On 101.07: Graham Jameson writes: The author establishes that

$$A^{1/3}G^{2/3} \leqslant L(a, b) \leqslant \tfrac{1}{3}A + \tfrac{2}{3}G, \tag{1}$$

where $A = A(a, b)$ and $G = G(a, b)$. It is a rather neat fact that this pair of inequalities contains, almost for free, the further information that the number $\frac{1}{3}$ is optimal on both sides. In other words, for any $r > \frac{1}{3}$, one can find a and b such that $L(a, b) < A^r G^{1-r}$, and for any $s < \frac{1}{3}$, one can find a and b such that $L(a, b) > sA + (1 - s)G$.

To show this, recall first, as stated in the Note, that for positive x, y and $0 < r < 1$, we have $x^r y^{1-r} \leqslant rx + (1 - r)y$. What we need is the following partial converse, showing that we cannot reduce the r on the right-hand side: if $0 < s < r < 1$, then there exists $\delta > 0$ such that whenever $1 < \frac{x}{y} < 1 + \delta$, we have

$$x^r y^{1-r} > sx + (1 - s)y. \tag{2}$$

To prove this, first take the case $y = 1$. Let $f(x) = x^r - sx - (1 - s)$. Then $f(1) = 0$ and $f'(1) = r - s > 0$. So for some $\delta > 0$, we have $f(x) > 0$, hence $x^r > sx + (1 - s)$, when $1 < x < 1 + \delta$. Statement (2) now follows by applying this to $\frac{x}{y}$.

Note next that if $a < b$, then $a < G < A < b$, so $1 < \dfrac{A}{G} < \dfrac{b}{a}$.

Now given $r > \frac{1}{3}$, apply (2) with $s = \frac{1}{3}$ and the right-hand side of (1): if $1 < \frac{b}{a} < 1 + \delta$ then $\frac{A}{G} < 1 + \delta$, so $A^r G^{1-r} > \frac{1}{3}A + \frac{2}{3}G \geqslant L(a, b)$. Similarly, given $s < \frac{1}{3}$, apply (2) with $r = \frac{1}{3}$ and the left-hand side of (1): if $1 < \frac{b}{a} < 1 + \delta$, then $sA + (1 - s)G < A^{1/3}G^{2/3} \leqslant L(a, b)$.

Maryam Mirzakhani (1977 – 2017)

Maryam Mirzakhani, who was born in Tehran, Iran, was, in 2014, the first woman to win the prestigious Fields Medal ‘for her outstanding contributions to the dynamics and geometry of Riemann surfaces and their moduli spaces’. Mourned by the mathematical community, she died of cancer on 14 July 2017 in hospital in California.

Mirzakhani took her bachelor's degree at Tehran's prestigious Sharif University and moved to Harvard in 1999. In 2004 she presented her doctoral thesis which secured her widespread recognition and a fellowship at the Clay Mathematics Institute. She rose rapidly from assistant professor to professor at Princeton University from 2004 to 2008 and then became professor at Stanford University.

For those interested in how young minds develop, Mirzakhani admitted that as a child she did rather poorly at mathematics and aspired to be a writer. Initially, it was an elder brother who fired her interest in mathematics and this was sustained by inspiring teachers at secondary school. Representing Iran, she won gold medals in the International Mathematical Olympiads of 1994 and 1995 with a perfect score in the latter.

https://doi.org/10.1017/mag.2017.152 Published online by Cambridge University Press

Maryam Mirzakhani speaking at the 2015 Clay Research Conference which was held 30 September 2015 in the Andrew Wiles Building, University of Oxford (*Mott Carter/Clay Mathematics Institute*)

As hinted above the core of Mirzakhani's work is the study of Riemann surfaces. Mathematicians are interested in these most general of surfaces as they can study analytic functions, complex analysis, geometry and algebra on them. The classification of such surfaces is facilitated by the remarkable uniformisation theorem which states that every Riemann surface is obtained from either Euclidean, spherical or hyperbolic geometry. Teichmüller spaces are examples of moduli spaces which are the geometric solution to the problem of classification. The study of the behaviour of geodesics on hyperbolic surfaces led Mirzakhani to number-theoretic and analytical considerations and required an understanding of a great variety of structures on the set of all surfaces of the same genus. Her work encompassed ergodic theory which involves the study of recurrence properties of measure – preserving transformations and symplectic geometry which plays an important part in understanding the actions of groups on manifolds.

Acknowledgement

The Editor wishes to thank the Clay Mathematics Institute for permission to use the photograph above.

10.1017/mag.2017.152

GRAHAM T. Q. HOARE
3 Russett Hill, Chalfont St Peter SL9 8JY

https://doi.org/10.1017/mag.2017.152 Published online by Cambridge University Press