Colloque de l'Union Astronomique Internationale
N° 37 – DECALAGES VERS LE ROUGE ET EXPANSION DE L'UNIVERS

HUBBLE DIAGRAM OF QUASARS

by
Vahé Petrosian and Mark A. Soldate
Stanford University
Stanford, California

Nous montrons que le diagramme de Hubble (densité de flux - décalage spectral) des quasars ne peut-être utilisé pour tester l'hypothèse que leur décalage vers le rouge est cosmologique. Toutefois, nous montrons que cette relation peut-être utilisée pour tester l'hypothèse locale dans laquelle le décalage cosmologique des quasars est supposé être petit et la majeure partie du décalage être due à quelque cause non cosmologique. Une telle hypothèse générale et locale peut-être rejetée avec une certitude supérieure à 95%. Nous montrons que ce pourcentage peut augmenter jusqu'à 99% en faisant des suppositions moins générales mais très plausibles.

I. Introduction

The purpose of this paper is to compare the observed relation between redshift z and (optical and radio) flux densities S_o, S_r of quasars (s-z relation) with the theoretically expected relations assuming first a cosmological origin for their redshift and then assuming that the bulk of their redshift is intrinsic (or some cause other than expansion of the universe) and that they are relatively local objects. Some aspects of this topic have been discussed previously by one of us (Petrosian, 1974; Paper I) and by others (McRea, 1972; Bahcall and Hills, 1973; Burbidge and O'Dell, 1973). These earlier analyses were concerned with S-z relation of the brightest quasars at which time only a few complete samples were known. As stressed in Paper I the completeness of the sample to limiting optical and radio flux densities S_o^0, S_r^0 is very important.

In this paper we analyze the combined data from three samples, totally containing 107 sources. Our analysis will be similar to that of Paper I (we refer the reader to Paper I for details) except that here we consider the relation between the redshift and the average fluxes rather than the

https://doi.org/10.1017/S0252921100053689 Published online by Cambridge University Press

fluxes of the brightest objects. We also include in our analysis the fact that there are two limiting fluxes.

II. Cosmological Hypotheses

If the redshift of quasars is cosmological, then the luminosities F_o and F_r are related to flux densities S_o and S_r through the relation $F_x = y^2 S_x$, where y^2 is 4π times the square of the luminosity distance and is a function of the redshift and the cosmological model. Representing the frequency distribution of quasar by a differential luminosity function $\psi(F_o, F_r, z)$, such that $\psi dF_o dF_r dz$ is the comoving number density of quasars with luminosities between F_o and $F_o + dF_o$, F_r and $F_r + dF_r$ and redshift between z and $z + dz$, we define three cumulative luminosity functions as follows:

$$\varphi_o(F_o, F_r, z) = \int_{F_o}^{\infty} \psi(F_o', F_r, z)\, dF_o', \quad \varphi_r = \int_{F_r}^{\infty} \psi(F_o, F_r', z)\, dF_r',$$

$$\varphi_{or} = \varphi_{ro} = \int_{F_r}^{\infty}\int_{F_o}^{\infty} \psi(F_o', F_r', z)\, dF_o' dF_r' . \tag{1}$$

It can then be shown that for a sample with limiting flux densities S_o^0 and S_r^0 the number of sources between redshift z and $z + dz$ and their total optical flux density are

$$dn(z) = dV(z)\, \varphi_{or}(S_o^0 y^2, S_r^0 y^2, z) \quad ,$$

$$dS_o(z) = dV(z) y^{-2} \int_{S_o^0 y^2}^{\infty} \varphi_r(F_o, S_r^0 y^2, z) F_o dF_o \quad , \tag{2}$$

where $V(z)$ is the comoving volume up to redshift z for the given cosmological model (exchanging o and r in this equation and in what follows would give the total radio flux density). Then the average flux density (normalized to the limiting flux) at redshift z or within a redshift bin centered on z is

$$\frac{dS_o(z)}{S_o^0 dn(z)} = \frac{\int_{S_o^0 y^2}^{\infty} \varphi_r(F_o, S_r^0 y^2 z) F_o dF_o}{y^2 S_o^0\, \varphi_{or}(S_o^0 y^2, S_r^0 y^2, z)} \quad , \tag{3}$$

https://doi.org/10.1017/S0252921100053689 Published online by Cambridge University Press

so that the average flux-redshift relationship is determined (for an assumed cosmological model) by the form of the luminosity function and its evolution. Since all our information about the luminosity function comes from the same S-z data, it is impossible to test the cosmological hypothesis. Any S-z data can be made compatible with the cosmological hypothesis and with any cosmological model through appropriate choice of the luminosity function (cf. paper I for some examples).

