VERY HIGH ENERGY COSMIC GAMMA RAYS (1)

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Résumé. — *Les collisions entre les particules de grande énergie du rayonnement cosmique et le gaz interstellaire produisent des rayonnements secondaires variés, dont des pions neutres qui se dégradent en rayons* γ *très énergétiques. On emploie le modèle hydrodynamique de* Landau-Milekhin *des chocs proton-proton pour calculer le spectre des pions produits qui correspondent à des rayons* γ *cosmiques d'énergie supérieure à* 10 *GeV. On obtient une fonction-source pour ces rayons en combinant les spectres de production et de dégradation des pions avec les données sur le flux de proton du rayonnement cosmique primaire. Le spectre* γ *résultant n'obéit pas à la même loi de puissance que les spectres déduits de l'hypothèse usuelle d'une raie unique pour les pions dans le système du centre de masse des deux protons. L'intensité des rayons* γ *énergétiques dans l'espace est calculée à partir d'un modèle simple d'Univers. On compare les résultats à des estimations plus anciennes de la photoproduction des protons et l'on trouve que les chocs proton-proton et proton-photon semblent contribuer de façon sensiblement égale à l'intensité du rayonnement* γ *intergalactique d'énergie supérieure à* 10^{15} *eV.*

Abstract. — *Collisions of high energy cosmic rays with intergalactic gas produce various secondaries, including neutral pions that decay into high energy* γ *rays. The* Landau-Milekhin *hydrodynamical model for proton-proton collisions is used to calculate the pion production spectrum corresponding to cosmic* γ *rays of energy above* 10 *Gev. A source function for these high energy* γ *rays in space is found by combining the pion production and decay spectra with the primary cosmic ray proton flux. The resulting* γ *ray spectrum follows a different power law than spectra based upon the usual assumption of a line spectrum for the pions in the center of mass system of the colliding protons. The high energy* γ *ray intensity in space is calculated for a simple model universe. By comparison with previous estimates for the proton photoproduction process, it is found that proton-proton and proton-photon collisions appear to contribute about the same order of magnitude to the intergalactic* γ *ray intensity above* $\sim 10^{15}$ *eV.*

Резюме. — Столкновения между частицами космического излучения большой энергии и межзвездным газом порождают различные вторичные излучения, среди которых нейтральные пионы переходящие в очень энергетические лучи γ. Применена гидродинамическая модель соударений протонпротон Ландау-Милехина для вычисления спектра порожданных пионов, соответствующих космическим лучам γ с энергией превышающей 10 гэв. Получена функция-источник для этих лучей, комбинируя спектры образованиям распада пионов с данными о протонном потоке первичного космического излучения. Вытекающий спекть не удовлетворяет тому же степенному закону, что спектры выведенные из обычной гипотезы единственной линии для пионов в системе центра массы из двух протонов. Интенсивность энергетических лучей γ в пространстве вычислена исходя из простой модели Вселенной.

Результаты сравнены с более давними оценками фотопорождения протонов и найдено, что соударения протон-протон и протон-фотон повидимому участвую в довольно равной мере в интенсивности межгалактического излучения γ с энергией превышающей 10^{15} эв.

1. Introduction.

Cosmic gamma rays with energies above 100 MeV are expected to be produced by the bremsstrahlung of high energy electrons against intergalactic gas, and by the (inverse) Compton scattering of high energy electrons with thermal photons in interstellar and intergalactic space. In addition, the collisions of high energy cosmic ray protons with gas nuclei and photons will produce gamma rays [1].

While high energy electrons dissipate energy more rapidly than protons, it has been estimated that gamma ray production by electrons is important in the " low " energy range 100 MeV-100 GeV [2]. The physics of these electron processes is well understood and the accuracy of the calculations of the gamma flux is limited only by the astrophysical data. However, the physics

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of high energy proton collisions is not well-known, and the calculations to date of the resulting gamma ray flux are based on very simple models.

In the present paper, a more exact treatment of the production of gamma rays by proton-proton collisions is given than in earlier simple calculations. The gamma rays result from the prompt decay of neutral pions produced in the collisions. The other significant proton collision process which produces gamma rays is photoproduction ; this occurs when protons undergo collisions with thermal photons in space [3] :

$$\gamma + p \rightarrow \pi^0 + p. \tag{1}$$

The nonelectromagnetic process considered here occurs when high energy protons collide with gas nuclei :

$$p + p \rightarrow p + p + a\pi^+ + b\pi^- + c\pi^0. \tag{2}$$

The relative importance of Eqs (1) and (2) depends, among other things, on the relative densities of photons and gas in interstellar and intergalactic space. In the present paper, it is assumed that production of gamma rays above 10 GeV occurs predominantly in intergalactic space. Recent downward revision [4] of the estimates of the thermal photon density in intergalactic space suggests that the $p - p$ process, Eq. (2), dominates the photoproduction process at lower energies.

