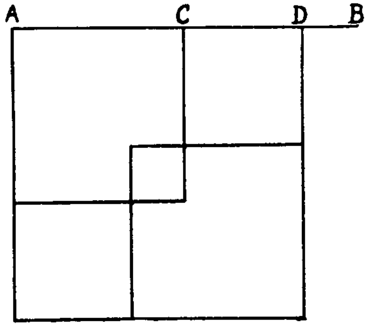
For the figure shows that each side is made up of the whole area plus the square in the centre.



An exactly similar figure gives the case of external section at D.

[This note is taken from a letter dated 4th May 1916 from Professor R. J. T. Bell, who remarks "I got it in some L.C. papers and it was new to me and seemed very neat."—Ed.]

A Link Slide Rule for the Mechanical Solution of Quadratic Equations.

[Note.—An earlier model of this slide rule was exhibited at a meeting of the Society in 1914. This note was held over during the war, and the instrument described now is an improved design.]

The instrument here described can be applied mechanically :

(a) to solve a numerical quadratic equation in the form
$$x^2 - ax + b = 0.$$

(b) to evaluate $x^2 - ax$ for assigned values of x and a and in particular to find its minimum value.

(c) to read the square or square root of a number.

(d) to multiply or divide two numbers.

(e) to evaluate the function $x + \dfrac{b}{x}$.

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1. *Principle and Description of Instrument.*

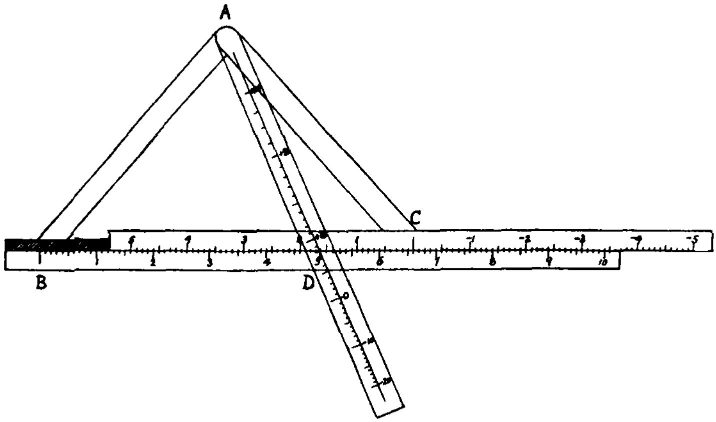
If AB, AC are the equal sides of an isosceles triangle and AD any line through A meeting the base in D, it is easy to show that

$$BD \,.\, DC = AB^2 - AD^2 \quad\quad\quad (1)$$

for any position of D if the signs of the segments BD, DC are taken into account. Putting x for BD, a for BC, p for AB, r for AD we obtain

$$x^2 - ax + b = 0$$

where
$$b = p^2 - r^2.$$



The diagram shows the isosceles triangle in the form of two equal links AB, AC pivoted at A, the base BC being formed by the edges of two slides capable of sliding along each other, the upper being connected at the back to the link AC and the lower to link AB. The slides dovetail into each other to prevent lateral separation. The line AD is embodied in a celluloid scale of indefinite length, pivoted at A, the connection with the links being by a collar which keeps the scale in the plane of the faces of the slides. A winged nut at A acts as a clamp. The slides are graduated equally with zeros at C and B respectively, and the celluloid scale is graduated according to the low

$$\text{distance from } A = \sqrt{p^2 - b},$$

and the corresponding value of b is marked at the point of graduation. Thus the zero of this scale is at distance p from A, points between A and the zero have b positive, points on the side of the zero remote from A have b negative.

2. *Method of Use.*

(a) The diagram shows the zero C of the upper slide set to $a = 6 \cdot 6$ on the lower slide and the celluloid scale rotated so that the graduation $b = +8$ falls on the common edge of the slides. The readings $+5 \cdot 00$, $+1 \cdot 60$ give the roots of

$$x^2 - 6 \cdot 6\, x + 8 = 0.$$

Hence the rule: set C opposite a on the lower slide and bring the point b of the celluloid scale on to the base line. The roots are then read off on the slides at that point.

(b) To evaluate $x^2 - ax$ set C to a, rotate AD to position x on the lower slide. The reading on the celluloid scale then gives $x(a-x)$.

(c) To read n^2 set C to $2n$ and read the celluloid scale in the position perpendicular to BC. To read $\sqrt{n}$ bring n on the celluloid scale into coincidence with BC in the perpendicular position and read the root on either slide. The coincidence of these values renders the reading very accurate.

(d) To multiply α by β first add the numbers and set C to the sum. Rotate AD to $x = \alpha$ and read the product on the celluloid scale. When the links are wide apart results comparable with those of a 10-inch slide rule are obtainable, but the design is uot adapted to accurate or speedy multiplication as a rule. Division is performed by setting the dividend b on AD against the divisor α on BC, and reading opposite C, which gives $\alpha + \beta$. Then subtract α.

(e) Values of $x + \dfrac{b}{x}$. Since the quadratic may be written in the form

$$a = x + \frac{b}{x}$$

we may regard a as a function of x and b; hence the rule mechanically evaluates this function at C, when assigned values of x and b are set together as in division.

3. *Accuracy of Results.*

Out of 20 trials at random the error per cent. of the roots was less than 0·1 in 6 cases, between 0·1 and 0·5 in 8 cases, between 0·5 and 1·0 in 3 cases, and above 1·0 in 3 cases. Obviously

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the instrument cannot yield good results when the roots are very near each other, especially if the links are close together. Change of unit will obviate this difficulty in most cases. For example, $x^2 - 0{\cdot}54\,x + {\cdot}06 = 0$ may be solved either by setting to

$$a = 0{\cdot}54,\ b = 0{\cdot}06 \text{ or to } a = 5{\cdot}4,\ b = 6.$$

In the former case the links are close together, and we read roots 0·13, 0·37; in the latter case the links are open, and we read 0·1564, 0·3834 with a greatly enhanced accuracy.

In constructing the instrument the graduation of the celluloid scale should be checked at every integral value by applying the multiplication test given in (*d*). This ensures the elimination of any systematic error due to faulty pivoting.

G. D. C. Stokes.

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