = \sum_{j\in N} A(K,L,j)\theta_{S_j,S_j}$ belongs to $\mathcal{K}(X_D)$. Thus $\phi(D_{00}) \subseteq \mathcal{K}(X_D)$ and, consequently,

$$\big\|\phi(a) - \mathcal{K}(X_D)\big\| \leqslant \big\|\phi(a) - \phi(D_{00})\big\| \leqslant \|a - D_{00}\|.$$

It sufficies to prove the reverse inequality for $a \in D_0$. So suppose that $a \in D_0 \setminus D_{00}$ is equal to $\sum_\nu \lambda_\nu P_{K_\nu,L_\nu}$, as in (1). Let $|\lambda_{\nu_0}| = \max\{|\lambda_\nu| \mid (K_\nu, L_\nu) \notin C_A\}$ for some ν_0 such that $(K_{\nu_0}, L_{\nu_0}) \notin C_A$, and let $i_\mu, j_\mu \in N$, $t_\mu \in \mathcal{C}$, and $b_\mu, d_\mu \in D$ for some finite set of indices μ. Since $P_{K_{\nu_0},L_{\nu_0}} S_k = S_k$ for infinitely many $k \in N$, there exists one such $k \in N$ different from all indices j_μ. We have

$$\begin{aligned}\Big\|\phi(a) - \sum_\mu t_\mu \theta_{S_{i_\mu} b_\mu, S_{j_\mu} d_\mu}\Big\| &\geqslant \Big\|\Big(\phi(a) - \sum_\mu t_\mu \theta_{S_{i_\mu} b_\mu, S_{j_\mu} d_\mu}\Big) S_k\Big\| \\ &= \Big\|\sum_\nu \lambda_\nu P_{K_\nu,L_\nu} S_k - \sum_\mu t_\mu S_{i_\mu} b_\mu d_\mu^* S_{j_\mu}^* S_k\Big\| \\ &= |\lambda_{\nu_0}| \geqslant \|a - D_{00}\|.\end{aligned}$$

Thus $\big\|\phi(a) - \mathcal{K}(X_D)\big\| \geqslant \|a - D_{00}\|$, as required. ∎

4.

We now recall the definitions of the Toeplitz algebra $\mathcal{T}_X$ and the Cuntz–Pimsner algebra $\mathcal{O}_X$ associated with the D-bimodule X [**11**]. We take the universal property approach of [**6**]. Thus, a Toeplitz representation (ψ, π) of the bimodule X in a C^*-algebra B consists of a linear map $\psi : X \to B$ and a C^*-algebra homomorphism $\pi : D \to B$ such that

$$\psi(xa) = \psi(x)\pi(a), \tag{3}$$

$$\psi(ax) = \pi(a)\psi(x), \tag{4}$$

$$\psi(x)^*\psi(y) = \pi(\langle x, y\rangle_D), \tag{5}$$

for any $x, y \in X$ and $a \in D$. Then $\mathcal{T}_X$ is a C^*-algebra generated by copies $\imath_X(X)$ of X and $\imath_D(D)$ of D, and universal for Toeplitz representations of X [**6**, Proposition 1.3]. That is, for any (ψ, π) satisfying (3) to (5), there exists a C^*-homomorphism $\psi \times \pi : \mathcal{T}_X \to B$ such that $\psi \times \pi\big(\imath_X(x)\big) = \psi(x)$ and $\psi \times \pi\big(\imath_D(a)\big) = \pi(a)$ for any $x \in X$ and $a \in D$. This Toeplitz algebra exists and is unique up to a canonical isomorphism. There exists a C^*-algebra homomorphism $\kappa : \mathcal{K}(X_D) \to \mathcal{T}_X$ such that $\kappa(\theta_{x,y}) = \imath_X(x)\imath_X(y)^*$. Let $\mathcal{J}_X$ be the closed 2-sided ideal of $\mathcal{T}_X$ generated by $\big\{\imath_D(a) - \kappa\big(\phi(a)\big) \mid a \in \phi^{-1}\big(\mathcal{K}(X_D)\big)\big\}$. The Cuntz–Pimsner algebra $\mathcal{O}_X$ of the bimodule X may be defined as the quotient $\mathcal{T}_X/\mathcal{J}_X$. We point out that we only consider Hilbert bimodules with injective left actions, as they suffice for our purposes (see Section 2).