Consequently, we shall not consider the cosmological hypothesis any further and concentrate on the local hypothesis in the remainder of this paper.

III. Local Hypothesis

If the observed redshift of quasars are partly cosmological in origin, then the problem is more complicated and we need an extra parameter in the description of the frequency distribution of the sources by a luminosity function. The luminosity function becomes $\psi(F_r, F_o, z_{in}, z_c)$ where z_c is the cosmological redshift and z_{in} is some other source of redshift: $(1+z) = (1+z_{in})(1+z_c)$. Clearly, this added degree of freedom would make such a general local hypothesis even less accessible to testing through the S-z relation than the cosmological hypothesis. However, if the cosmological redshift is comparable to the intrinsic redshift, none of the so-called difficulties with the cosmological hypothesis is alleviated. Consequently, we consider here the only hypothesis of interest which is the local hypothesis where the cosmological redshifts are small; $z_c \ll z_{in} \approx z$. According to this hypothesis only local quasars are detected because their luminosities are small. It is, therefore, unlikely that there will be any strong variation of the intrinsic properties (such as luminosity, density or intrinsic redshift) over the small cosmological redshift range. Thus we can eliminate the dependence of the luminosity function on z_c and set $z_{in} = z$.

For small cosmological redshifts, $z_c \ll 1$, we can use the Euclidean approximation, so that the luminosity of a source at a distance r is given by $F_x = S_x(1+z)^2 4\pi r^2$. Such a source would be in a sample limited to S_x^0 up to a maximum volume $V_m = \frac{4\pi}{3} r_m^3 = \frac{4\pi}{3}\left[F_x / S_x^0(1+z)^2 4\pi\right]^{3/2}$.

Because of the existence of two limiting fluxes S_o^0 and S_r^0, the limiting volume for some sources will be determined by the optical limit and for some sources by the radio limit. Sources with $F_o/F_r = S_o/S_r \equiv \xi > S_o^0/S_r^0 \equiv \xi_o$ would be radio limited (RL) and sources with $\xi < \xi_o$ will be optically limited (OL). The number of sources in each of these two categories are then given by

$$\frac{dn_o(z)}{dz} = (1+z)^{-3}/(3\sqrt{4\pi})\int_0^\infty dF_r\int_0^{\xi_o F_r} dF_o\psi(F_o,F_r,z)(F_o/S_o^0)^{3/2}, \text{ OL},$$

$$\frac{dn_r(z)}{dz} = (1+z)^{-3}/(3\sqrt{4\pi})\int_0^\infty dF_r(F_r/S_r^0)^{3/2}\int_{\xi_o F_r}^\infty \psi(F_o,F_r,z)dF_o, \text{ RL}. \quad (4)$$

The contribution to the optical flux density of sources with luminosities F_o and F_r and redshift z is proportional to the integral from 0 to r_m of $\psi S_o 4\pi r^2 dr$ which is equal to $\psi F_o r_m/(1+z)^2$. Integrating this over all luminosities we find for the total optical flux in z to $z + dz$

$$\frac{dS_o(z)}{S_o^0 dz} = \int_0^\infty dF_r\int_0^{\xi_o F_r} \psi(F_o,F_r,z)(F_o/S_o^0)^{3/2} dF_o/\sqrt{4\pi}\,(1+z)^3, \text{ OL}$$

$$\frac{dS_o(z)}{S_o^0 dz} = \int_0^\infty (F_r/S_r^0)^{\frac{1}{2}} dF_r\int_{\xi_o F_r}^\infty \psi(F_o,F_r,z)\left(\frac{F_o}{S_o^0}\right) dF_o/\sqrt{4\pi}(1+z)^3, \text{ RL}. \quad (5)$$

Similar relations can be found for the radio fluxes. It is evident from the first of equations (4) and (5) that the average optical (or radio) flux density within different redshift bins of optically (or radio) limited sources is a constant equal three times the optical (or radio) limiting flux density; $< S_o >/S_o^0 = dS_o(z)/S_o^0 dn(z) = 3$. This is a very general result independent of the details of the luminosity function and the limiting fluxes of the sample. Thus we can combine the objects in different samples and test the constancy of the ratio $< S_o >/S_o^0$ or $< S_r >/S_r^0$ for the optically and radio limited objects, respectively.