The source function for the production of gamma rays in intergalactic space by proton-proton collisions, q_γ (E_γ) cm^{-3} sterad^{-1} Gev^{-1} s^{-1}, is given by

$$q_\gamma(E_\gamma) = n\,\sigma^t_{pp} \int_{E_p^L(E_\gamma)}^{E_p^U(E_\gamma)} dE_p \,.\, j(E_p) \,.\, m_\pi(E_p)$$

$$\times \int_{E_\pi^L(E_p,E_\gamma)}^{E_\pi^U(E_p,E_\gamma)} dE_\pi \,.\, f_{p\pi}(E_p, E_\pi)\; f_{\pi\gamma}(E_\pi, E_\gamma) \tag{3}$$

where n protons cm^{-3} is the intergalactic gas density, σ^t_{pp} cm^2 is the total cross section for production of pions in proton-proton collisions (σ^t_{pp} is approximately energy independent at high energies),

$$j(E_p) = K_p\,E_p^{-\eta} \text{ cm}^{-2} \text{ Sterad}^{-1} \text{ Gev}^{-1} \text{ s}^{-1}$$

is the intergalactic cosmic ray proton intensity, $m_\pi(E_p)$ is the pion multiplicity, $f_{p\pi}(E_p, E_\pi)\, dE_\pi$ is the fraction of pions produced in the range E_π to $E_\pi + dE_\pi$ by cosmic ray protons with energy E_p, and $f_{\pi\gamma}(E_\pi, E_\gamma)$ is the energy spectrum of the pion decay. Both $f_{p\pi}$ and $f_{\pi\gamma}$ are normalized to unity. The functional forms of the integration limits depend explicitly on the shape of the pion energy distribution, $f_{p\pi}$ [5]. In the present paper, it is assumed that the intergalactic cosmic ray intensity above about 100 GeV is comparable to the measured galactic cosmic ray intensity.

In Section 2 the Landau-Milekhin hydrodynamical model solution for $f_{p\pi}$ will be briefly discussed and the exact model solution of Eq. (3) will be presented along with an approximate form which is convenient for examining the energy spectrum. In Section 3 a relativistic Euclidian world model, with absorption, is used as the framework in which the calculations of the high energy γ ray background radiation are made.

2. PION AND γ RAY PRODUCTION IN SPACE.

The Landau hydrodynamical model of high energy nucleon-nucleon collisions describes the nucleons as colliding relativistic fluids, and their subsequent decay as the expansion and decomposition of the composite nucleonic fluid into pions, nucleons, kaons, etc.... This model is theoretically plausible and, in the light of present cosmic ray data, very acceptable in the energy region above 50-GeV incident proton energy. This can be seen by reviewing some of the predictions for the model [5 through 8]. The pion multiplicity is predicted to follow an $E_p^{1/4}$ law, where E_p is the cosmic ray proton energy. This is in agreement with a large portion of available data [9]. Observational and experimental data on the asymmetrical angular distribution of the produced pions can be reasonably fitted by the hydrodynamical model [5 and 10]. Lastly, observational data indicate that a small fraction of the collision products carries off a large fraction of the available kinetic energy. This action can also be explained by using the hydrodynamical model.

It can be shown [5] that the hydrodynamical model predicts the pion energy spectrum to be of the form

$$f_{p\pi}(E^p, E_\pi) \,\alpha\, \frac{1}{E_\pi}\, e^{-\frac{1}{2L}\ln^2\frac{E\pi}{2E_0}}, \tag{4}$$

where

$$L = 0.56 \ln \frac{E_p}{M_p} + 1.6$$

and

$$E_0 = \mu c^2 \left[\frac{1 + E_p/M_p}{2}\right]^{1/2},$$

where μ is the pion mass and M_p the proton mass. Using this form of $f_{p\pi}$, along with the well-known square pulse shaped energy spectrum of the π^0 decay, the γ ray source function is found to be given by