Theorem 5. *If $A = \big[A(i,j)\big]_{i,j \in N}$ is a 0–1 matrix with no zero rows, D is the C^*-subalgebra of $\mathcal{O}_A$ generated by $\{S_i^* S_i \mid i \in N\}$, and $X = \overline{\mathrm{span}}\{S_i a \mid i \in N, a \in D\}$, as in Section 2, then the Cuntz–Krieger algebra $\mathcal{O}_A$ and the Cuntz–Pimsner algebra $\mathcal{O}_X$ are canonically isomorphic.*

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PROOF: Since the natural inclusions of X and D into $\mathcal{O}_A$ give a Toeplitz representation of X there exists a homomorphim $\tilde{f} : \mathcal{T}_X \to \mathcal{O}_A$ such that $\tilde{f}\big(\imath_X(x)\big) = x$ and $\tilde{f}\big(\imath_D(a)\big) = a$ for any $x \in X$, $a \in D$. It follows from (EL4) that $\imath_D(a) - \kappa\big(\phi(a)\big) \in \ker \tilde{f}$ for any $a \in D_{00}$. Thus Lemma 3 implies that $\mathcal{J}_X \subseteq \ker \tilde{f}$ and, consequently, $\tilde{f}$ induces a homomorphism $f : \mathcal{O}_X \to \mathcal{O}_A$. Since the family $\big\{\imath_X(S_i) + \mathcal{J}_X \bigm| i \in N\big\} \subseteq \mathcal{O}_X$ satisfies (EL1) to (EL4), there exists a homomorphism $g : \mathcal{O}_A \to \mathcal{O}_X$ such that $g\big(\imath_X(S_i)\big) = \tilde{S}_i + \mathcal{J}_X$ for any $i \in N$. Since, clearly, both $fg = \mathrm{id}$ and $gf = \mathrm{id}$, the C^*-algebras $\mathcal{O}_A$ and $\mathcal{O}_X$ are isomorphic, as claimed. ∎

6.

Theorem 5 allows application of general techniques developed for Cuntz–Pimsner algebras to the case of Cuntz–Krieger algebras corresponding to infinite matrices. In particular, one can apply to $\mathcal{O}_A$ the six-term exact sequences for KK-groups found by Pimsner [**11**]. As a special case of these we obtain an exact sequence for the K-groups. Namely, [**11**, Theorem 4.9], natural isomorphisms $KK_*^{\mathrm{nuc}}(\mathbb{C}, M) \cong K_*(M)$ for any C^*-algebra M, $K_1(M) = 0$ for any AF-algebra M, Lemma 3 and Theorem 5 yield an exact sequence

$$0 \to K_1(\mathcal{O}_A) \to K_0\big(\overline{D}_{00}\big) \xrightarrow{\beta_X} K_0(D) \to K_0(\mathcal{O}_A) \to 0, \tag{6}$$

with $\beta_X = \otimes\big(\mathrm{id} - [X]\big)$. Consequently, we obtain a description of the K-groups of $\mathcal{O}_A$ as

$$K_0(\mathcal{O}_A) \cong \operatorname{coker} \beta_X,$$
$$K_1(\mathcal{O}_A) \cong \ker \beta_X.$$

Of course, it is not difficult to write the map β_X more explicitly in terms of the matrix A. Namely, we can identify $K_0(D)$ with the subring of $\mathbf{Z}^N$ generated by the rows of A (see [4]), and $K_0\big(\overline{D}_{00}\big)$ with the subring of $K_0(D)$ consisting of finitely supported elements of $\mathbf{Z}^N$. Then β_X is identified with the map $\mathrm{id} - A^t$. This gives a different description of the K-groups of $\mathcal{O}_A$ from the one produced by Exel and Laca in [4]. Namely, our map is defined by the same formula as in [4], but between smaller Abelian groups.

7.

In Theorem 5 we used a bimodule X which was defined in Section 2 with help of some elements from the C^*-algebra $\mathcal{O}_A$. In fact it is not difficult to construct D and X directly from the matrix A. Namely, let $A = \big[A(i,j)\big]_{i,j \in N}$ be a 0–1 matrix with no zero rows. Let D_A be any C^*-algebra generated by a family $\{P_i \mid i \in N\}$ of commuting projections such that for any finite subsets K, L of N

$$A(K, L, j) = 0,\ \forall j \in N \ \Leftrightarrow\ \prod_{k \in K} P_k \prod_{l \in L} (I - P_l) = 0. \tag{7}$$

In the above formula we take I to be the identity of the multiplier algebra of D_A. If D is the C^*-subalgebra of $\mathcal{O}_A$ generated by $\{S_i^* S_i \mid i \in N\}$, as defined in Section 2, then it is not difficult to verify that there exists a C^*-algebra isomorphism $D_A \to D$ such that $P_i \mapsto S_i^* S_i$ for any $i \in N$. Thus, in particular, D_A is canonically determined by the matrix A.

