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THE GROUPS OF ORDER DIVIDING 256

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The central topic of the thesis is a description of the techniques used in a computer assisted determination of the groups of order dividing 256. There are 56,092 isomorphism types of groups of order 256.

The determination is carried out using an implementation of an algorithm for generating finite p-groups. A theoretical description of the basic algorithm was given by Newman [3]. The theory of this algorithm is developed and a detailed description of its implementation is given.

In carrying out the determination of the groups of order 128 and 256, practical difficulties arise in respect of space and computational time requirements. In order to resolve these difficulties, two important extensions of the basic algorithm are developed. The theory of these extensions and their implementations are also described. These techniques are used in calculating the orbits of a permutation group of degree 178,940,587.

Tree diagrams are used to display results obtained by applying the algorithms. The duplication by machine of the work of Hall and Senior [1] on the groups of order dividing 64 is discussed and the necessary algorithms for such a project developed. In this context, an algorithm is described that takes as input a p-group of small order and determines a permutation group of least degree that represents it.

The 2,328 isomorphism types of groups of order 128 are determined using the implementation. This calculation takes about 8 minutes of CPU time on a VAX 8700. The isoclinism families for the groups of order 128 are also determined. Using this material, previous work by Rodemich [5] on these groups is corrected. Tables are included which give invariants and other information on the isoclinism families.

The provision of access to the group descriptions obtained is addressed. A CAYLEY library for the groups of order dividing 128 and a database, developed to provide access to certain properties of the groups, are described. The CAYLEY library allows access to a group presentation in at most 0.4 seconds of CPU time on a VAX 8700. This library and some of its applications are also described in O'Brien [4] and Newman and O'Brien

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[3]. A similar library has been prepared for the groups of order 256. In determining presentations of the groups, descriptions of their automorphism groups are obtained. These descriptions are also included in the CAYLEY libraries.

The history of group determinations is briefly reviewed and, in light of this, there is a detailed discussion of the accuracy of the results obtained. In addition, two appendices are supplied. The first is a microfiche listing of the program used in carrying out the determination of the groups. The program is written in FORTRAN 77 and is available to all users of the data. The second appendix provides a summary of the data used as input to the program and a summary of the results. This appendix allows a user of the results to check, by hand or other techniques, their correctness.

References

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