MAGNETIC MULTIPOLE TRANSITION PROBABILITIES

R. H. GARSTANG

Joint Institute for Laboratory Astrophysics, University of Colorado and National Bureau of Standards, Boulder, Colorado 80302, U.S.A.

Abstract. A review is given of the occurrence and transition probabilities of spectrum lines arising from magnetic quadrupole radiation. Magnetic dipole radiation in relativistic quantum mechanics is also discussed.

1. Introduction

In the classical theory of radiation the electromagnetic quantities are expressed in terms of multipole expansions. There are two sets of such quantities, electric dipole, electric quadrupole, electric octupole, ... and magnetic dipole, magnetic quadrupole, ... These expansions are carried over into the quantum theory of radiation. It is a standard result that when a whole spectrum line is considered (but not the individual Zeeman components of a line separately) the quantum mechanical transition probability is the arithmetical sum of the transition probabilities of the individual multipoles. Each multipole has its own set of selection rules. When electric dipole radiation is allowed it usually predominates, and other multipoles may be neglected. When electric dipole radiation is not allowed by exact selection rules the spectral line is called forbidden. Forbidden lines can arise from any of the higher multipoles. In astrophysics forbidden lines have been known for many years, from the work of Bowen, Edlén, and many others, in the spectra of the night sky and aurorae, solar corona, peculiar stars and planetary nebulae. These lines are due to magnetic dipole radiation, or to electric quadrupole radiation, or to a mixture of the two. The transitions take place between states of the same parity and electric dipole transitions between the states are rigorously forbidden. In this paper we shall review the possible occurrence of magnetic quadrupole radiation, and in addition discuss the occurrence of magnetic dipole radiation in a higher order theory than that usually considered.

2. Magnetic Quadrupole Radiation

It was pointed out some years ago by Mizushima (1964) that although magnetic dipole radiation is forbidden in *LS*-coupling for intercombination lines, magnetic quadrupole radiation is allowed for $\Delta S = \pm 1$ transitions. The transition probability of a magnetic quadrupole line was calculated by Mizushima and by Garstang (1967). When expressed in terms of a line strength S it is

$$A = \frac{1}{2J+1} \frac{256\, \pi^6 \nu^5}{135\, hc^5} S \tag{1}$$

De Jager (ed.), Highlights of Astronomy, 555–560. *All Rights Reserved Copyright* © 1971 *by the IAU*

https://doi.org/10.1017/S1539299600000587 Published online by Cambridge University Press

where

$$S = \left|\left\langle J \left\| \frac{e}{2mc} \sum_i \nabla (r^2 C^{(2)}) \cdot (\mathbf{L} + 3\mathbf{S}) \right\| J' \right\rangle\right|^2 . \tag{2}$$

Here J and J' are the total angular momenta of the upper and lower energy level involved in the transition, $C^{(2)}$ is a spherical harmonic tensor operator as defined by Racah (1942), and the sum is over all the electrons i in the atom. r, $\mathbf{L}$ and $\mathbf{S}$ are the coordinate, orbital and spin angular momenta of one electron, ν is the frequency of the transition, and e, m, c and h have their usual meanings. From the application of Equation (2) to the LS-coupling case it can be shown that the selection rules for magnetic quadrupole radiation are:

(1) Parity change
(2) $\Delta J = 0, \pm 1, \pm 2 (0 \not\leftrightarrow 0, 0 \not\leftrightarrow 1, \frac{1}{2} \not\leftrightarrow \frac{1}{2})$
(3) $\Delta M = 0, \pm 1, \pm 2$
(4) $\Delta S = 0, \pm 1$
(5) (a) If $\Delta S = 0$, $\Delta L = 0, \pm 1, \pm 2$
(5) (b) if $\Delta S = \pm 1$, $\Delta L = 0, \pm 1 (0 \not\leftrightarrow 0)$.