There are various statistical tests that could be carried out. We present here only the results from linear regression analysis on 90 sources from various samples [3CR sample of Schmidt (1968), the 4C sample of Lynds and Wills (1972), and a new larger but yet unpublished sample of 4C sources kindly made available to us by Lynds and Wills]. Most of the sources are radio limited. The filled circles in figure (1) show the variation of $\log(\langle S_r\rangle / S_r^0)$ with $\log\langle z\rangle$ when the data is divided into six redshift bins, each containing 13 or 12 sources. The 95% confidence interval for the true slope and the confidence level for rejection of the slope zero expected from the local hypothesis are also indicated. In general, this confidence level is different from different subsamples and bin numbers. In all cases tested by us we find that the local hypothesis can be rejected at the confidence level greater than 95% [1].

It is evident from equations (4) and (5) that if

$$\int_{\xi_o F_r}^{\infty} F_o \psi(F_o, F_r, z)\, dF_o \propto \xi_o F_r \int_{\xi_o F_r}^{\infty} \psi(F_o, F_r, z)\, dF_o \;, \qquad (6)$$

which would be the case for a power law luminosity function ($\psi \propto F_o^{-n}$, independent of z), then the ratio $dS_o(z)/S_o^0 dn(z)$ for the radio limited objects would also be independent of redshift. Similarly, if the radio and optical luminosities are statistically correlated (which is evident from the statistical correlation between radio and optical flux densities), then the luminosity function can be written, for example, as $\psi(F_o, F_r, z) = \psi(F_r, z) f(F_o/F_r)$ in which case it can be shown that for radio limited objects the average optical flux

$$\left\langle \frac{S_o(z)}{S_o^0} \right\rangle = dS_o(z) \Big/ S_o^0 dn(z) = 3 \int_{\xi_o}^{\infty} f(x)\, x\, dx \Big/ \xi_o \int_{\xi_o}^{\infty} f(x)\, dx \;, \qquad (7)$$

is also a constant whose value depends only on ξ_o. Thus for samples with the same value of ξ_o we can combine both the optically and radio limited

---

[1] Note we have not carried out the test whether or not the average flux is three times the limiting flux. Such a test is similar to the V/V_m analysis, whereby one expects $\langle V/V_m\rangle = \langle (S/S^0)^{-3/2}\rangle = \frac{1}{2}$, which was discussed by Rowen-Robinson (1969) and Lynds and Wills (1972). Their test is not as general as the one discussed here because they did not distinguish between the radio and optically limited sources. Furthermore, the present test is more general since absence of correlation could rule out also the local hypothesis with a power law cosmological density evolution which is independent of the intrinsic luminosities and redshift. For example, for a density evolution $n(r) = n_o(r/r_o)^{\alpha}$, the ratio of $\langle S_x\rangle / S_x^0$ is still expected to be independent of redshift but its expected value for the whole sample is $(3+\alpha)/(1+\alpha)$ instead of 3 and the expected value of $\langle V/V_m\rangle = (3+\alpha)/(6+\alpha)$ instead of $\frac{1}{2}$.

https://doi.org/10.1017/S0252921100053689 Published online by Cambridge University Press

