Introduction to Set Theory and Topology, by K. Kuratowski. (Internat. Series of Monographs in Pure and Applied Mathematics, Vol. 13. Translated by L. F. Boron.) Pergamon Press, 1961. 283 pages. $7.50.

Contents are as follows: Part 1 (Set Theory): Propositional calculus, algebra of sets, propositional functions, finite and infinite set operations, the concept of power, cardinal numbers, ordering and well-ordering, order types and ordinal numbers; Part 2 (Topology): metric spaces, closure and limits, basic definitions, continuous and homeomorphic maps, separability, completeness and compactness, connectedness and local connectedness, continua, dimensions, an introduction to homology and homotopy, and a proof of the Jordan theorem.

Each chapter is followed by problems, the harder ones of which have hints to the solution (and introduce many new topics) and brief discussions. There is an index, a symbol list and two reading lists.

The book appears to be a careful and thoughtful contraction of the author's well-known treatise on topology and of his Polish book (written jointly with A. Mostowski) on the theory of sets. The logical notation is not that used on this continent but that should present no difficulty. No errors or misprints were found. The translation is smooth and the book reads very pleasantly, --something of a boon to advanced undergraduate and beginning graduate students of the subject.

Z.A. Melzak, McGill University

Mathematisches Wörterbuch, mit Einbeziehung der theoretischen Physik, edited in collaboration with many specialists by Josef Naas and Hermann Ludwig Schmid for the Institut of Pure Mathematics of the Deutsche Akademie der Wissenschaften. Pergamon Press, Oxford, London, New York, Paris, 1961. Two volumes; xii + 1043 and viii + 952 pages. Price L40/-/-.

This is an encyclopedia, listing alphabetically technical terms and their definitions, theories, theorems, principles of pure and applied mathematics and theoretical physics, including their history and main representatives. Each item is followed by an article of variable length and individual conception with large numbers of cross references. The names of eminent mathematicians of the past occur with the dates of their birth and death; their main places of activity and their most important fields of research are mentioned in all cases, mostly only in a few words. But taking into account the large number of enumerated mathematical objects called after Euler, we find exactly nine pages on him and his contributions to mathematics; similarly eleven pages on Gauss, but only five pages on Hilbert, one and a half pages on H. Weyl, and less than half a page on G. Cantor.

Naturally this is not a book to be read from beginning to end, though it is quite pleasant to read in this book, starting at a certain point and letting oneself be directed by the numerous references from one item to another one. Used in this way the work will provide useful entertainment for any length of time. Some of the articles are long and really instructive, others are only brief suggestions. Frequently they are supplemented by more or less extensive bibliographies on their subject.

In view of the complexity of modern mathematics the decision as to what to take into the book and what to leave out must have been exceedingly difficult in many cases. The average mathematician will probably expect the book to be a help in reviewing mathematical works which contain material not entirely within his own field of knowledge. The present reviewer has used the Wörterbuch in this way for some time and indeed found it reasonably useful in a number of cases. Similarly as with other "tools", one has to learn to use it. And this may well take a year or more.

This of course was not the only purpose for which the work has been compiled. As a matter of fact, the editors must have had in mind several disjoint classes of users. There are plenty of articles which by the nature of their subject as well as by their style are definitely meant for the accomplished mathematician or theoretical physicist. On the other hand, however, let us consider the article "Gerade(Linie)": Apart from a few remarks (7 half-lines) on the straight line in the foundations of some branches of geometry, this is a relatively detailed and entirely primitive exposition of about 100 half-lines on elementary analytical geometry of the straight line in the plane and in space, to be understood by every first-year or high school student who has learnt his course. There is no reference to pencils or bundles and not even a cross reference to the corresponding articles. An article of this kind is a waste of space from the point of view of the mathematician. There are others in the same vein.

