4.4 ON THE SOLAR MODEL AND THE PRECESSION OF THE EQUINOXES IN THE ALPHONSINE ZĪJ AND ITS ARABIC SOURCES

Julio Samsó
University of Barcelona, Barcelona, Spain

Alphonse X, King of Castille (1252-1284), sponsored astronomical work previous to the compilation of the Alphonsine Tables, carried out by his two Jewish collaborators Yehudah ben Mosheh and Isaac ben Sid. On the one hand, these tables can be considered the first European attempt to develop original research in astronomy, but on the other hand, an analysis of this work should be concerned, first of all, with its Islamic precedents. This is why I tend to consider the Alphonsine Tables as another zīj which should be studied in the light of what we know concerning the development of Islamic zījes in Medieval Spain. Of these, two seem to be well documented as having been known and used by the Alphonsine collaborators. One of them is al-Battānī's zīj, the canons of which were translated into Spanish (Bossong, 1978), and which may have been used to compute the solar positions for the end of each month appearing in four Alphonsine works (Astrolabio redondo, Cuadrante para rectificer, Relogio dell agua, and Lámina Universal). These positions have been computed, however, with the Alphonsine Tables themselves. The second zīj known in the Alphonsine circle was al-Zarqalluh's Toledan Tables, used -- as O. Gingerich has established -- to compute the solar and planetary positions in the horoscope which was cast to establish the propitious moment to start the Latin translation of Ibn Abī-l-Rijāl's Kitāb al-bāri^c fī aḥkām al-nujūm (Hilty, 1954, lxii-lxiii).

The two aforementioned zījes might have influenced the Alphonsine work. The Toledan Tables could have been the model for the first draft of the Alphonsine zīj, of which we know only the Spanish canons (Rico IV, 119-183), for in this latter text it is clearly established that the tables were used to compute sidereal longitudes. On the other hand, al-Battānī's zīj might have caused a change of attitude in the Alphonsine collaborators as reflected in the Latin numerical tables, the known canons of which seem to be the work of Parisian astronomers of the fourteenth century, used to compute tropical longitudes (Poulle, 1984).

The prologue to the Spanish canons establishes clearly that observations were made during the period 1263-1272. In spite of this fact it is easy to see that most of the planetary parameters used derive either from Ptolemy or from al-Battānī. The solar parameters seem more interesting however, and this should be related to the insistence of the Spanish canons on solar observations made during more than one year using both the Ptolemaic method (passage of the sun through the equinoxes and solstices) and the method established by the astronomers of the Caliph al-Maʾmūn (passage of the sun through the mid-points

https://doi.org/10.1017/S0252921100106013 Published online by Cambridge University Press

between equinoxes and solstices). These Alphonsine observations may have been the origin, at least in part, of the solar parameters appearing in the *Alphonsine Tables* or in other Alphonsine sources. I will review them briefly

1. *Tropical year*: there is no need to insist on this new parameter (365 days 5;49,15,58,58,56,38,24 hours), for Price (1955,*a* and *b*, pp. 104-107) clearly established its origin.

2. *Solar equation*: 2;10° is the maximum solar equation in the Alphonsine *zīj* and the characteristics of the table imply that it was computed using Ptolemaic methods and not according to the old Indian solutions by sines and by declinations. Attempts to recalculate the table do not, however, give good results: only two values correspond exactly to the recomputed ones, and in fourteen instances the error amounts only to 1". In the rest of the table the error is greater, with a maximum of 30". The Alphonsine astronomers do not seem to have done very good work here.

The Alphonsine maximum solar equation (2;10°) is an original parameter which may have resulted from observation. Jean de Murs, however, seems to consider that the aforementioned parameter is only an adaptation of the value used by al-Zarqālluh in the *Toledan Tables* (1;59,10°) due to the fact that the latter tables give sidereal longitudes, while the longitudes computed with the Latin *Alphonsine Tables* are tropical. (Poulle, 1980, 262-263). I cannot see much sense in Jean de Murs' argument, but I do not feel too sure about the originality of the Alphonsine parameter, for it does not differ too much from others we find in the Hindu-Iranian astronomical tradition: 2;13°, and 2;14° in the *Zīj al-Shāh* (Kennedy, 1958, p. 259) as well as 2;14° in al-Khwārizmī's *zīj* (Neugebauer, 1962 *a*, pp. 95-96); on the other hand we also know that, in the 8th c., al-Fazārī used 2;11,15° and 2;14° and al-Bīrūnī ascribes to the *Sindhind* tradition the use of 2;10,46,40° a parameter that might be related to the 2;10,31° used in the *Paitāmahasiddhānta* (Pingree, 1970, pp. 110-114 and 1968, pp. 103-104). We could, therefore, have here a revival of an old Indian parameter.