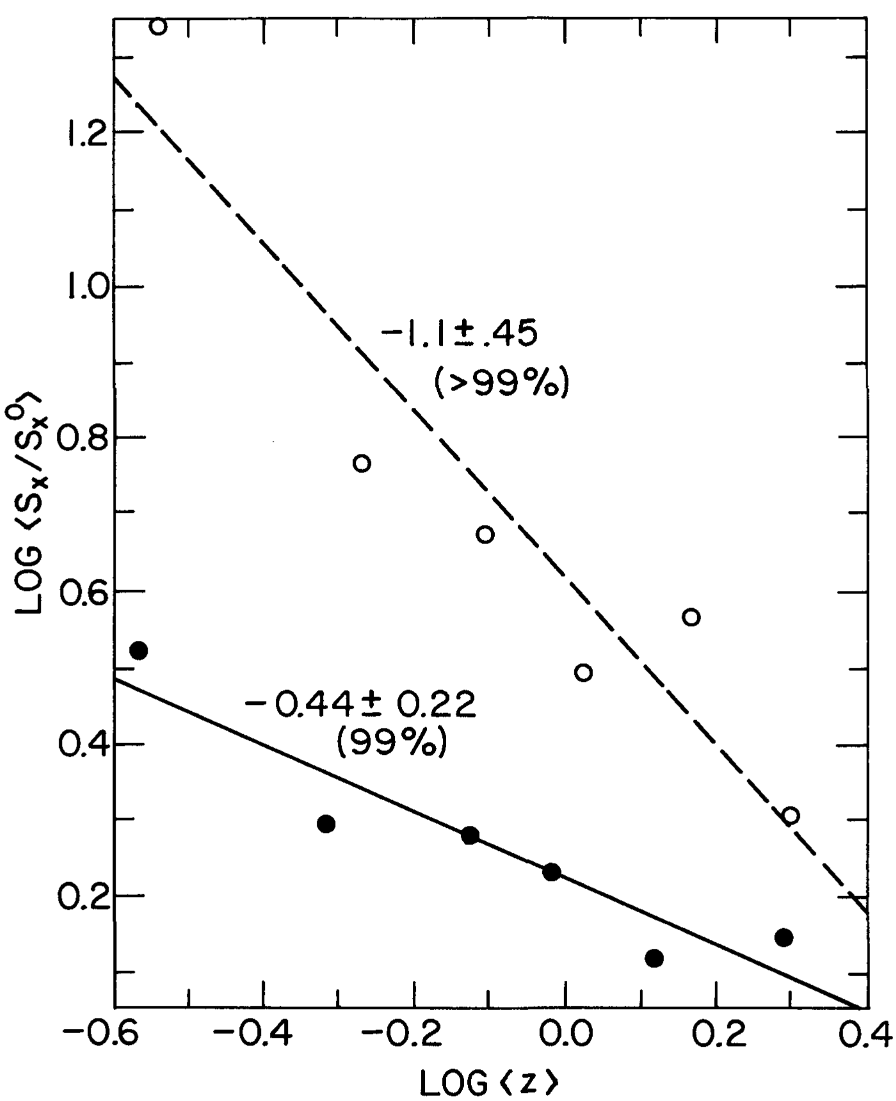


Figure 1

Correlation of average flux density with average redshift. Solid line and filled circles for radio flux density of radio limited objects (75 objects). Dashed line and open circles for optical flux of all sources (optically and radio limited) from samples with $\xi_o = S_o^0 / S_r^0 = 0.23$ (93 sources). The numbers indicate the 95% confidence interval for the true slope and (in parenthesis) the confidence level for rejection of the zero slope.

https://doi.org/10.1017/S0252921100053689 Published online by Cambridge University Press

objects and look for correlation between $<S_o>/S_o^0$ (or $<S_r>/S_r^0$) and redshift.

From the sample of sources described above we have chosen the sub-samples which all have $\xi_o = 0.229$ and carried out a linear regression analysis of the $\log (<S_o>/S_o^0) - \log <z>$ data.

The open circles and the dashed lines show the result along with the 95% confidence interval of the true slope and the confidence level for rejection of the zero slope. For different sub-samples and bin numbers tested by us this confidence level is mostly >99.9 %, and is >99% for all cases.

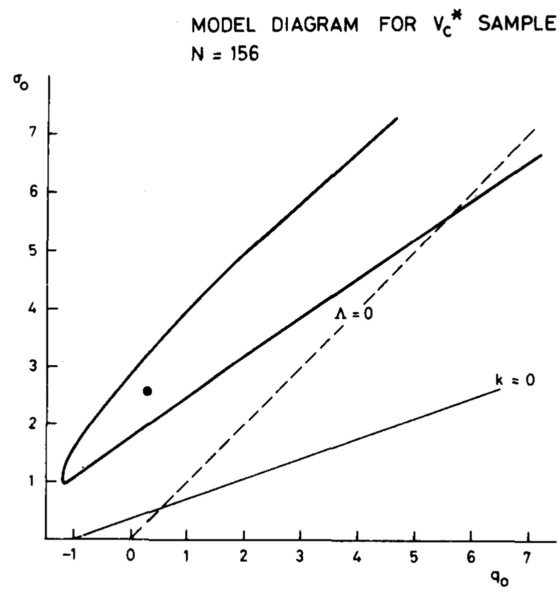
References

1. Bahcall, J. N., and Hills, R. E. 1973, Ap. J., 179, 699.
2. Burbidge, G. R., and O'Dell, S. L. 1973, Ap. J., 183, 759.
3. Lynds, R., and Wills, D. 1972, Ap. J., 172, 531.
4. McCrea, W. H. 1972, in "External Galaxies and Quasi-stellar Objects" (IAU Symposium No. 44), ed. D. S. Evans (Dordrecht:Reidel), p. 283
5. Petrosian, V. 1974, Ap. J., 188, 443.
6. Rowan-Robinson, M. 1969, Nature, 224, 1094.
7. Schmidt, M. 1968, Ap. J., 151, 393.

