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The Mass–Ratio Distribution of Short–Period Binaries

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ABSTRACT: We present a new algorithm to derive the mass–ratio distribution of an observed sample of spectroscopic binaries. The algorithm replaces each binary of unknown inclination by an ensemble of virtual systems with a distribution of inclinations. We show that contrary to a widely held assumption the orientations of each virtual ensemble should *not* be distributed randomly in space. A few iterations are needed to find the true mass–ratio distribution. Numerical simulations clearly demonstrate the advantage of the new algorithm over the classical method. We have applied the new algorithm to the recent large sample of G-dwarf spectroscopic binaries, and got a uniform or perhaps a slightly rising linear mass–ratio distribution. This result suggests that the mass–ratio distributions of short–period and long–period binaries are substantially different. It also indicates that the mass distribution of the secondary stars is not the same as that of the single stars.

1. INTRODUCTION

Large samples of spectroscopic binaries have been recently detected by precise systematic radial–velocity surveys (e.g., Latham *et al.* 1988; Duquennoy *et al.* 1991). These samples can help to resolve the long standing controversy over the mass–ratio distribution of short–period binaries (e.g., Abt & Levy 1976; Halbwachs 1987; Trimble 1990). We might be able to find out whether the distribution is different for long– and short– period binaries (Abt & Levy 1976; Duquennoy & Mayor 1991), and whether the mass distributions of the primaries and the secondaries are similar to that of the single stars (Abt & Levy 1976; Tout 1991). Unambiguous answers to these questions can provide important clues to the formation of close binary systems (e.g., Bodenheimer *et al.* 1992).

The mass–ratio distribution of any sample of spectroscopic binaries cannot be derived directly, because the mass ratio of the single–line systems cannot be deduced; only the mass function can be derived directly from the observations. For a binary system with a primary mass M_1 and a secondary mass M_2, the mass function is

$$f(M_2) = M_1 \frac{q^3}{(1+q)^2} \sin^3 i \,, \tag{1}$$

where q is the mass ratio ($= M_2/M_1$) and i is the inclination of the binary orbit relative to our line of sight. Since the inclination is not known, the mass ratio can *not* be derived, even when the primary mass can be estimated from its spectral type.

The problem of unknown inclinations was already addressed back in the 1920s (e.g., Aitken 1935), and since the 1970s by many workers (see Halbwachs 1987, Tout 1989, and Trimble 1990 for reviews). One statistical approach was to assign the expected value of $\sin^3 i$ to *all* single–line spectroscopic binaries of the sample. Aitken quoted Campbell and Schlesinger, who had suggested using an average value of 0.589 for $\sin^3 i$ in an ideal unbiased sample of binaries. They derived this value by averaging $\sin^3 i$ over all possible angles. However, this method fails to reconstruct the true mass–ratio distribution, because one of its basic assumptions is incorrect. We demonstrate the failure of the Campbell and Schlesinger (CS) method by using some simulated samples, and point out the specific reasons for its failure.

We present here a new iterative statistical algorithm (Mazeh & Goldberg 1992a,b), which replaces each binary with an ensemble of virtual systems of nonrandom orientations. Numerical simulations of ideal samples, without any observational effects, show the success of the new method to reconstruct the correct mass–ratio distribution. We also present the results of a recent analysis (Mazeh *et al.* 1992) that uses the new algorithm to derive the mass–ratio distribution of the nearby G-type dwarfs. The analysis is based on the results of the systematic radial–velocity survey of nearby G-dwarfs accomplished recently by Duquennoy & Mayor (1991). Finally, we comment on a very preliminary analysis of a sample of halo binaries, taken out of the Carney–Latham large survey (Carney & Latham 1987).

2. THE FAILURE OF THE CAMPBELL & SCHLESINGER METHOD

In order to demonstrate the failure of the CS method, we applied it to a *simulated* sample of 2000 binaries, with a uniform mass–ratio distribution and with random orientations. We set the period distribution of the sample to be uniform in $\log P$, between 1 and 1000 days, and the primary mass to be $1\mathcal{M}_\odot$. We then calculated the mass function for each binary by using Equation 1. The simulated sample was then analyzed, using the only information available for each binary in real samples — the primary mass and the mass function, ignoring the information about the mass ratio of each binary.