3. *Solar apogee, precession, and trepidation*: The *Alphonsine Tables* give us a set of solar and planetary apogees which do not seem to derive from any of the tables in use in Medieval Spain. They should, therefore, be considered original unless the contrary can be proved and this, of course, applies specially to the solar apogee. This set of longitudes has, however, a peculiar characteristic for they are neither sidereal nor tropical. It is easy to prove that they incorporate the constant term of precession based on a revolution of 360° in 49 000 years which is equivalent to a precession of about 26.45" per year. Therefore, if we want to know the actual tropical longitude of the solar apogee for the Alphonsine era (midday of 31.05.1252) we should add to the longitude given in the table of *Radices* (Poulle, 1984, p. 124) the amount corresponding to the trepidational term of precession according to the Alphonsine model:

Radix	80;37,66°
Trepidation	8;03,06°
Solar apogee	88;40,06°

We should remark here that the *editio princeps* (1483) of the *Tables* gives the value of the trepidational term for each one of the different eras used, and that it states that the corresponding value for the Alphonsine era amounts to

https://doi.org/10.1017/S0252921100106013 Published online by Cambridge University Press

8;04,01^{0}. This small mistake will acquire a certain interest later on.

The trepidation theory used in the Latin version of the Alphonsine Tables has been studied by Delambre (1819), Dreyer (1920), Price (1955, a and b, p. 104-107) Dobrzycki (1965), and Mercier (1967-77) and does not need to be explained here. It is enough to say that it uses exactly the same model we find in the Liber de motu octave spere traditionally ascribed to Thābit ibn Qurra. It does not use, however, the same parameters: the period of revolution of the head of Aries along the small equatorial epicycle is 7 000 years (in the Liber de motu, approximately 4 077 Julian years: see Neugebauer, 1962, b, p. 297) and the maximum value of the equation in the Tabula equationum motus accessus et recessus octave sphere amounts to 9^{0} (10;45^{0} in the Liber de motu). No more information can be gathered directly from the Alphonsine Tables, but it can be obtained otherwise.

It is a well known fact since Goldstein (1964) that the approximation 10;45^{0} sin i does not give good results for recalculating the values of the equation of trepidation in the Liber de motu. The same thing happens with the Tabula equationum of the Alphonsine Tables if we use 9 sin i. A much better approximation method was suggested by Dobrzycki (1965 p. 23) with sin 9^{0} sin i, and results which are almost as good can be obtained if we use another approximate formula, given by Mercier (1976 p. 205):

$$\sin \Delta\lambda = \sin i \ \frac{\mathrm{tg}\ r}{\sin \epsilon_0}$$

Mercier's formula has the advantage of allowing us, here, to obtain two new parameters; after a certain number of attempts I have deduced that the best results in the recomputation of the table will be obtained using 23;33^{0} for the mean obliquity of the ecliptic (ϵ_0 , the same parameter as in the Liber de motu) and 3;34,35^{0} for the value of r, the radius of the small equatorial epicycle (4;18,43^{0} in the Liber de motu). These two parameters will be useful, later on, to consider the problem of the obliquity of the ecliptic according to the Alphonsine trepidation model.

Dreyer (1920, p. 247), Price (1955 b, pp. 104-107), and Mercier (1976-77) have underlined the fact that the Alphonsine precession-trepidation model gives good results for the value of precession in the second half of the thirteenth century. I would like, once again, to insist on this point: we have already seen that the longitude of the solar apogee at the beginning of the Alphonsine era (midday of 31.05.1252) is 88;40,06^{0}. It will be easy to compute the position of the solar apogee for the 16.05.16 A.D., the date at which the value of the equation of trepidation was 0^{0} as Mercier (1977, p. 59) has established (not the 18.05.15 A.D. as Dreyer, 1920, p. 247 pretended). The solar apogee for this latter date will be at 71;32,10^{0}. Therefore, in a period of time of, practically, 1236 years the solar apogee has advanced 17;07,56^{0}, a value which corresponds to the increase of longitude due to precession between the two aforementioned dates; this gives us an excellent mean value of precession which amounts to 49.90" per year.

