problem is of three degrees of freedom. One of the integrals in the ordinary case is a substitute for the Jacobi integral.

Now suppose that the distribution of asteroids considered as a function of the Jacobi constant has no singularity, in the same way as Klose's. Brouwer found that the distribution function for L^*, which is the transform of L, is flat in the ordinary case, but becomes v-shaped in the commensurability case.

In the neighbourhood of a commensurability of a lower order, that is, in the ratio of two small integers, there exist an infinite number of commensurabilities in the ratios of very large integers, each with the v-shaped distribution functions. The higher order commensurabilities have adequate room for the v-shaped distribution in the vicinity of the 2/1 commensurability and also for commensurabilities such as 3/1, 5/2, 7/3, all with larger mean motions than 2/1. Brouwer says that for the 3/2 and even more for the 4/3 commensurability little room is available free from interference by other commensurabilities. This would, according to Brouwer, indicate the reason why the asteroids in this part of the belt seem to seek the commensurability region rather than avoid them. Brouwer thinks this conjecture is confirmed by the fact that the asteroids in the region between 440″ and 575″ mean motion show a distribution that is closely correlated with the distribution of the commensurabilities of comparatively lower orders. It can be understood that in the latter types of commensurabilities the v-shaped distribution may become almost flat due to the superposition or overlapping of the v-shaped distribution corresponding to the various commensurabilities of higher orders nearby. But the question is left unanswered why the asteroids in this part of the belt seem to seek the commensurability region. There is also the question of the relative density in the distribution of the commensurabilities of higher orders in the neighbourhood of the two types of the lower commensurabilities.

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TROJAN ASTEROIDS

Rabe (1) carried out the actual determination on electric computers of periodic libration orbits about the equilateral triangular equilibrium points of the restricted problem and their stabilities. He at first describes the general method developed for finding such periodic orbits and then deals with orbits of various dimensions including one of the horse-shoe shaped in the natural Trojan case (1) and, together with Schanzle, in the case of the Earth-Moon restricted problem (3). Rabe (2) obtained the Fourier series representation of all orbits for asteroids of the Trojan group by numerical harmonic analysis with an electronic computer. The convergence of the Fourier expansions is very satisfactory, according to Rabe, up to amplitudes of the order of those of the actual Trojan asteroids with the largest libration amplitudes. All orbits computed appear to be stable, even the horse-shoe shaped periodic orbit enclosing both the equilateral triangular equilibrium points L_4 and L_5 and the collinear equilibrium point L_3 opposite to Jupiter from the Sun. This latter class of orbit, anticipated by Brown, is of particular interest, because with their increasing amplitude but decreasing periods these orbits link the equilibrium points L_4 and L_5 with the satellite region of Jupiter. This settles the question as to the existence of the horse-shoe shaped orbits in Brown's conjecture. Similar computation has been carried out by Colombo, Lautman and Munford (4) in the elliptical problem by taking Jupiter's eccentricity into account, with application to the Earth-Moon system in mind.

Rabe later began to use the periodic solutions in the form of the Fourier series obtained by the harmonic analysis of the periodic orbits previously computed as reference or intermediary orbits for the analytical representation of non-periodic librational motions in the restricted as well as in the elliptical restricted problem with Jupiter's orbital eccentricity. This is facilitated in the Trojan cases by the availability of a sufficiently dense net of periodic solutions, so that we can interpret for any desired periodic reference orbit. In the elliptical problem the reference or the intermediary orbit is a suitable combination of a periodic solution of the restricted problem with a short-period scale factor corresponding to Jupiter's variable distance from the Sun. Here, of course, the resulting intermediary orbit is already non-periodic in nature, but is a useful basis for the description of librational motions on which Jupiter tends to impress its own elliptical terms. The results are of interest not only as far as the non-periodic motions of a librational nature are concerned, but also because they afford a discussion of the stability of the periodic reference orbit to a higher order than the first-order stability results from Hill's equation, which neglects the second and higher order terms.

Stumpff (5) extended Thüring's theory in 1929–31 on the mathematical treatment of the librational motion for approximating the periodic orbits for all amplitudes. Thüring's solution provides a starting point for an exact theory of the plane long-period Trojan orbits according to the method of the variation of constants. Special attention is devoted to the border line case in Brown's conjecture on the horse-shoe shaped orbit.

