A FEW FAMILIES OF CAYLEY GRAPHS AND THEIR EFFICIENCY AS COMMUNICATION NETWORKS

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Cayley graphs are highly attractive structures for communication networks because of their many desirable properties, including vertex-transitivity and efficient routing algorithms [4]. The families of circulants and cube-connected graphs are among the most popular Cayley graphs for efficient communication networks [11, 12]. The diameter, forwarding and optical indices, bisection width and Wiener index of a network are among the most important parameters to measure the efficiency of the network [2, 5–7].

Circulant graphs and, in particular, circulant graphs with small degrees are interesting models for communication networks [1]. However, our knowledge of many of their parameters, including the arc-forwarding index, edge-forwarding index, directed and undirected optical indices, are very limited, except for very few special cases. We study the family of circulant graphs of degree 4 and obtain lower and upper bounds for their forwarding and optical indices. We give approximation algorithms for the corresponding problems of the forwarding indices and optical indices with a small constant performance ratio. Our results on the family of circulant graphs of degree 4 are published in [3].

The family of recursive cubes of rings has received a lot of attention for communication networks [10], but many aspects of them have remained unknown. We study this family of graphs by redefining each of them as a Cayley graph on the semidirect product of an elementary abelian group by a cyclic group in order to facilitate the study of them by using algebraic tools. We give an algorithm for computing shortest paths and obtain the exact value of their diameters. We obtain

sharp bounds on the Wiener index, vertex-forwarding index, edge-forwarding index and bisection width of recursive cubes of rings. The cube-connected cycles and cube of rings are special recursive cubes of rings, and our results apply to these well-known networks. Our results on this topic are published in [9].

We introduce cube-connected circulants as a new family of cube-connected Cayley graphs and study their efficiency for communication networks [8]. We give an algorithm for computing shortest path routing and the exact value of the diameter of a cube-connected circulant. We observe that while recursive cubes of rings are special cube-connected circulants, these two families of cube-connected graphs have significantly different routing behaviours in general. Hence we develop results for 'proportional' graphs which will be useful in obtaining bounds for the edge-forwarding index of cube-connected circulants. We give sharp lower and upper bounds for the Wiener index, vertex-forwarding and edge-forwarding indices of cube-connected circulants. We study the embedding of cube-connected circulants into hypercubes and the embedding of hypercubes into cube-connected circulants. We show that cube-connected circulants outperform a few well-known network topologies in many aspects.

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