17;07,56^{0} can, on the other hand, be rounded to 17;08^{0} which is precisely the amount by which the longitudes of stars in Ptolemy's Almagest (VII,5) have been increased in the four Libros de la Ochava Espera as well as in the star catalogue of the Alphonsine Tables. No convincing explanation has been given for the use of this parameter, and I would like to suggest here a new hypothesis: The Alphonsine astronomers confused the first year of the reign of Antoninus (137 A.D.), which is the radix date of Ptolemy's catalogue of stars, with the first year of the reign of Tiberius (which, in fact, started in year 14 A.D.) for the Alphonsine Primera Crónica General de España (ed. Menendez Pidal, 1977 I,111) states that at the beginning of the reign of this latter Emperor took place in year 16 A.D. If this hypothesis could be proved to be true, it would definitely support the Alphonsine character of the precession-trepidation model used in the Alphonsine Tables. Let us also remember that the combination of variable trepidation with constant precession seems to continue a traditional idea in Muslim Spain: in the eleventh century Ṣāᶜid al-Andalusī (Blachère, 1935, p.86) and al-Zarqālluh (Millás, 1943-50, pp. 275-276), as well as al-Biṭrūjī in the twelfth century (Goldstein, 1971 I, 89-91 and II, 173-181), when they trace the history of the precession/trepidation theories, state that Theon of Alexandria (4th c. A.D.) had already combined precession and trepidation.

We should mention, finally, that the Alphonsine astronomers not only succeeded in obtaining a good value of precession for their time, but that their model gave good approximations to a certain number of historical determinations of the longitude of the solar apogee. I will only mention here three positions of the solar apogee which are quoted in Ibn Yūnus' Ḥākimī zīj: for year 450 A.D. this source gives 77;55^{o} (Kennedy-van der Waerden, 1963 p. 325) and we obtain 78;08,32^{o} with the Alphonsine Tables (difference -0;13,32^{o}); for year 632 (beginning of the Yazdijird era) Ibn Yūnus gives 80;44,19^{o}, while we can compute 80;46,28^{o} with the Alphonsine trepidation model (the difference amounts only to -0;02,19^{o}) (Caussin, 1804, p. 134). The third instance, however interesting, is less impressive: it corresponds to Ibn Yūnus determination of the longitude of the apogee in 1004 (Caussin, 1804, p. 216) which amounts to 86;10^{o} while we obtain 85;45,35^{o} with the Alphonsine model (difference $\pm$ 0;24,25^{o}). All this leads me to the suspicion that the Alphonsine astronomers had a copy of Ibn Yūnus' zīj and that they used its historical data to adjust the results obtained with their precession/trepidation model.

4 Obliquity of the ecliptic: One year, at least, of solar observations should, in principle, imply a determination of the value of the obliquity of the ecliptic. In a most striking way, however, the three Latin editions of the Alphonsine Tables which I have been able to examine (1483, 1524, 1553) do not contain a table of the solar declinations, whilst the table of right ascensions seems to have been copied from the Toledan Tables (Toomer, 1968,pp.34-35) or from al-Battānī's zīj (Nallino 1907, II, 61-64) which used an obliquity of 23;35^{o}. On the other hand, the Castillian canons (Rico IV, 153) mention the parameter 23;33^{o}, which is the one used by al-Zarqālluh, and allude to a table of solar declinations (Rico IV, 136 and 179-180).

All this seems most discouraging until one realizes that, in another Alphonsine treatise, the Cuadrante para rectificar its author, Isaac ben Sid mentions an unreported value of the obliquity (23;32,29^{o}) together with two values of the solar declination which are perfectly compatible with the aforementioned parameter. In fact, there is a complete table of solar declinations which has been preserved in another Alphonsine work whose author is also Isaac ben Sid, the

https://doi.org/10.1017/S0252921100106013 Published online by Cambridge University Press

Libro del relogio de la piedra de la sombra (Rico IV, 6). Here the obliquity is 23;32,30° (an obvious rounding of 23;32,29°) add the values of the solar declinations, apart from three obvious mistakes, are, in general well computed. On the other hand the text states that the table is based on observations made "in our time". Therefore it seems that we have here an original parameter which is extremely precise for Newcomb's formula gives us an obliquity of 23;32,11° for 1262 (approximately the beginning of Alphonsine observations) and 23;32,04° for 1277 (the date in which the Cuadrante para rectificar was written). We may face here the first determination of the obliquity of the ecliptic made in Christian Europe: A little later, in 1290, Guillaume de Saint Cloud obtained a much worse value of 23;34° (Poulle, 1980, 261-262). On the other hand we may have here a possible explanation for the origin of the parameter 23;32° which Copernicus in the De Revolutionibus ascribes to Prophatius Iudaeus although it is not used in his Almanach (Swerdlow-Neugebauer, 1984, 133).