Deprit of the Louvain University is working at Cincinnati on the continuation of the horse-shoe shaped periodic Trojan orbits toward even larger amplitudes for establishing the end of this family as the immediate vicinity of Jupiter and of the collinear equilibrium points L_1 and L_2 is approached. For this purpose Deprit has to use regularizing variables on his computer program. Brouche (6) of the Louvain University computed 392 periodic orbits on an electronic computer in the Earth-Moon case in a systematic survey.

Danby (8) discusses the stability of the triangular points in the elliptic restricted three-body problem. Message's work on the librations of the Trojan asteroids will be published shortly in the lecture note of the Summer Institute on Dynamical Astronomy at Cornell in 1963. Sehnal (9) discussed the stability of Liapounov of an Earth satellite at the equilateral triangular point with the Moon and the Earth.

Jeffreys (7) argued against the supposition of Kuiper, that the Trojans had been satellites of Jupiter before Jupiter lost its mass to the present value, on the basis of the nature of L_4 and L_5. He says that, if Jupiter had lost mass, then the amplitude of the libration of the satellite should have increased. Further, Jeffreys attributes the secular acceleration of the natural satellites Jupiter V, Mars I, Saturn I to the interaction between satellites either to viscous or turbulent drag, the formation of shock waves or accretion, after showing that the mutual influence of satellites through tidal friction after Darwin is insufficient.

On the other hand Rabe (10) showed that, as far as the crowding of the Trojans in the neighbourhood of the centres is concerned, if no mass decrease of Jupiter is considered, there is no theoretical explanation for the preference of orbits with small amplitude and with small Jacobi constant. The result of Rabe's investigation as well as empirical fact, according to Rabe, gives considerable support to the suggested origin of the Trojans and the mass loss of Jupiter. Huang (11) advocates that a Trojan can escape from the Jovian system even without the mass decrease of Jupiter.

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LUNAR THEORY

Hori (1) developed Brouwer's project for the lunar theory on the basis of von Zeipel's method. As a first attempt he neglected the orbital inclination and obtained the solution in powers of m, but in a closed form with respect to the eccentricity. He computed as far as the order m^4 for the periodic terms and as far as m^5 for the secular terms. He has just completed the computation in closed forms with respect to both the eccentricity, the inclination and the solar eccentricity as far as the order m^4.

Stumpff's lunar theory (2) is based on the use of his so-called invariants and the main equation in his theory of the two-body problem. He considers the differential equations for Hill's variational orbit in his lunar theory in two dimensions, and obtains a relation of the form $f(\dddot{r}, \ddot{r}, \dot{r}, r; C) = 0$. The equation can be integrated numerically by iteration if the Moon's orbit is considered to be a disturbed Keplerian ellipse with any value of the eccentricity, for a certain fairly extended time interval in the vicinity of the initial epoch. The iteration process is limited to the solution of the transcendental main equation. In the second paper Stumpff expanded the co-ordinates in powers of Hill's m in a manner different from Hill's.

Schubart (3) published his work on the extension to three dimensions of the family of periodic orbits of Hill's lunar problem of two dimensions. For representing the third co-ordinate a new variable is introduced. The series obtained are proved to be convergent by the procedure of Siegel.

In 1961, at Berkeley, Eckert reported that Brown's harmonic series for the co-ordinates of the Moon had been substituted into the differential equations and the residuals obtained with precision, and outlined a method for the solution of the variation equations which would overcome the difficulties arising from the small divisors. Eckert wrote me that the method had been developed since and the programmes necessary to generate and solve the variation equations on the 7094 computer had been completed and applied to about 3500 residuals. The results appear to be completely satisfactory according to Eckert. The corrections obtained from the solution are now being applied to the initial series and the corrected series substituted in the differential equations. Eckert plans to repeat the entire process at least once with appropriate variations to estimate the stability of the results. He says that his machine programmes permit a complete solution with little effort compared to that already expanded on the problem.

Van der Waerden (4) considered the secular terms and fluctuations in the motions of the Sun and the Moon from two causes. The one is due to the tidal friction and it causes the secular retardation of the motion of the Moon. The other is the irregular rotation of the surface of the Earth due to random currents in the interior of the Earth.

Kovalevsky (5) is continuing his work on the theory of motion of the eighth satellite of Jupiter, according to the principle of successive approximations, but is obliged to wait until a more powerful computer can be utilized. He is also working on the long-period terms in the

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