We denote by $\widetilde{N}$ a complex vector space with a basis $\{\tilde{i} \mid i \in N\}$. $\widetilde{N} \otimes D_A$ is a right D_A-module with the action $\left(\tilde{i} \otimes a\right)b = \tilde{i} \otimes ab$ for $i \in N$, $a, b \in D_A$. We define a possibly degenerate D_A-valued inner product on $\widetilde{N} \otimes D_A$ by $\langle \tilde{i} \otimes a, \tilde{j} \otimes b \rangle_{D_A} = \delta_{i,j} a^* P_i b$ for $i, j \in N$, $a, b \in D_A$. We define $\widetilde{X}_A$ as the quotient of $\widetilde{N} \otimes D_A$ by its submodule (by virtue of the Cauchy–Schwarz inequality [**12**, Lemma 2.5]) $\left\{x \in \widetilde{N} \otimes D_A \,\middle|\, \langle x, x \rangle_{D_A} = 0\right\}$. $\widetilde{X}_A$ is naturally a right inner product D_A-module [**12**, Definition 2.1]. We denote by X_A the completion of $\widetilde{X}_A$. For $i \in N$, $a \in D_A$ we denote by $i \otimes a$ the canonical image of $\tilde{i} \otimes a$ in X_A. For any $i \in N$ there is a projection $Q_i \in \mathcal{L}(X_A)$ such that $Q_i(j \otimes a) = A(i,j) j \otimes a$ for any $j \in N$, $a \in D_A$. Since the projections $\{Q_i \mid i \in N\}$ commute and satisfy (7) there exists a homomorphism $\phi_A : D_A \to \mathcal{L}(X_A)$ such that $\phi_A(P_i) = Q_i$ for any $i \in N$. With the left action of D_A given by ϕ_A, X_A becomes a Hilbert D_A-bimodule.

PROPOSITION 8. *Let $A = \left[A(i,j)\right]_{i,j \in N}$ be a 0–1 matrix with no zero rows, and let X and X_A be the corresponding Hilbert D and D_A bimodules, respectively, as defined in Sections 2 and 7. With the canonical identification of D with D_A, as in Section 7, the Hilbert bimodules X and X_A are unitarily equivalent.*

PROOF: We identify D_A with D, as in Section 7, so that $P_i = S_i^* S_i$ for any $i \in N$. We define a map $U : \widetilde{X}_A \to X$ by

$$U\left(\sum_\nu \lambda_\nu i_\nu \otimes a_\nu\right) = \sum_\nu \lambda_\nu S_{i_\nu} a_\nu$$

with $\lambda_\nu \in \mathcal{C}$, $i_\nu \in N$, $a_\nu \in D_A$ for some finite set of indices ν. We have

$$\begin{aligned}
\Big\| \sum_\nu \lambda_\nu i_\nu \otimes a_\nu \Big\|^2 &= \Big\| \sum_{\nu,\mu} \overline{\lambda}_\nu \lambda_\mu \langle i_\nu \otimes a_\nu, i_\mu \otimes a_\mu \rangle_{D_A} \Big\| \\
&= \Big\| \sum_\nu |\lambda_\nu|^2 a_\nu^* P_{i_\nu} a_\nu \Big\| \\
&= \Big\| \sum_{\nu,\mu} \overline{\lambda}_\nu \lambda_\mu \langle S_{i_\nu} a_\nu, S_{i_\mu} a_\mu \rangle_D \Big\| \\
&= \Big\| \sum_\nu \lambda_\nu S_{i_\nu} a_\nu \Big\|^2
\end{aligned}$$

which shows that U is a well-defined isometry. Thus, U extends to an isometry from X_A into X, still denoted U. It is clear that U is a surjective map commuting with the right actions of $D_A \cong D$. Furthermore, for any $i, j \in N$, $a \in D_A$ we have

$$U\left(P_i(j \otimes a)\right) = A(i,j) U(j \otimes a) = A(i,j) S_j a = S_i^* S_i U(j \otimes a)$$

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and, hence, U commutes with the left actions as well. ∎

As shown in [11], Proposition 8 implies that the C^*-algebras $\mathcal{O}_{X_A}$ and $\mathcal{O}_X$ are canonically isomorphic.

9.

Let X be the Hilbert bimodule associated with a matrix A, as in Section 2, and let $\mathcal{T}_X$ be the corresponding Toeplitz algebra [11, 6]. Similarly to $\mathcal{O}_X \cong \mathcal{O}_A$, $\mathcal{T}_X$ admits a definition in terms of generators and relations. Indeed, let $\{T_i \mid i \in N\}$ be a family of partial isometries. We say that this family satisfies condition (T) if for any finite subsets K, L of N

$$\text{(T)} \qquad A(K,L,j) = 0,\ \forall j \in N \ \Rightarrow \prod_{k\in K} T_k^* T_k \prod_{l \in L} (I - T_l^* T_l) = 0.$$

This is precisely condition (1.5) from [5].