Rule (5b) is the unexpected rule. Transitions satisfying $\Delta S = \pm 1$ can take place in many cases by electric dipole transitions made possible by spin-orbit or by nuclear-spin electron-orbit interaction. The transition $2s^2\,2p^2\;{}^3P_2$–$2s\,2p^3\;{}^5S_2$ in C I is an example of a case in which both magnetic quadrupole radiation and spin-orbit-induced electric dipole radiation occur. Many transitions of this type were studied by Mizushima (1966). Generally magnetic quadrupole radiation turns out to be numerically unimportant.

There are a few transitions in which only magnetic quadrupole radiation can occur. The most interesting one is $s^2\;{}^1S_0$–$sp\;{}^3P_2$. The only magnetic multipole which is allowed is the quadrupole (dipole is forbidden because $\Delta J = 2$, octupole and higher multipoles are forbidden because of $0 \not\leftrightarrow 2$ rules). All electric multipoles are also forbidden (dipole because $\Delta J = 2$, quadrupole by parity and higher multipoles by $0 \not\leftrightarrow 2$ rules). Nuclear-spin-induced electric dipole radiation is, however, possible for atoms whose nuclei have a non-zero spin. Garstang (1967) made a series of calculations on magnetic quadrupole radiation and calculations on nuclear-spin-induced electric dipole radiation (Garstang, 1962). The results are illustrated in Table I (results for some additional atoms may be found in the references mentioned). We see that in Hg I magnetic quadrupole radiation is unimportant. In Cd I magnetic quadrupole radiation forms about 10 per cent of the whole. There is some evidence from studies of the laboratory intensity ratio (${}^1S_0$–${}^3P_2/{}^1S_0$–${}^3P_0$) in samples of cadmium with enriched odd isotope abundances that magnetic quadrupole radiation is indeed present (see Garstang, 1967). The Zn I line at λ3040 was observed by Fukuda (1926) and Foote *et al.* (1925); they deserve the credit for being the first to observe a magnetic quadrupole line in the laboratory. The Mg I line at λ4562.5 was observed by Bowen (1960) in the planetary nebula NGC 7027. This was the first observation of a magnetic

https://doi.org/10.1017/S1539299600000587 Published online by Cambridge University Press

quadrupole transition in a cosmical source. One other laboratory observation of magnetic quadrupole radiation is the work of Catz (1970) on angular correlation between K and L X-rays in lead in cascade emission, in which the observed correlation is in good agreement with theoretical predictions only when a small admixture of magnetic quadrupole radiation is taken into account.

We turn to a consideration of magnetic quadrupole radiation in highly ionized atoms. For transitions in which there is a change $\Delta n(\neq 0)$ of principal quantum number the magnetic quadrupole transition probability varies along an isoelectronic sequence as Z^8. If $\Delta n=0$ the variation is as Z^3. This suggests that, at least for $\Delta n \neq 0$, magnetic quadrupole radiation might be important for large Z. Garstang (1969) calculated a number of transitions, some of the form $s^2\,{}^1S_0$–$sp\,{}^3P_2$ and others of the forms $p^6\,{}^1S_0$–$p^5\,s\,{}^3P_2$, $p^5\,d\,{}^1D_2$ or $p^5\,d\,{}^3D_2$. Some examples are listed in Table II

TABLE I

Transition probabilities (sec^{-1}) for $ns^2\,{}^1S_0$–$nsnp\,{}^3P_2$ lines

Atom[a]	λ	n	A (magnetic quadrupole)	A (nuclear spin electric dipole)
Mg I	4562	3	1.8×10^{-4}	1.6×10^{-5}
Zn I	3040	4	8.7×10^{-4}	2.3×10^{-4}
Cd I	3141	5	9.6×10^{-4}	6.9×10^{-3}
Hg I	2270	6	3.6×10^{-3}	0.15

[a] All calculations refer to atoms with the natural isotope abundance ratios.

TABLE II

Transition probabilities[a] for ${}^3P_2$ de-excitation[b]

Atom	Wavelength (Å)	Transition	Type[c]	A (sec^{-1})
He I	591	$1s\,2p\ {}^3P_2$–$1s^2\ {}^1S_0$	mq	0.22
O VII	21.8	$1s\,2p\ {}^3P_2$–$1s^2\ {}^1S_0$	mq	3.0×10^5
Ar XVII	3.99	$1s\,2p\ {}^3P_2$–$1s^2\ {}^1S_0$	mq	3.1×10^8
Fe XXV	1.88	$1s\,2p\ {}^3P_2$–$1s^2\ {}^1S_0$	mq	6.5×10^9
Fe XXV	384	$1s\,2p\ {}^3P_2$–$1s\,2s\ {}^3S_1$	ed	5.1×10^8
Fe XXIII	250	$2s\,2p\ {}^3P_2$–$2s^2\ {}^1S_0$	mq	2.1×10^1
Fe XXIII	1060	$2s\,2p\ {}^3P_2$–$2s\,2p\ {}^3P_1$	md	1.1×10^4
Fe XVII	17.1	$2p^5\,3s\ {}^3P_2$–$2p^6\ {}^1S_0$	mq	2.3×10^5
Fe XV	394	$3s\,3p\ {}^3P_2$–$3s^2\ {}^1S_0$	mq	1.8
Fe XV	7059	$3s\,3p\ {}^3P_2$–$3s\,3p\ {}^3P_1$	md	3.8×10^1
Fe IX	249	$3p^5\,3d\,{}^3P_2$–$3p^6\ {}^1S_0$	mq	7.1×10^1
Fe IX	18700	$3p^5\,3d\,{}^3P_2$–$3p^5\,3d\,{}^3P_1$	md	2.1

[a] From Garstang (1967, 1969), where additional results may be found.

[b] The principal alternative de-excitation mechanisms are listed only for the various stages of ionized iron. Electric dipole radiation to the $1s\,2s\,{}^3S_1$ state is important throughout the helium sequence and is dominant up to S XV.

[c] e = electric, m = magnetic, d = dipole, q = quadrupole.

https://doi.org/10.1017/S1539299600000587 Published online by Cambridge University Press

(others may be found in the original paper). The first four lines in Table II show the increase of A along an isoelectronic sequence, in the case of $\Delta n \neq 0$. The increase of A for transitions with $\Delta n = 0$ is much smaller, as we expect from the Z^3 dependence instead of Z^8.

The important question for solar physics is whether magnetic quadrupole radiation contributes to the de-excitation of the $^3P_2$ levels. Where there are competing radiative de-excitation processes we have listed their transition probabilities in Table II. In Fe IX we see that magnetic quadrupole radiation is more important than magnetic dipole radiation in de-exciting the $3p^5\,3d\,^3P_2$ level. In Fe XV magnetic quadrupole radiation is not an important de-excitation mechanism for the $3s\,3p\,^3P_2$ level; it does not significantly affect the intensity of the well-known forbidden line $^3P_2$–$^3P_1$. In Fe XVII magnetic quadrupole radiation provides the only radiative de-excitation mechanism for the $2p^5\,3s\,^3P_2$ level, and should certainly be included in calculations of the statistical equilibrium of that level. In Fe XXIII magnetic quadrupole radiation is not a significant contributor to the de-excitation of the $2s\,2p\,^3P_2$ level. Finally, in Fe XXV (and in all ions of the He I isoelectronic sequence above S XV) magnetic quadrupole radiation to the $1s^2\,^1S_0$ level is a more important mechanism for the de-excitation of the $1s\,2p\,^3P_2$ level than *electric* dipole radiation to the $1s\,2s\,^3S_1$ level. This remarkable result points to a general moral, that when dealing with very highly ionized atoms quantities which are negligible for neutral atoms may become significant and even dominant if they have a large positive power Z dependence.

A recent confirmation of the calculations of magnetic quadrupole transition probabilities has come from R. Marrus and his group in Berkeley. In work as yet unpublished they have succeeded in exciting the $1s\,2p\,^3P_2$ state in Ar XVII by beam-foil spectroscopy and in measuring the lifetime of this state by direct observation of its decay. Their (as yet preliminary) result is in rough agreement with the lifetime calculated from the sum of the magnetic quadrupole transition probability to the $1s^2\,^1S_0$ state (given in Table II) and the electric dipole transition probability (the present reviewer estimates $A = 1.7 \times 10^8\ \text{sec}^{-1}$) to the $1s\,2s\,^3S_1$ state. Neglect of the magnetic quadrupole contribution would lead to a discrepancy of a factor three between theory and experiment.

3. Relativistic Magnetic Dipole Transitions

It was pointed out by Gabriel and Jordan (1969a) that a series of lines in the ultraviolet solar spectrum could be ascribed to the transition $1s^2\,^1S_0$–$1s\,2s\,^3S_1$ in C V, O VII, Ne IX, Na X and Mg XI. Gabriel and Jordan (1969b) identified the corresponding line in Si XIII. In work as yet unpublished R. Marrus and R. W. Schmieder have observed in their laboratory in Berkeley the same transition in Si XIII and the corresponding transitions in S XV and Ar XVII using beam-foil spectroscopy.

The transition is a very interesting one. It is forbidden by parity for electric dipole, magnetic quadrupole, electric octupole, ... radiation. It is forbidden for electric quadrupole radiation (and for all higher multipoles) by the $0 \nleftrightarrow 1$ rule on J. Magnetic

https://doi.org/10.1017/S1539299600000587 Published online by Cambridge University Press

dipole radiation is the only possibility (a one-photon process is required to produce a discrete spectrum line). The transition is forbidden in LS-coupling by the $\Delta S=0$ rule and by the rule that the principal quantum number does not change. Accordingly we must look to departures from LS-coupling. One possibility is spin-other-orbit interaction, but Griem (1969) showed that this gives a very small transition probability. Griem pointed out that if relativistic wave functions were considered a much larger transition probability would be obtained. The transition becomes possible because in relativistic theory electron spin, and spin-orbit interaction, is automatically introduced.

The transition $1s$–$2s$ in hydrogen can also take place by relativistic magnetic dipole radiation. This was shown by Breit and Teller (1940). If we define a magnetic dipole line strength S_m in the usual way

$$S_m = |\langle J_1 \| U^{(1)} \| J_2 \rangle|^2 \tag{3}$$

then the relativistic magnetic dipole tensor operator for one electron is

$$\mathbf{U}^{(1)} = \tfrac{1}{2} e\mathbf{r} \times \boldsymbol{\alpha} \tag{4}$$

where $\boldsymbol{\alpha}$ is the vector of the Dirac 4×4 matrices. For the $1s\ ^2S_{1/2}$–$2s\ ^2S_{1/2}$ transition in hydrogen-like atoms with nuclear charge Z we find by inserting Dirac wave functions into Equation (3) that

$$S_m(1s\ ^2S_{1/2}\text{–}2s\ ^2S_{1/2}) = \frac{64}{243}\left(\frac{e\hbar}{2mc}\right)^2 (\alpha Z)^4 \tag{5}$$

where $(\alpha Z)^6$ and higher powers are neglected. Numerically we find from Equation (5) that the transition probability is

$$A_m(1s\ ^2S_{1/2} \leftarrow 2s\ ^2S_{1/2}) = 5.62 \times 10^{-6}\, Z^{10}\ \text{sec}^{-1} \tag{6}$$

in agreement with Breit and Teller for $Z=1$. (Griem quoted Z^8 but this must be an error.) In hydrogen it turns out that two photon de-excitation has a probability $A=8.23Z^6\ \text{sec}^{-1}$ (Spitzer and Greenstein, 1951, and Breit and Shapiro, 1959) and so is more important than relativistic magnetic dipole radiation for small Z, and indeed up to Fe XXVI and beyond. Relativistic magnetic dipole transition probabilities contribute over 10% to the total probability from Ca XX onwards along the isoelectronic sequence, and should be allowed for in studies of the statistical equilibrium of energy levels.

In helium-like ions the situation is quite different, in that the two photon transition probabilities are very small (Dalgarno and Drake, 1969), in large measure due to the intercombination nature of the transition. Relativistic magnetic dipole radiation is dominant. Griem (1969) gave an approximation to the transition probability

$$A(1s^2\ ^1S_0 \leftarrow 1s\,2s\ ^3S_1) = 3.75 \times 10^{-6}\left(\frac{\nu}{\nu_Z}\right)^3 Z^9 (Z+\tfrac{1}{2}) \tag{7}$$

where ν is the frequency of the transition in the helium-like ion, with $Z=1$ for He I,

https://doi.org/10.1017/S1539299600000587 Published online by Cambridge University Press

$Z=2$ for Li II, and so on. ν_Z is the frequency of the Lyman-α transition in a hydrogen-like ion with the same degree of ionization (e.g. N VII if we want O VII). (We have corrected Griem's result by a factor Z^2: the Z dependence must be as Z^{10}.)

A more precise calculation of the transition probability of the $1s^2\,{}^1S_0$–$1s\,2s\,{}^3S_1$ transition would be valuable. Attempts are in progress in several laboratories (R. H. Garstang at Boulder, I. P. Grant at Oxford, and perhaps others) but the calculation is difficult and an adequate treatment has not been obtained as yet. Gabriel and Jordan and their colleagues at the Culham Laboratory have attempted to reverse the calculations on solar line intensities and from observations of the ${}^1S_0$–${}^3S_1$ transition in C V they deduce a value of the constant in Equation (7) above which is about one-half of Griem's value. It is perhaps a fair deduction that Griem's formula is not in error by much more than a factor 2 or 3.

Acknowledgement

The preparation of this review was supported by National Science Foundation Grant GP-20696.

References

Bowen, I. S.: 1960, *Astrophys. J.* **132**, 1.
Breit, G. and Shapiro, J.: 1959, *Phys. Rev.* **113**, 179.
Breit, G. and Teller, E.: 1940, *Astrophys. J.* **91**, 215.
Catz, A. L.: 1970, *Phys. Rev. Letters* **24**, 127.
Dalgarno, A. and Drake, G. W. F.: 1969, *Mém. Soc. Roy. Sci. Liège, Ser. V* **17**, 69.
Foote, P. D., Takamine, T., and Chenault, R. L.: 1925, *Phys. Rev.* **26**, 165.
Fukuda, M.: 1926, *Sci. Papers. Inst. Phys. Chem. Res. Tokyo* **4**, 171 (No. 55).
Gabriel, A. H. and Jordan, C.: 1969a, *Nature* **221**, 947.
Gabriel, A. H. and Jordan, C.: 1969b, *Monthly Notices Roy. Astron. Soc.* **145**, 241.
Garstang, R. H.: 1962, *J. Opt. Soc. America* **52**, 845.
Garstang, R. H.: 1967, *Astrophys. J.* **148**, 579.
Garstang, R. H.: 1969, *Publ. Astron. Soc. Pacific* **81**, 488.
Griem, H. R.: 1969, *Astrophys. J.*, **156**, L103.
Mizushima, M.: 1964, *Phys. Rev.* **134**, A883.
Mizushima, M.: 1966, *J. Phys. Soc. Japan* **21**, 2335.
Racah, G.: 1942, *Phys. Rev.* **62**, 438.
Spitzer, L. and Greenstein, J. L.: 1951, *Astrophys. J.* **114**, 408.

DISCUSSION

A. Underhill: Does your work contribute a possible mechanism by which an atom or ion in a $1s$ state collides with a free electron, forms a $1sns$ complex, and then radiates by relativistic magnetic dipole radiation to form a $1s^2$ stable ground state?

R. H. Garstang: I do not know if that is possible, but the transition probability is small, so that the process would be unlikely.

https://doi.org/10.1017/S1539299600000587 Published online by Cambridge University Press