The above-mentioned parameter for the obliquity of the ecliptic (23;32,30°) seems to be known only through one other source: Prof. E. S. Kennedy tells me that it appears in an Arabic zīj preserved in Arabic manuscript 6040 of the Bibliothèque Nationale de Paris. The author's name is Abū Muḥammad ᶜAṭāᵓ b. Aḥmad... Ġāzī al-Samarqandī al-Sanjufīnī, and the date of the manuscript is 17.12.1366. It was written in Ho-chou (modern Han-Chia), in the modern Kansu province, on the upper reaches of the Yellow River. I cannot give now any convincing reason for such coincidence although the hypothesis brought forward by my master Professor J. Vernet, (1984) on possible contacts between King Alphonse and the astronomers of the Marāgha school should be considered here for further study. In any case, I would not like to finish this paper without paying some attention to the fact that the Alphonsine parameter for the obliquity of the ecliptic cannot be the result of computation with the three well-known trepidation models in use in Spain: I am referring to the trepidation models in the Liber de motu, in al-Zarqālluh's Treatise on the movement of the Fixed Stars (Millás, 1943-50, pp. 239-243; Goldstein, 1964; Samsó, 1985, u) and in the Alphonsine Tables themselves.

It is easy to disregard entirely al-Zarqālluh's trepidation model for, as Goldstein clearly explained, it gives values of the obliquity of the ecliptic comprised between 23;33° and 23;53°. The Alphonsine parameter is, therefore, below the minimum zarqāllian value. The values of the obliquity of the ecliptic ε implied in the Liber de motu are not so easy to establish, for the preserved Latin text of the work does not explain any straightforward method to compute the value of ε : Mercier (1976, p. 212 and 1977, p. 39) has provided us with two exact formulae to calculate the obliquity according to two possible interpretations of the model of the Liber de motu: in one of them (Goldstein, 1964, and North, 1967) the point of intersection of the movable and fixed ecliptics is kept at a distance of 90° from the centre of the small equatorial epicycle; according to the second one (Mercier, 1976, p. 210) the intersection of the two ecliptics is kept at a distance of 90° from the moving head of Aries. Using Mercier's formulae it is easy to establish that Goldstein-North's constraint gives good approximations for the values we should expect for the time of Ptolemy (23;51,20°) as well as for the time of the Caliph al-Maᵓmūn (23;33,52° according to Bīrūnī, 1962, p. 91). On the other hand, if we use Mercier's constraint, the results will only be acceptable for the time of al-Maᵓmūn. This

https://doi.org/10.1017/S0252921100106013 Published online by Cambridge University Press

I think, is a good reason to prefer Goldstein and North's hypothesis according to which the minimum value for the obliquity of the ecliptic is 23;33° which is attained towards 887 A.D. After this date the value of increases, reaching about 23;40° in year 1252 A.D., which corresponds to the beginning of the Alphonsine era. We can, therefore, conclude that the Liber de motu was not used to compute the Alphonsine parameter for the obliquity of the ecliptic.

A third possibility is, as we have seen, to use the trepidation model and parameters embedded in the Alphonsine Tables. I prefer here again to use the Goldstein-North constraint and the corresponding Mercier formula, although we would reach similar conclusions with the other hypothesis. The results of my calculations have been plotted against time in Fig. 1 for a period between 16 A.D: (in which, as we have seen $i_o = 0^o$) until the middle of the 14th century. It is easy to see not only that the model was not used to compute the Alphonsine parameter but also that the curve only gives us a fairly good approximation to the historical determination of the value of the obliquity made in the time of al-Maʾmūn. The values established by Ptolemy, al-Zarqālluh (23;33°) and the Alphonsine astronomers themselves seem not to have been taken into consideration. This is a serious drawback if we compare it with the results one can get with the Liber de Motu and with al-Zarqālluh's trepidation model. In fact I am inclined to suspect that the coincidence with al-Maʾmūn's value could be purely casual, and that King Alphonse's astronomers designed their trepidation model having only in mind some historical determinations of the positions of the solar apogee as well as the value of precession established in their time. They seem to have regarded entirely the variation of the obliquity of the ecliptic.