The results of this exercise are presented in Figure 2a. Clearly, the CS method is drastically biased toward lower mass ratios. In Figure 2b we present a similar simulation with a mass–ratio distribution which rises linearly toward unity. The intrinsic bias of the CS method is very prominent again. Many of our simulations yielded similar results.

To understand the reasons for the failure of the CS method, we must first clarify its basic assumptions. Explicitly, the method assumes that the orbital planes of the sample are randomly oriented. However, this assumption is not enough. Actually, the CS method implicitly assumes random orientations within certain subsets of the sample — subsets with a constant value of the mass function. Otherwise, it would not be possible to assign the same value of 0.589 to every binary of the sample.

This further assumption is not correct, because the mass function is not an independent variable of the sample. Our basic assumption is that the independent variables are the mass ratio, the inclination, and possibly the period of the binaries. The mass function depends on these variables through Equation 1, and therefore is a dependent variable. To demonstrate this point we plot in Figure 2 some constant mass–function contours on the $q - \sin i$ plane, for primary mass of $1\mathcal{M}_\odot$. The two axes represent independent variables, while clearly the con-

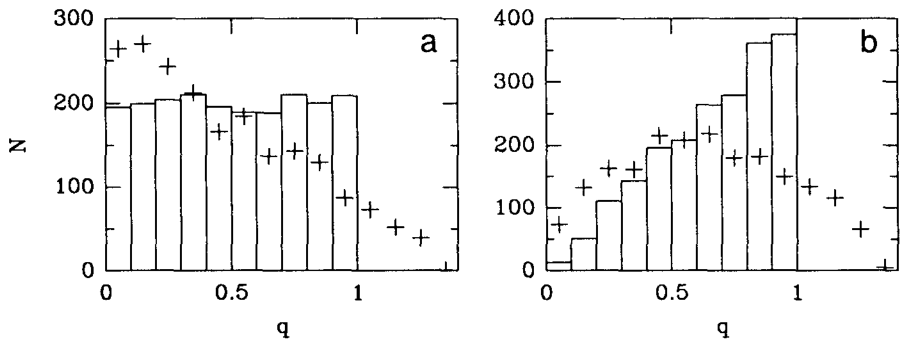

FIGURE 1. Numerical simulations to test the classical Campbell & Schlesinger method. The histogram shows the true distribution of the simulated sample. The pluses represent the results of the method. (a) A simulation with a uniform distribution. (b) A simulation with a monotonously increasing distribution of $N(q) = 2q$.

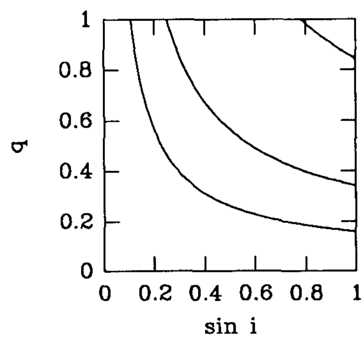

FIGURE 2. Contours of constant mass function in the mass–ratio – $\sin i$ plane.

tours do not. As can be seen from Figure 2, once we choose the mass function to equal some value, the two variables i and q are not independent anymore. Consequently, if a sample includes only systems with mass ratio smaller than unity, then $\sin i$ has a lower limit, which is seen clearly in Figure 2. Averaging over $\sin^3 i$ should be carried out only from this lower limit up to unity.

Moreover, the probability of a system with a given mass–function value to have a certain inclination i depends on the mass–ratio distribution as well as on the $\sin i$ distribution. To illustrate this point consider an extreme situation where all the binaries of the sample have only one q value – q_0. In such a case the probability of a system with a mass–function f_0 to have an inclination i is different from zero only for i_0, which solves Equation 1 for given q_0 and f_0. Thus, the inclination distribution is not random, and depends on the specific value of f and on the q distribution. In order to take into account the dependence on the mass–ratio distribution, which is exactly the distribution we are looking for and is therefore unknown, we need some iterative approach.