$$(5)\quad q_\gamma(E_\gamma) \propto \frac{1}{\beta}\left\{\frac{e^{-4\beta/4\beta+1}}{\sqrt{4\beta+1}}\cdot\left[1-\Phi\left(u\sqrt{4\beta+1}+\frac{1}{\sqrt{4\beta+1}}\right)\right] + \frac{e^{-4(u^2+u)\sqrt{\beta}}}{2}\left[1-\Phi\left(u\left\{\sqrt{4\beta}-1\right\}-1\right)\right] - \frac{e^{+4(u^2+u)\sqrt{\beta}}}{2}\left[1-\Phi\left(u\left\{\sqrt{4\beta}+1\right\}+1\right)\right]\right\}$$

where

$$\beta = 1.5(\eta - 1)$$

$$u \approx -1 + \left[1 + 2\left[\ln\frac{E_\gamma}{\sqrt{2}\mu c^2} + 1.6\right]\right]^{\frac{1}{2}}$$

and

$$\Phi(x) \equiv \sqrt{\frac{2}{\pi}}\int_0^x e^{-t^2}\,dt.$$

A log plot (see Fig. 1) of $q_\gamma(E_\gamma)$ versus energy E_γ, as given by Eq. (5), suggests that the complicated expression given by Eq. (5) might be approximated by the simple power law

$$(6)\qquad q_\gamma(E_\gamma) \propto E_\gamma^{-\nu},$$

where ν is a slowly varying function of the γ ray energy. Over a large part of the γ ray energy spectrum, it is possible to make the approximation

$$(7)\qquad \nu \approx 3.2,$$

when η is given as 2.6 [4]. The present results can be compared directly with those of GINZBURG and SYROVATSKII [4], who made a calculation similar to the above, except that they made some rather crude approximations about the pion's energy in order to obtain $f_{p\pi}$. The approximations used by GINZBURG and SYROVATSKII are in lieu of a pion production model. Employing their parameters [4], Eq. (6) can be written as

$$(8)\qquad q_\gamma(E_\gamma)\,dE_\gamma \approx 4n\sigma_{pp}^t\,E_\gamma^{-3.2}\,dE_\gamma,$$

whereas they find

$$(9)\qquad q_\gamma(E_\gamma)\,dE_\gamma \approx \frac{n\sigma_t^{pp}}{40}\,E_\gamma^{-2.8}\,dE_\gamma.$$

It is seen that the present, more detailed calculation suggests a more rapidly changing energy spectrum. It should be noted that Eqs (8) and (9) give about equal γ ray fluxes at energies of 10^{12} eV, whereas Eq. (8) gives a flux which is reduced by a factor of 1000 from that of the GINZBURG- SYROVATSKII model at 10^{19} eV.

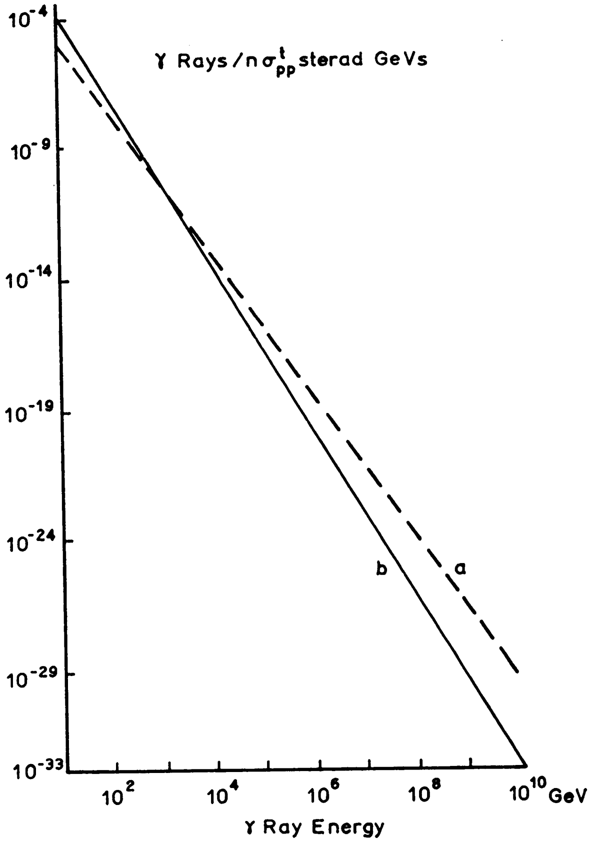

FIG. 1. — γ Ray Production Spectrum.
(*a*) GINZBURG-SYROVATSKII model ; (*b*) Hydrodynamic model.