5 A few concluding remarks: I have tried to underline in these pages a few instances in which the Alphonsine Tables seem to depend on an Arabic astronomical tradition which may go back to the origins of Islamic astronomy and draw on Eastern sources giving information on observation techniques established in the time of the Caliph al-Maʾmūn. It is also possible that the king's collaborators had a copy of Ibn Yūnus' Ḥākimī zīj, and I do not think there is much need to prove the influence of the Liber de motu, the Eastern or Western origin of which is being discussed nowadays. On the other hand, one can appreciate a certain maturity in the Alphonsine treatment of the subjects: new parameters are established and some of them may be the result of new observations. Sometimes one can appreciate a certain originality in the Alphonsine models: such is the case with the combination of precession and trepidation, an idea which, as we have seen, has clear Andalusian precedents, but which never, to my knowledge, produced a set of tables before the time of King Alphonse. In this sense it is most interesting to remark that Naṣīr al-Dīn al-Ṭūsī seems to refer to such a combination in a not very clear passage of his al-Tadhkira fī ʿilm al-hayʾa (Ragep 1982, pp. 216-218 and 31-32 of the translation). Here again we have a possibility of a link between Alphonsine astronomy and the Marāgha school.

https://doi.org/10.1017/S0252921100106013 Published online by Cambridge University Press