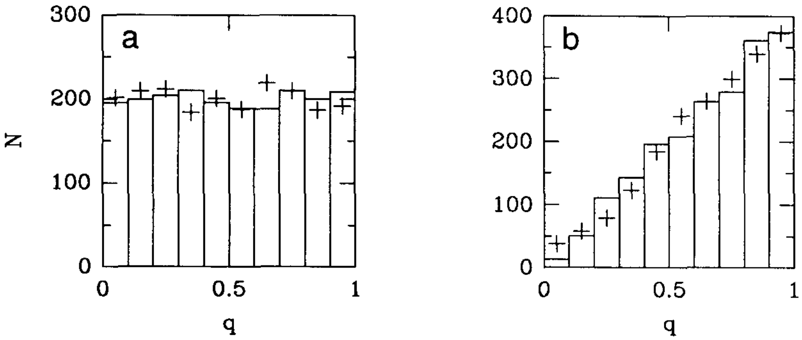

FIGURE 3. Numerical simulations to test the proposed new algorithm. The histogram shows the true distribution of the simulated sample. The pluses represent the results of the algorithm. The figure should be compared with Figure 1. (a) A simulation with a uniform distribution. (b) A simulation with a monotonously increasing distribution of $N(q) = 2q$.

3. THE PROPOSED ALGORITHM

The proposed algorithm considers each observed binary as drawn from a large subset of binaries with different inclinations, but with the same period, primary mass, and mass function. To take into account the special shape of the inclination distribution, the algorithm replaces each binary with an ensemble of N virtual systems, which mimics the parent subset of the binary. For normalization purposes, each of the virtual systems represents $1/N$ binaries. We know the mass function, the primary mass, and the inclination of each virtual system, and therefore can solve for its mass ratio. The mass–ratio histogram of the ensembles of all binaries included in the sample represents our best estimation of the mass–ratio distribution of the observed sample.

Each binary is replaced by an ensemble of binaries whose inclinations are distributed between some lower limit, $i_{\rm min}$, and $90°$. In order to account for the dependence of the inclination distribution on the unknown mass–ratio distribution, the algorithm performs few iterations. To begin, we assume a uniform distribution of q, and assign each virtual ensemble the resulting inclination distribution. We then derive the q distribution of the whole sample. This new distribution is the first–order approximation derived by the algorithm. It is then used as the input for calculating the second–order iteration, and so on. This process is continued till the nth order approximation is statistically indistinguishable from the $(n-1)$th one. (A detailed description of the algorithm and its subtleties can be found in Mazeh & Goldberg 1992a,b.)

To test the proposed new algorithm we applied it to the same simulated samples that were used to demonstrate the drawbacks of the CS method. The results are presented in Figure 3. A comparison of Figures 1 and 3 shows the advantage of the proposed algorithm over the CS method. Numerous tests with different samples yielded the same conclusion.

4. THE DISTRIBUTION OF THE NEARBY G-DWARF BINARIES

Duquennoy & Mayor (1991) have published recently the results of a careful radial–velocity survey of the nearby G-dwarfs. This survey is unique in its high precision, long time coverage, and completeness. Together with Duquennoy & Mayor, we applied the new algorithm to this sample, in order to get the true mass–ratio distribution of the nearby close binaries (Mazeh *et al.* 1992).

Duquennoy & Mayor found 37 spectroscopic binaries in their complete sample, with period range of up to 30,000 days. We focused on the study of short–period binaries, and therefore included in the analysis only systems with periods shorter than 3000 days. We were left with 23 binaries, all listed in Mazeh *et al.* (1992).

Many kinds of observational selection effects can limit and bias the mass–ratio distribution of a binary sample. Some famous examples are the bias introduced by binaries detected by visual means, and the distortion caused by a magnitude limited survey (Öpik 1924; Branch 1976; Halbwachs 1987; Trimble 1990). Another selection effect that can distort completely the high end of the distribution is the relative weight of the double–line binaries (Trimble 1990). The present binary sample is free of all these selection effects (Mazeh *et al.* 1992). The only selection effect that limits this sample is the inability to detect low amplitude binaries. This effect was corrected for by a process described by Mazeh & Goldberg (1992a,b).

Figure 4 presents our results. Tokovinin (1992b) obtained a similar distribution by using his maximum likelihood technique (Tokovinin 1992a). The figure implies a uniform mass–ratio distribution, or perhaps a slightly rising distribution. A formal linear fit to the data yields

$$N(q) = (2.6 \pm 1.4) + (2.9 \pm 2.3)\, q\,, \tag{2}$$

where q is the mass ratio and $N(q)$ is the number of binaries in each bin. The best linear fit is also plotted in Figure 4. A uniform distribution, which is a linear function with a slope equal to zero, is only 1.25σ away from the best fit, and therefore is still possible. We note that any possible local feature could not have been detected by the present work, because of the small sample and consequently the small number of bins. Nevertheless, the simple shape of the distribution is intriguing.