THEOREM 10. *Let $A = \left[A(i,j)\right]_{i,j\in N}$ be a 0–1 matrix with no zero rows, and let X be the corresponding Hilbert bimodule, as constructed in Section 2. Then the Toeplitz algebra $\mathcal{T}_X$ is naturally isomorphic to a universal C^*-algebra $\mathcal{T}$ generated by partial isometries $\{T_i \mid i \in N\}$ satisfying (EL1) to (EL3) and (T).*

PROOF: A standard argument, involving a direct sum of suitable cyclic representations, shows that such a universal C^*-algebra $\mathcal{T}$ indeed exists. Since the partial isometries $\left\{\imath_X(S_i) \mid i \in N\right\}$ satisfy (EL1) to (EL3) and (T) (see Section 7), there exists a homomorphism $f : \mathcal{T} \to \mathcal{T}_X$ such that $f(T_i) = \imath_X(S_i)$ for any $i \in N$. This in particular shows that projections $\{T_i^* T_i \mid i \in N\}$ satisfy (7) and, thus, they generate a C^*-subalgebra of $\mathcal{T}$ naturally isomorphic to D. Hence, there is a homomorphism $\pi : D \to \mathcal{T}$ such that $\pi(S_i^* S_i) = T_i^* T_i$ for any $i \in N$. For λ_ν, $i_\nu \in N$, $a_\nu \in D$, indexed by a finite set, we define

$$\psi\left(\sum_\nu \lambda_\nu S_{i_\nu} a_\nu\right) = \sum_\nu \lambda_\nu T_{i_\nu} \pi(a_\nu).$$

A short calculation similar to the one from the proof of Proposition 8 shows that this is an isometry and, hence, extends to a map $\psi : \mathcal{T}_X \to \mathcal{T}$ such that $\psi(S_i) = T_i$ for any $i \in N$. Clearly, (ψ, π) is a Toeplitz representation of the bimodule X inside $\mathcal{T}$. Indeed, the very definition of ψ implies (3), (4) follows from (EL3), and (5) follows from (EL2). Considering the corresponding homomorphism $\psi \times \pi : \mathcal{T}_X \to \mathcal{T}$ we see that $f(\psi \times \pi) = \mathrm{id}$ and $(\psi \times \pi) f = \mathrm{id}$, which completes the proof. ∎

In view of Theorem 10 it would only be natural to denote $\mathcal{T}_X$ by $\mathcal{T}_A$ and call it the Toeplitz algebra associated with the matrix A. However, this notation has been already reserved in [3] for a different object. Namely, Exel and Laca define the Toeplitz algebra $\mathcal{T}_A$ as the universal C^*-algebra for (EL1) to (EL3).

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REFERENCES

[1] T. Bates, D. Pask, I. Raeburn and W. Szymański, 'The C^*-algebras of row-finite graphs', (preprint).

[2] J. Cuntz and W. Krieger, 'A class of C^*-algebras and topological Markov chains', *Invent. Math.* **56** (1980), 251–268.

[3] R. Exel and M. Laca, 'Cuntz–Krieger algebras for infinite matrices', *J. reine angew. Math.* **512** (1999), 119–172.

[4] R. Exel and M. Laca, 'The K-theory of Cuntz–Krieger algebras for infinite matrices, K-Theory' (to appear).

[5] N.J. Fowler, M. Laca and I. Raeburn, 'The C^*-algebras of infinite graphs', *Proc. Amer. Math. Soc.* (to appear).

[6] N.J. Fowler and I. Raeburn, 'The Toeplitz algebra of a Hilbert bimodule', *Indiana Univ. Math. J.* **48** (1999), 155–181.

[7] T. Kajiwara, C. Pinzari and Y. Watatani, 'Ideal structure and simplicity of the C^*-algebras generated by Hilbert bimodules', *J. Funct. Anal.* **159** (1998), 295–322.

[8] T. Kajiwara, C. Pinzari and Y. Watatani, 'Hilbert C^*-bimodules and countably generated Cuntz–Krieger algebras', (preprint).

[9] A. Kumjian, D. Pask, I. Raeburn and J. Renault, 'Graphs, groupoids, and Cuntz–Krieger algebras', *J. Funct. Anal.* **144** (1997), 505–541.

[10] P.S. Muhly and B. Solel, 'On the simplicity of some Cuntz–Pimsner algebras', *Math. Scand.* **83** (1998), 53–73.

[11] M. Pimsner, 'A class of C^*-algebras generalizing both Cuntz–Krieger algebras and crossed products by **Z**', *Fields Inst. Commun.* **12** (1997), 189–212.

[12] I. Raeburn and D.P. Williams, *Morita equivalence and continuous-trace C^*-algebras*, Math. Surveys and Monographs **60** (Amer. Math. Soc., Providence, R.I., 1998).

[13] W. Szymański, 'Simplicity of Cuntz–Krieger algebras of infinite matrices', (preprint).

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