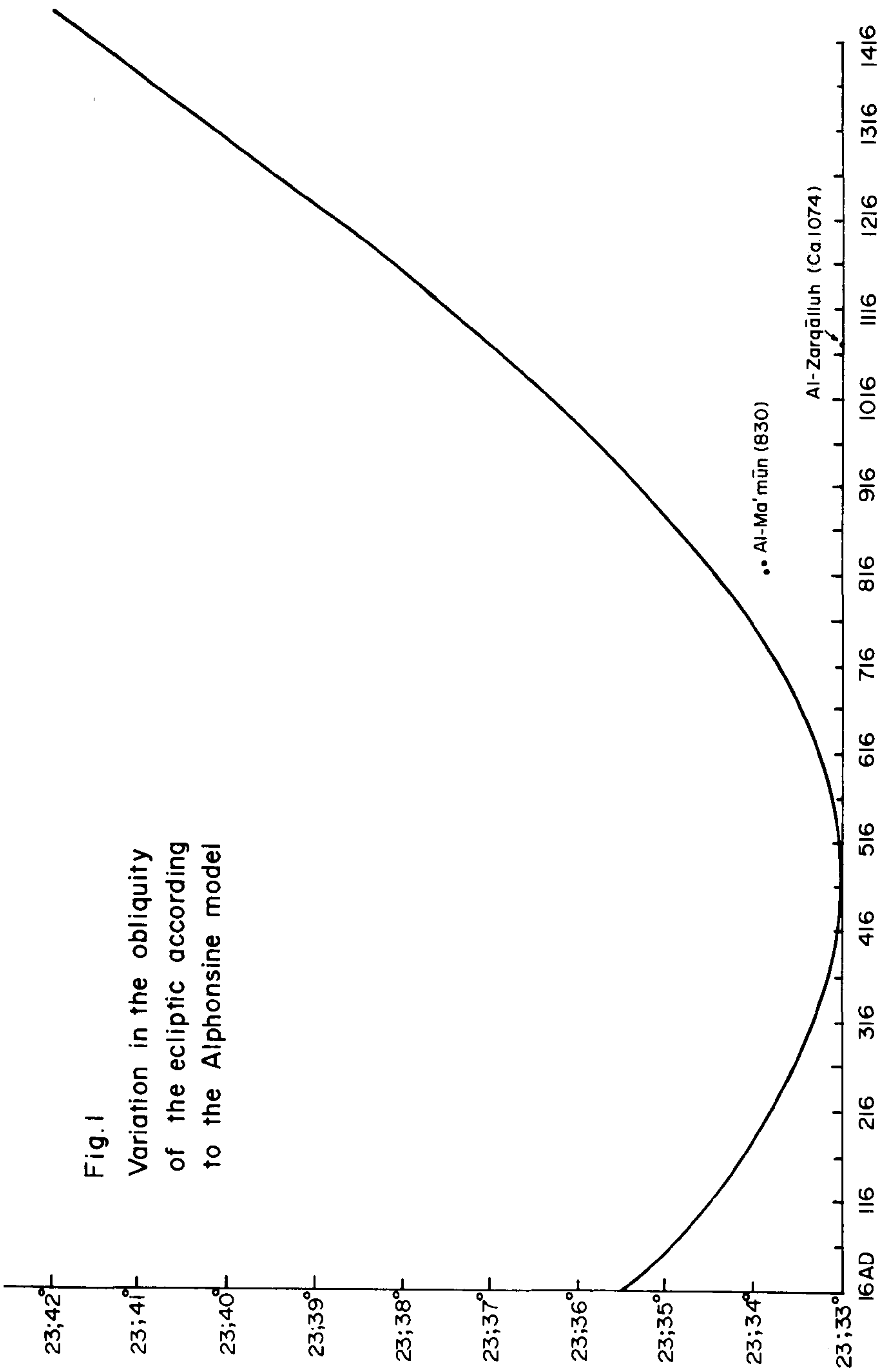


Fig. I
Variation in the obliquity of the ecliptic according to the Alphonsine model

https://doi.org/10.1017/S0252921100106013 Published online by Cambridge University Press

References

al-Bīrūnī (1962). Taḥdīd nihāyāt al-amākin li-taṣḥīh masāfāt al-masākin, ed. P. Boulgakov. Revue de l'Institut des Manuscrits Arabes, 8.
Blachère, R. (1935). Ṣāᶜid al-Andalusī, Kitāb Ṭabaqāt al-Umam (Livre des Catégories des Nations). Pairs: Larose.
Bossong, Georg (1978). Los Canones de Albateni. Tübingen: Max Niemeyer.
Caussin de Perceval, A. P. (1804). Le livre de la grande table Hakémite. Notices et extraits des manuscrits de la Bibliothèque Nationale, [Paris], 7, 16-240.
Delambre, J.B. (1819). Histoire de l'astronomie du Moyen Age. Paris: Courcier. Repr. Johnson 1965.
Dobrzycki, Jerzy (1965). Teoria orecesji w astronomii sredniowiecznej. Studia i Materialy Dziejow Nauki Polskiej, Seria Z,Z. 11, 3-47. (summary in English)
Dreyer, J.L.E. (1920) . The original form of the Alfonsine Tables. Monthly Notices of the Royal Astronomical Society, 80, 243-262.
Goldstein, Bernard R. (1964). On the Theory of Trepidation according to Thābit b. Qurra and al-Zarqāllu and its Implications for Homocentric Planetary Theory. Centaurus, 10, 232-247.
Goldstein, Bernard R. (1971). Al-Biṭrūjī: On the Principles of Astronomy. An Edition of the Arabic and Hebrew versions with translation, analysis and an Arabic-Hebrew-English Glossary. 2 vols. New Haven: Yale University.
Hilty, Gerold (ed.) (1954). Aly Aben Ragel, El Libro Conplido en los Iudizios de las Estrellas. Madrid: Real Academia Española.
Kennedy, E. S. (1958). The Sasanian Astronomical Handbook Zīj-i Shāh and The Astrological Doctrine of "Transit" (Mamarr). Journal of the American Oriental Society, 78, 246-262. Repr. in E.S.Kennedy et al., Studies in the Islamic Exact Sxiences. Beirut: American University of Beirut. 319-335.
Kennedy, E. S. ; B. L. van der Waerden (1963). The World-Year of the Persians. Journal of the American Oriental Society, 83, 315-327. Repr. in (1983) Kennedy et al., Studies in Islamic Exact Sciences, pp. 338-350.
Menendez Pidal, Ramón (ed.) (1977). Primera Crónica General de España. 2 vols. Madrid.
Mercier, Raymond (1976-77). Studies in the Medieval Conception of Precession. Archives Internationales d'Histoire des Sciences, 26, 197-220; 27, 33-71.
Millás Vallicrosa, J. M. (1943-50) Estudios sobre Azarquiel. Madrid-Granada: Escuelas de Estudios Árabes.
Nallino, C. A. (1907). Al-Battani sive Albatenii, Opus Astronomicum. Vol. III, Mediolani Insubrum, 1907. Repr. Hildesheim: Olms.
Neugebauer, O. (1962a). The Astronomical Tables of al-Khwārizmi. Translation with Commentaries of the Latin Version edited by H. Suter supplemented by Corpus Christi College MS 283. Hist.-filosofiske Skrifter udgivet af Det Kongelige Danske Videnskabernes Selskab. 4, no. 2, Köbenhavn: Munksgaard.
Neugebauer, O. (1962b). Thābit ben Qurra "On the Solar Year" and "On the Motion of the Eighth Sphere". Proceedings of the American Philosophical Society, 106, 264-299.

https://doi.org/10.1017/S0252921100106013 Published online by Cambridge University Press

North, John D. (1967). Medieval Star Catalogues and the Movement of the Eighth Sphere. Archives Internationales d'Histoire des Sciences, 17, 73-83.

Pingree, David (1968). The Fragments of the Works of Ya^cqūb ibn Ṭāriq. J. of Near Eastern Studies, 29, 103-123.

Poulle, E. (1980). Jean de Murs et las Tables Alphonsines. Archives d'Histoire Doctrinale et Littéraire du Moyen Age, 241-271.

Poulle, E. (1984). Les Tables Alphonsine avec les canons de Jean de Saxe. Edit. trad. et commentaire. Paris: C.N.R.S.

Price, Derek J. (1955a). A Medieval Footnote to Ptolemaic Precession. Vistas in Astronomy, 1, 66.

Price, Derek J. (1955b). The Equatorie of the Planetis. Cambridge: University.

Ragep, Faiz Jamil (1982). Cosmography in the "Tadhkira" of Naṣīr al-Dīn al-Ṭūsī. Unpublished Ph.D. dissertation. Harvard University.

Rico y Sinobas, Manuel (1866). Libros del Saber de Astronomía del rey D. Alfonso X de Castilla. Vol. IV, Madrid: Impres. ... Real Casa.

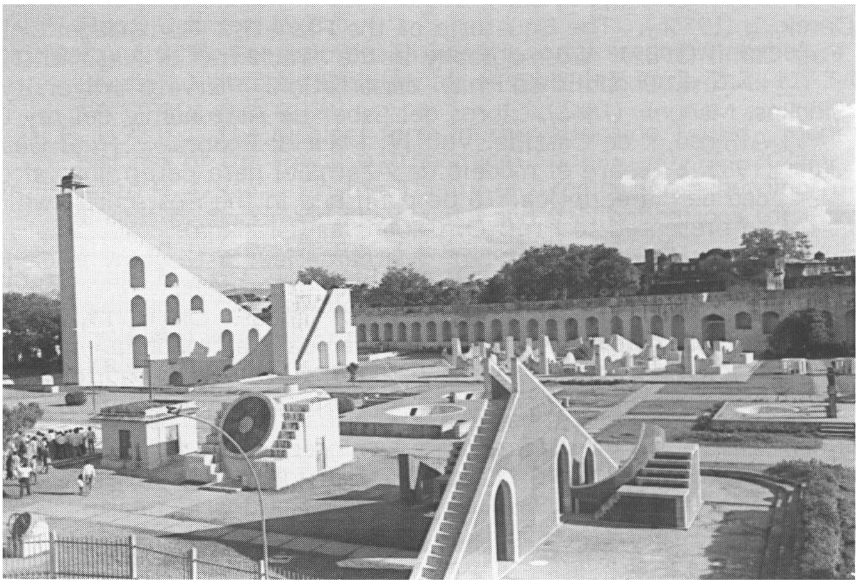
Samsó, Julio (1985u). Sobre el modelo de Azarquiel para determinar al oblicuidad de la eclíptica. To be published in the Festschrift which will be presented to Prof. D. Cabanelas (Granada).

Swerdlow, Noel; O. Neugebauer (1984). Mathematical Astronomy in Copernicus' De revolutionibus. 2 vols. New York: Springer.

Toomer, G. J. (1968). A Survey of the Toledan Tables. Osiris, 15, 5-174.

Vernet, Juan (1984). Alfonso X el Sabio: mecánica y astronomía. Conmemoración del Centenario de Alfonso X el Sabio. Madrid: Real Academia de Ciencias Exactas, Físicas y Naturales, 23-32.

https://doi.org/10.1017/S0252921100106013 Published online by Cambridge University Press



Jaipur Observatory. The observatory, popularly known as the Jantar Mantar, was completed in 1734 and has the largest number of Jai Singh's instruments.

https://doi.org/10.1017/S0252921100106013 Published online by Cambridge University Press