The mass–ratio distribution of the short–period binaries obtained here is significantly different from the corresponding distribution of the long–period binaries found in the same sample of stars (Duquennoy & Mayor 1990, 1991). While the long–period distribution rises toward small q, with a possible drop off at q equals 0.1 or less, the short–period distribution is uniform and might even rise toward unity. Abt & Levy (1976) have noticed a very similar difference between the short and the long period binaries.

The primaries of the present sample all have about the same mass, close to $1\mathcal{M}_\odot$. Therefore, the obtained mass–ratio distribution represents also the mass distribution of the secondaries. The linear, possibly uniform, distribution for the secondary mass is very different from the mass distribution of single stars. All the functions suggested to describe the mass distribution of single

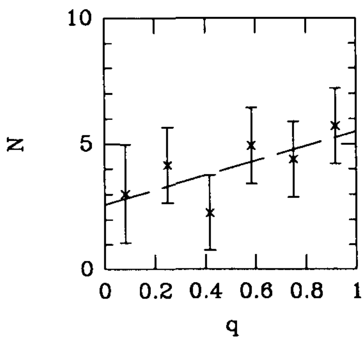

FIGURE 4. **The mass–ratio distribution of the nearby short–period binaries with G-dwarf primaries.**

stars (e.g., Kroupa *et al.* 1991) include a substantial drop when the stellar mass changes from, say, 0.3, to $1\mathcal{M}_\odot$. We find, to the contrary, a uniform and even possibly rising distribution in this range. Apparently, some mechanism which acts during the formation of short–period binaries affects the secondary mass.

Finally, we would like to report on an on–going study in which we try to derive the mass–ratio distribution of the halo binaries (Mazeh, Goldberg, Latham, Carney, & Torres, in preparation). We are using the results of the large Carney & Latham (1987) survey of proper–motion stars. Although the study is only in its *very* preliminary stages, it seems that the mass–ratio distribution of the short–period halo binaries is not substantially different from that of the nearby G-dwarfs, except for a possible excess of white dwarf secondaries. This possible result, if confirmed, is of particular interest. It might indicate that the uniform mass–ratio distribution is perhaps a common feature of the short–period binaries.

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7. DISCUSSION

QUIRRENBACH: Your methd assumes basically that you know the primary masses exactly (e.g., from its spectral type). How large are the errors induced by errors in the adopted primary masses in practical cases?

MAZEH: The estimate for the error of the primary mass is 10% for the G dwarfs of Duquennoy and Mayor. These error induce $10\%(n_b)^{1/2}$ error in each bin, where n_b is the number of systems in the bin. It turns out that this error is very small relative to the errors caused by the fact that the number of systems in each bin is small in the G-dwarf sample.

TOKOVININ: How did you estimate the error bars in the distributions that you've shown?

MAZEH: We ran a few hundred simulations with exactly the same number of binaries as the sample, with the same mass-ratio distribution as we have found, generated by random numbers.

ABT: I am glad to see how well you and I agree. Where you find for solar-type stars N = constant + $2.9q$, we found N = constant + $2.5q$. But I do not understand your plot of q *versus* $\sin i$; q cannot be a function of $\sin i$.

MAZEH: Indeed, our result agrees with your work, published years ago. The quantity q is certainly not a function of $\sin i$, however, the mass-function is a function of the two variables.

ZINNECKER: Would you care to tell us a little more about how your new approach differs from the previous work of J.L. Halbwachs that you mentioned?

MAZEH: I should point out that J.L. Halbwachs' paper is a seminal one and gave us many important clues. I should also point out that Tokovinin developed also a different algorithm and applied it to the G dwarf sample, and he got a very similar result.

SCARFE: What do your distributions look like for single-line binaries only?

MAZEH: I do not have it here. However, we put a lot of effort to combining correctly the SB1 and SB2 binaries. What we have done is to analyze separately the SB1 with the iterative algorithm and then to add the SB2. Obviously, for the SB2, we do not need to generate the ensemble of virtual binaries as the mass-ratio is known.

HEACOX: What you are doing is statistical modelling of the affects of inclination, in order to infer an estimate of the true distribution of mass ratios from an observed distribution of mass function. I believe this is correct, and suggest that similar modelling is required to translate from observed distribitions of binary separations to those of semi-major axes. The method of doing so is discussed in the poster paper of Heacox and Gathright.

MAZEH: I agree.