3. THE HIGH ENERGY γ RAY BACKGROUND IN SPACE.

In order to calculate the high energy γ ray background of intergalactic space, it is necessary to employ a definite cosmological model, so that γ ray production from distant sources may be taken into account in a reasonable manner. For the present estimate, it is sufficient to consider a universe which is expanding uniformly. In addition, the theoretical description of the high energy γ ray flux requires an examination of the processes by which γ rays are attenuated. In an expanding universe model, γ rays are attenuated by the relative motion of the sources and the observer, as well as by their interaction with particles and waves in intergalactic space.

First consider the attenuation due to scattering.

The Compton scattering cross section can be approximated by

$$(10) \qquad \sigma_i(E_\gamma) = \pi r_i^2 \frac{m_i}{E_\gamma}\left(\ln \frac{2E_\gamma}{m_i} + \frac{1}{2}\right),$$

where

$$r_i = q_i/4\pi m_i$$

and

$$m_i = \text{mass of particle with charge } q_i.$$

Table I presents the absorption coefficient k for γ rays of energy 2×10^{10} eV, and the optical depth τ over a distance $R = 10^{28}$ cm = Hubble radius. The unimportance of attenuation by Compton scattering of γ rays with energy of 2×10^{10} eV can be clearly seen. Eq. (10) shows that γ rays with energy greater than 2×10^{10} eV will be even less attenuated by Compton scattering.

TABLE I

COMPTON SCATTERING LOSSES

	n	k	τ
Electrons...	10^{-5} cm^{-3}	7×10^{-34} cm^{-1}	10^{-5}
Protons ...	10^{-5} cm^{-3}	1×10^{-37} cm^{-1}	10^{-9}

The scattering of γ rays by static potentials has been studied with regard to both pion production and elastic (DELBRUCK) scattering [11]. Unfortunately, all of these calculations are for the electrostatic Coulomb field of a nucleus [11 and 12]. In intergalactic space only magnetic fields are expected to be appreciable, and since a magnetic field is described by a vector potential field, a direct comparison cannot be made to the electric field scattering. However, an order of magnitude calculation [5] indicates that this type of scattering will be reduced by a factor of 10^{25} (using a conservatively high estimate of the intergalactic magnetic field of 10^{-5} gauss) from elastic photon-photon scattering.

Investigations of γ rays scattered by photons have indicated that elastic scattering is considerably less important than electron pair production scattering [5, 12 and 13]. The major problem for calculating the absorption of γ rays by photons is the complete lack of information concerning some portions of the electromagnetic spectrum in space [13], and the disagreement on the hypothesized electromagnetic energy density [4 and 12]. NIKISHOV [12], using photon densities of the order of 0.1 eV cm^{-3}, and GOLDREICH and MORRISON [13], using measured radio wave intensities, find absorption coefficients of the order of 10^{-26} cm^{-1} for some ranges of gamma ray energy; cf. Table II. If the thermal photon density 10^{-3} eV cm^{-3} suggested by GINZBURG and SYROVATSKII [4] is used, photon scattering of γ rays may be negligible in the 10^{12} eV region, but is probably appreciable for gamma rays above 10^{18} eV.

TABLE II

PHOTON-PHOTON SCATTERING LOSSES *

E_γ(eV)	10^{11}	10^{12}	10^{13}	10^{18}	10^{19}	10^{20}
$10^{27}\, k$(cm^{-1})	0.05	7	2	0.5	2.5	5
τ	0.005	0.7	0.2	5	25	50

(*) The first two lines adapted from NIKISHOV (Ref. 12) and from GOLDREICH and MORRISON (Ref. 13). The third line gives the optical depth for an intergalactic thermal photon energy density of 10^{-3} eV cm^{-3} and R = 10^{28} cm.

The combined effect of the cosmological red shift and scattering will be considered now in a simple expanding Euclidian universe in which the red shift energy and number effects, but not solid angle effects, will be considered. At a distance x from the observer, the photon-photon scattering absorption coefficient of γ rays produced at a distance $r (r > x)$ is defined by means of the integral

$$(11) \qquad k(E_r, x) = \int_0^\infty d\varepsilon_x\, n_x(\varepsilon_x)\, \sigma(E_r, \varepsilon_x),$$

where E_r is the energy a γ ray must have when produced at r in order to be measured as having energy E_0 at the observer:

$$E_0 = \frac{E_r}{1 + Hr}$$

with H = Hubble constant $\approx 10^{-28}$ cm^{-1}. In Eq. (11) $n_x(\varepsilon_x)$ is the number of photons of energy ε_x cm^{-3} at x, and $\sigma\,(E_r, \varepsilon_x)$ is the photon-photon scattering cross section.