https://doi.org/10.1017/S0252921100053689 Published online by Cambridge University Press

DISCUSSION

M.F. BARNOTHY AND J.E. SOLHEIM: The four complete samples (3CR; 4C of Lynds and Wills; 4C of Schmidt and Schmidt's optical samples), as well as a sample of 141 quasars with photoelectrically determined UBV magnitudes, when corrected for the loss of faint objects at high redshifts, reveal a magnitude - logz relation with a slope of 3.58, and 3.55, respectively, magnitude per decade redshift. The difference from the value of 5, found for galaxies, has a confidence level better than 99 %. We determined with a computer program containing all general relativistic models, the best model fit to the data. The figure shows in the (q_o, σ_o), plane, the range of the cosmological models within 65 % confidence level that agree with the observational data. Within these limits none of the relativistic models with zero cosmological constant can reproduce the experimental results, unless quasars have a very large luminosity evolution, or the mass density in the universe exceeds by orders of magnitude the critical mass density.



Model diagram with "error ellipse", representing 65 % confidence level fit to the data of the four complete samples.

V. PETROSIAN: As I mentioned, if one assumes that redshifts are cosmological, then the slope of the logz -m relation is determined not only by the cosmological model but also by the slope and evolution of the luminosity function. Therefore, I disagree that the slope 3.5 to 3.8 is evidence for non-zero cosmological constant. I would also like to stress the importance of completeness of the sample used in any analysis of the Hubble diagram.

https://doi.org/10.1017/S0252921100053689 Published online by Cambridge University Press

J.P. VIGIER: Of course, if one assumes that all quasars are local, one finds your result i.e. rejection of the local assumption. The result is completely different if one assumes, as first suggested by Dr. Rowan-Robinson, that we are dealing with two QSO families, compact-flat and steep-(spectra) extended. In that case the result is completely different and a preliminary utilization of methods of Bell and Fort yield a different result; the compact-flat could be local, the steep-extended would be more or less at their cosmological distances.

V. PETROSIAN: We have not carried out this analysis for flat and steep spectrum sources separately. Such a separation would of course reduce the sample size and lead to a less significant result. The Bell and Fort model assumes no cosmological evolution of intrinsic properties and can be ruled out by this analysis unless the cosmological redshifts become sizable.

E.L. TURNER: John Bahcall and I have recently analyzed the Hubble diagram for the most luminous QSO s in a complete sample of 112 objects provided by Schmidt. After correction for sampling biases and the Scott effect, the radio and optical Hubble diagrams are fit very well by a standard cosmological model with $q_o = +1$, $\Lambda = 0$, and no luminosity evolution. The upper limit to luminosity evolution (for luminous QSOs) in this model is one or two e-foldings over a period ($\sim H_o^{-1}$) spanning $10^2 - 10^3$ typical QSO lifetimes and during which the QSO space density evolves by $\sim e^{10}$.

V. PETROSIAN: The analysis of the Hubble diagram does not say anything about density evolution but can be utilized for setting limits on pure luminosity evolution. However, such a limit would depend on the assumed cosmological model. Of course any observed luminosity evolution can be alteratively interpreted with evolution of the shape of the luminosity function.

J. TERRELL: I am pleased to see local quasars being considered by Dr. Petrosian. However, his assumption of no correlation between distance and source strength is not suitable for the case of very local quasars, emitted from the center of our own Galaxy.

For this very local model it would be expected that the strength

https://doi.org/10.1017/S0252921100053689 Published online by Cambridge University Press

of the synchrotron radiation, presumably resulting from relativistic motion through the gas in the vicinity of our Galaxy, would depend both on gas density, i.e. on present (retarded) distance of quasars from the center of our Galaxy, and on velocity through this gas, i.e. on Doppler redshift, again correlated with distance on this basis (Science 154, 1281, 1966; Astrophys. J. 147, 827, 1967).

V. PETROSIAN: The assumption of absence of cosmological evolution of intrinsic parameters is the simplest and most plausible assumption for the general class of local models. For local models which predict specific relationships between the intrinsic properties of quasars and their distances these relations must and could be taken into account in the analysis of the logz-magnitude relationship. Whether the model proposed by Dr. Terrell agrees with the Hubble diagram depends on the result from such an analysis.

https://doi.org/10.1017/S0252921100053689 Published online by Cambridge University Press