In other cases one might subjectively differ in as to what an article should contain or not contain. Some articles provide excellent and up-to-date information; others (as for instance the article "Schlichte Funktion") tell us nothing about recent results (that is, after 1923). Naturally a certain amount of material is duplicated; on the other hand, different theories concerning one and the same subject are dealt with under different headings (e. g. "Schwingungsgleichung")and "Helmoltzsche Schwingungsgleichung"); but these are defects (if we may call them so) which are probably unavoidable. More serious appears to be a certain non-homogeneity in the matter of bibliographic references. What is meant may be explained by examples: In the article "Erzeugende Funktion in der Wahrscheinlichkeitsrechnung" there is no reference at all to the literature; under "Erweiterung von Gruppen" which now-a-days is dealt with in all good

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books on group theory, only the original papers by O. Schreier are quoted. The article "Charakteristisches Polynom einer Matrix" quotes only a work by M. Parodi "La localisation des valeurs caractéristiques des matrices et ses applications" Paris 1959, As a matter of contrast: The article on "Homologiealgebra" consists of 30 references only; the basic papers and all the recent books on the subject are quoted; at least cross references to some of the fundamental notions could have been given as far as they actually occur in the Wörterbuch. It goes without saying that in many articles a reference to literature would be out of place.

Some of the articles are more or less naturally very hard to understand for the non-specialist (and the specialist does not need them). But in some cases a few words would have been helpful. As an example we consider the article "Leere Menge (Nullmenge)". It says: "The set which contains no element; sign $\emptyset$. It has the cardinal and order type zero; sign 0. The sets $\emptyset$, $\{\emptyset\}$, $\{\{\emptyset\}\}$ ... are mutually different and together form the elements of an infinite set (axiom of infinity)." A more digestible statement of the "Axiom of infinity" can then be found under "Unendlichkeitsaxiom" to which word, however, no reference sign "$\wr$" is added in the "Leere Menge"-article.

The lexicographical order is sometimes questionable: "Liesche Algebra" followed by "Liebmannsches Mittelungsverfahren", "Liesche charakteristische Funktion", "Lieferungsqualitätsgrenze", "Liesche Geometrie der Kugeln", ..., "Lienardsche Differentialgleichung", ..., "Liesche Quadrik", etc. The system behind this apparent irregularity consists in that the adjectivating syllable "sche" (and other similar ones) is not counted, as is shown in the exhibited titles on top of the pages. For the non-German user of the work this is certainly disturbing.

There are not too many misprints except in the biographical articles. We find for instance that Hamilton lived from 1905 until 1865; that Weyl worked on "Theorie der Algebra", etc.

As with many other things it is naturally difficult to say which mathematicians should be honored by a biographical note in the Wörterbuch, and it is impossible to state which ones should not have been mentioned. But this reviewer has missed a note on R. Remak and one on Sylow; and he found the notes on E. Hellinger and on N. Tschebotaröw rather poor.

Altogether this is a useful book which should have a place at least in every University--or department--library. Many users will then find that they are getting used to it and wish to have their own copy.

Suggestions for a second edition: (1) Let the contributors sign their own articles. (2) Enhance the usefulness of the Wörterbuch by adding to each "Stichwort" the corresponding word in English, French, Italian, Russian.

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Infinitistic methods, Proceedings of the symposium on foundations of mathematics, Warsaw 1959. Pergamon Press. 362 pages. £5 net.

Under the influence of Hilbert's program to provide a sound basis for mathematics by purely finitary consistency proofs, logicians have worked in metamathematics for many years with one hand tied behind their backs. To mark the recognition of a new era, we have here the proceedings of a symposium on the foundations of mathematics with no holds barred: Everything goes, from Zorn's lemma to formulas of infinite length. The results are tremendously interesting and illuminate many different branches of mathematics.

The volume under review contains research papers that were presented at the 1959 Warsaw Symposium by the following authors:

J. Łoś, S. MacLane, R. Montague, C. Spector, G. Kreisel, A. Mostowski, L. Henkin, A. Heyting, Dana Scott, A. Robinson, R. L. Vaught, L. Kalmar, all in English.

P. Bernays, G. H. Müller, P. Lorenzen, R. MacDowell and E. Specker, R. Péter, all in German.

A.S. Ésénine-Volpine, L. