CORRESPONDENCE.

ON MR. G. F. HARDY'S PLAN FOR THE COMPUTATION OF MR. WOOLHOUSE'S METHOD OF GRADUATION.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—Having recently been led to give some attention to the different methods which have from time to time been proposed for applying Mr. Woolhouse's formula to the actual graduation of a table of mortality, I have carefully perused the account of the discussion upon Mr. J. A. Higham's paper on this subject, read before the

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Institute on 24 April 1882, and have discovered one or two misprints in the report of Mr. G. F. Hardy's speech, which I think it may be as well to point out for the benefit of other readers. On page 352 of the *Journal*, vol. xxiii, line 10 from the top, the last term in that line should be $14u_9$, in place of $14u_7$; and again, four lines lower down, the last two terms of the formula there given should be $-2u_{12}-3u_{13}$, in place of $-2u_{11}-3u_{12}$.

I have also discovered two errors* in the table given by Mr. Ackland in his paper on the same subject, which immediately follows Mr. Higham's. On page 355 of the same volume of the *Journal*, the figures opposite ages 72 and 77 respectively, in col. (5), should be 30,697 and 28,915, in place of 30,597 and 29,015. It follows that, in col. (7), the numbers should be 41,911 and 39,310; in col. (10), 31,294 and 29,554; and in col. (11), 2,503·5 and 2,364·3. The sums of the columns will not be altered, as the errors balance one another. It was in the course of working out Mr. G. F. Hardy's method of applying Mr. Woolhouse's formula to the H^M Table, that I was led to the discovery of these discrepancies in Mr. Ackland's results. I append the entire working by that method, from which it will be seen that columns (7), (8), and (9), agree exactly, from age 17 onwards, with Mr. Ackland's columns (5), (10), and (11). I have also added, in column (11), the numbers-living as they appear in Mr. Woolhouse's final adjustment of the table; and a comparison of this column with the previous one will at once show the amount of the further adjustments made by him. It will thus be seen that at age 70 the difference is as much as 12. At age 77 it is 10, and at a number of other points adjustments of a smaller amount have been made. The various differences of a single unit may be disregarded, being evidently due in some way to the decimals. So far as I can gather from Mr. Woolhouse's remarks on page xci of the *Institute of Actuaries' Life Tables*, he seems to have differenced the numbers-living, as derived directly from his formula, as far as second differences, and then, noticing the points where any marked irregularities occurred, removed them by the application of the rule given in his paper on "Interpolation", in the *Journal*, vol. xii, p. 140. To illustrate the application of this rule, I have differenced the numbers-living as given in col. (10), and the result is shown in the following table:—

* These errors were pointed out by Mr. J. A. Higham, in a note to his paper "On the Adjustment of Mortality Tables", p. 44, *ante*.—Ed.

x	l_x	Δl_x	$\Delta^2 l_x$	x	l_x	Δl_x	$\Delta^2 l_x$
17	97,624			57	63,649		
		− 379				1,524	
18	97,245		− 87	58	62,125		68
		466				1,592	
19	96,779		90	59	60,533		75
		556				1,667	
20	96,223		53	60	58,866		80
		609				1,747	
21	95,614		34	61	57,119		82
		643				1,829	
22	94,971		7	62	55,290		88
		650				1,917	
23	94,321		+ 12	63	53,373		84
		638				2,001	
24	93,683		16	64	51,372		77
		622				2,078	
25	93,061		5	65	49,294		58
		617				2,136	
26	92,444		− 1	66	47,158		62
		618				2,198	
27	91,826		10	67	44,960		45
		628				2,243	
28	91,198		32	68	42,717		31
		660				2,274	
29	90,538		13	69	40,443		33
		673				2,307	
30	89,865		21	70	38,136		76
		694				2,383	
31	89,171		12	71	35,753		43
		706				2,426	
32	88,465		15	72	33,327		78
		721				2,504	
33	87,744		2	73	30,823		49
		723				2,553	
34	87,021		17	74	28,270		26
		740				2,579	
35	86,281		15	75	25,691		+ 52
		755				2,527	
36	85,526		26	76	23,164		53
		781				2,474	
37	84,745		21	77	20,690		109
		802				2,365	
38	83,943		20	78	18,325		110
		822				2,255	
39	83,121		15	79	16,070		120
		837				2,135	
40	82,284		12	80	13,935		115
		849				2,020	
41	81,435		3	81	11,915		137
		852				1,883	
42	80,583		14	82	10,032		164
		866				1,719	
43	79,717		21	83	8,313		175
		887				1,544	
44	78,830		24	84	6,769		197
		911				1,347	
45	77,919		39	85	5,422		209
		950				1,138	
46	76,969		46	86	4,284		198
		996				940	
47	75,973		45	87	3,344		167
		1,041				773	
48	74,932		39	88	2,571		157
		1,080				616	
49	73,852		46	89	1,955		121
		1,126				495	
50	72,726		34	90	1,460		87
		1,160				408	
51	71,566		33	91	1,052		80
		1,193				328	
52	70,373		42	92	724		74
		1,235				254	
53	69,138		52	93	470		58
		1,287				196	
54	67,851		51	94	274		54
		1,338				142	
55	66,513		61	95	132		59
		1,399				83	
56	65,114		66	96	49		43
		1,465				40	
			59	97	9		

From the above, it will be seen that at several points there are marked irregularities in the progression of the differences, and at these points Mr. Woolhouse made a further adjustment. To take an example :—The three second differences of which that opposite age 57 is the central one, are −66, −59, and −68. The rule in question requires the first and third of these numbers to be separately subtracted from the second, and the two remainders added together; then one-sixth of the sum will be the correction to be applied to the number-living. In this case we shall have 7+9=16, which divided by 6 gives +3 as the correction; and this, being added to the number derived from

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the formula, as per column (10), will be found to give exactly the number in the H^{M} Table, as per col. (11). At some of the other points in the table, however, the application of this rule gives a result differing by a unit in excess or defect from the correction which Mr. Woolhouse has actually made use of; and at certain ages, such as at 70 and 72, the effect of the several corrections gets involved in a manner which renders it difficult to discover the exact method by which Mr. Woolhouse produced his final results.

Yours, &c.,

D. CARMENT.

Sydney, N.S.W.,
17 May 1883.

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(1) x	(2) l_x Unadjusted.	(3) $\alpha = l_x - l_{x+5}$	(4) $\beta = \alpha_x - \alpha_{x+5}$	(5) $\gamma = \beta_x - \beta_{x+5}$	(6) $\Sigma\gamma$	(7) $\Sigma^2\gamma$	(8) $\Sigma^2\gamma - 3\gamma_{x-1}$	(9) Col. (8) $\div$ 12·5 $= d_{x+6}$	(10) $\Sigma d_{x+6} = l_{x+6}$	(11) l_{x+6} Woolhouse.	(12) $x+6$
10	10,000	194	+ 4	+133	− 569	+ 4,564	...	...	...	...	...
11	9,921	137	− 87	0	702	5,133	+ 4,734	+ 378·72	97,624	97,624	17
12	9,921	137	154	−137	702	5,835	5,835	466·80	97,245	97,245	18
13	9,881	138	171	171	565	6,537	6,948	555·84	96,779	96,779	19
14	9,846	162	161	177	394	7,102	7,615	609·20	96,223	96,223	20
15	9,806	190	129	138	217	7,496	8,027	642·16	95,614	95,614	21
16	9,784	224	87	62	79	7,713	8,127	650·16	94,971	94,971	22
17	9,784	291	17	+ 12	17	7,792	7,978	638·24	94,321	94,321	23
18	9,743	309	0	42	29	7,809	7,773	621·84	93,683	93,683	24
19	9,684	323	+ 16	62	71	7,838	7,712	616·96	93,061	93,061	25
20	9,616	319	9	61	133	7,909	7,723	617·84	92,444	92,444	26
21	9,560	311	− 25	− 2	194	8,042	7,859	628·72	91,826	91,826	27
22	9,493	308	29	+ 3	192	8,236	8,242	659·36	91,198	91,192	28
23	9,434	309	42	− 17	195	8,428	8,419	673·52	90,538	90,538	29
24	9,361	307	46	9	178	8,623	8,674	693·92	89,865	89,865	30
25	9,297	310	52	12	169	8,801	8,828	706·24	89,171	89,171	31
26	9,249	336	23	+ 30	157	8,970	9,006	720·48	88,465	88,465	32
27	9,185	337	32	21	187	9,127	9,037	722·96	87,744	87,748	33
28	9,125	351	25	27	208	9,314	9,251	740·08	87,021	87,021	34
29	9,054	353	37	− 2	235	9,522	9,441	755·28	86,281	86,281	35
30	8,987	362	40	12	233	9,757	9,763	781·04	85,526	85,524	36
31	8,913	359	53	19	221	9,990	10,026	802·08	84,745	84,745	37
32	8,848	369	53	18	202	10,211	10,268	821·44	83,943	83,943	38
33	8,774	376	52	3	184	10,413	10,467	837·36	83,121	83,122	39
34	8,701	390	35	+ 39	181	10,597	10,606	848·48	82,284	82,284	40
35	8,625	402	28	61	220	10,778	10,661	852·88	81,435	81,436	41
36	8,554	412	34	62	281	10,998	10,815	865·20	80,583	80,582	42
37	8,479	422	35	78	343	11,279	11,093	887·44	79,717	79,717	43
38	8,398	428	49	57	421	11,622	11,388	911·04	78,830	78,830	44
39	8,311	425	74	23	478	12,043	11,872	949·76	77,919	77,919	45
40	8,223	430	89	4	501	12,521	12,452	996·16	76,969	76,969	46
41	8,142	446	96	7	505	13,022	13,010	1040·80	75,973	75,973	47
42	8,057	457	113	− 12	512	13,527	13,506	1080·48	74,932	74,932	48
43	7,970	477	106	+ 14	500	14,039	14,075	1126·00	73,852	73,850	49
44	7,886	499	97	47	514	14,539	14,497	1159·76	72,726	72,726	50

(1) x	(2) l_x Unadjusted.	(3) $\alpha = l_x - l_{x+5}$	(4) $\beta = \alpha_x - \alpha_{x+5}$	(5) $\gamma = \beta_x - \beta_{x+5}$	(6) $\Sigma\gamma$	(7) $\Sigma^2\gamma$	(8) $\Sigma^2\gamma - 3\gamma_{x-1}$	(9) Col. (8) $\div 12{\cdot}5 = d_{x+6}$	(10) $\Sigma d_{x+6} = l_{x+6}$	(11) l_{x+6} Woolhouse.	(12) $x+6$
45	7,793	519	93	59	561	15,053	14,912	1192·96	71,566	71,566	51
46	7,696	542	103	47	620	15,614	15,437	1234·96	70,373	70,373	52
47	7,600	570	101	59	667	16,234	16,093	1287·44	69,138	69,138	53
48	7,493	583	120	47	726	16,901	16,724	1337·92	67,851	67,852	54
49	7,387	596	144	30	773	17,627	17,486	1398·88	66,513	66,513	55
50	7,274	612	152	51	803	18,400	18,310	1464·80	65,114	65,114	56
51	7,154	645	150	53	854	19,203	19,050	1524·00	63,649	63,652	57
52	7,030	671	160	41	907	20,057	19,898	1591·84	62,125	62,125	58
53	6,910	703	167	24	948	20,964	20,841	1667·28	60,533	60,533	59
54	6,791	740	174	10	972	21,912	21,840	1747·20	58,866	58,866	60
55	6,662	764	203	− 32	982	22,884	22,854	1828·32	57,119	57,119	61
56	6,509	795	203	65	950	23,866	23,962	1916·96	55,290	55,289	62
57	6,359	831	201	95	885	24,816	25,011	2000·88	53,373	53,374	63
58	6,207	870	191	66	790	25,701	25,986	2078·88	51,372	51,373	64
59	6,051	914	184	90	724	26,491	26,689	2135·12	49,294	49,297	65
60	5,898	967	171	63	634	27,215	27,485	2198·80	47,158	47,156	66
61	5,714	998	138	0	571	27,849	28,038	2243·04	44,960	44,960	67
62	5,528	1,032	106	+ 53	571	28,420	28,420	2273·60	42,717	42,717	68
63	5,337	1,061	125	− 58	624	28,991	28,832	2306·56	40,443	40,443	69
64	5,137	1,098	94	50	566	29,615	29,789	2383·12	38,136	38,124	70
65	4,931	1,138	108	199	516	30,181	30,331	2426 48	35,753	35,753	71
66	4,716	1,136	138	302	317	30,697	31,294	2503·52	33,327	33,320	72
67	4,496	1,138	159	400	15	31,014	31,920	2553·60	30,823	30,823	73
68	4,276	1,186	67	315	+ 385	31,029	32,229	2578·32	28,270	28,269	74
69	4,039	1,192	44	329	700	30,644	31,589	2527·12	25,691	25,691	75
70	3,793	1,246	+ 91	213	1,029	29,944	30,931	2474·48	23,164	23,164	76
71	3,580	1,274	164	174	1,242	28,915	29,554	2364·32	20,690	20,700	77
72	3,358	1,297	241	142	1,416	27,673	28,195	2255·60	18,325	18,326	78
73	3,090	1,253	248	185	1,558	26,257	26,683	2134·64	16,070	16,068	79
74	2,847	1,236	285	192	1,743	24,699	25,254	2020·32	13,935	13,930	80
75	2,547	1,155	304	156	1,935	22,956	23,532	1882·56	11,915	11,915	81
76	2,306	1,110	338	126	2,091	21,021	21,489	1719·12	10,032	10,032	82
77	2,061	1,056	383	38	2,217	18,930	19,308	1544·64	8,313	8,313	83
78	1,837	1,005	433	+ 77	2,255	16,713	16,827	1346·16	6,769	6,768	84
79	1,611	951	477	174	2,178	14,458	14,227	1138·16	5,422	5,422	85
80	1,392	851	460	204	2,004	12,280	11,758	940·64	4,284	4,284	86
81	1,196	772	464	262	1,800	10,276	9,664	773·12	3,344	3,343	87
82	1,005	673	421	249	1,538	8,476	7,690	615·20	2,571	2,570	88
83	832	572	356	184	1,289	6,938	6,191	495·28	1,955	1,955	89
84	660	474	303	147	1,105	5,649	5,097	407·76	1,460	1,460	90
85	541	391	256	136	958	4,544	4,103	328·24	1,052	1,052	91
86	424	308	202	106	822	3,586	3,178	254·24	724	723	92
87	332	252	172	92	716	2,764	2,446	195·68	470	469	93
88	260	216	172	128	624	2,048	1,772	141·76	274	274	94
89	186	171	156	141	496	1,424	1,040	83·20	132	135	95
90	150	135	120	105	355	928	505	40·40	49	49	96
91	116	106	96	86	250	573	258	20·64	9	9	97
92	80	80	80	80	164	323	65	5·20	...	...	...
93	44	44	44	44	84	159	− 81	− 6·48	...	...	...
94	15	15	15	15	40	75	57	4·56	...	...	...
95	15	15	15	15	25	35	10	·80	...	...	...
96	10	10	10	10	10	10	35	2·80	...	...	...
97	...	...	...	...	...	...	30	2·40	...	...	...

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