The intergalactic γ ray intensity is then given by

$$(12) \quad j_\gamma(E_\gamma)\, dE_\gamma = \int_0^{1/2\,H} dr\, \frac{q_\gamma(E_r)}{1 + Hr} \exp\left(-\int_0^r k(E_r, x)\, dx\right) dE_\gamma,$$

where q_γ is given in Eq. (8). The upper limit for the distance, one-half the Hubble radius, limits the calculation to γ rays from the nearer part of the observable universe, and, hopefully, increases the validity of the present nonrelativistic calculation.

Using the fact that the photon-photon pair production cross section has a strong maximum near threshold, and assuming that the intergalactic photon density is approximately constant over the time $R/2c \approx 5 \times 10^9$ years, the γ ray flux can be evaluated approximately. Of course, the approximate spectrum follows the same power law, $E_\gamma^{-3.2}$, as the source function.

The results of our calculation are compared in Figure 2 (attenuation above 10^{18} eV is neglected) with those of Hayakawa and Yamamoto [3] for photoproduction. Their original fluxes, based on a thermal photon density of 0.2 eV cm^{-3}, are presented as well as the scaled down fluxes based upon a density of 10^{-3} eV cm^{-3}. The low and high energy ends of the spectrum calculated by Hayakawa and Yamamoto depend on the density of hard and soft thermal photons, respectively. While cosmological considerations enter into these photon densities, it is usually assumed that these densities are not particularly high. Thus, it appears that with large uncertainties due to the paucity of information about intergalactic matter and radiation, the proton-proton production process dominates photo-production below $\sim 10^{15}$ eV, while above 10^{15} eV the contributions of the two processes appear to be of the same order of magnitude.

The γ ray estimates are compared with the extensive air shower measurements by Burbidge and Gould [2]. A comparison of our calculations with the results of Chudakov and Jelley, and details of the calculations outlined here are presented in [5].

In conclusion, it is noted again that the model of pion production used is an extremely important factor in the calculation of the γ ray spectrum. Thus, using a reasonable model of the strong interaction between protons leads to a cosmic γ ray production spectrum with a different spectrum law from that usually obtained by simple approximations.

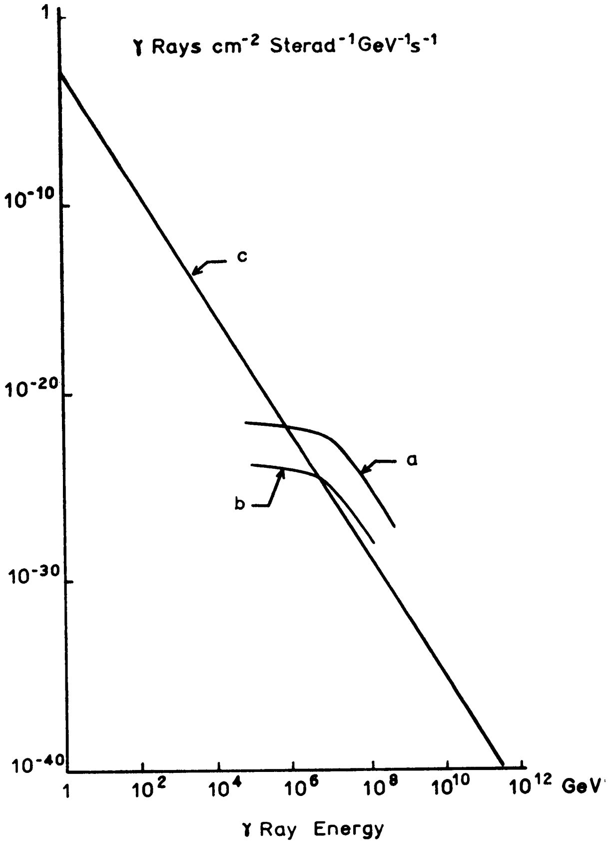

Fig. 2. — γ Ray Flux in Space. (*a*) Hayakawa-Yamamoto calculation with $n_{ph} = .2$; (*b*) Same with $n_{ph} = .001$; (*c*) γ rays from proton-proton scattering.

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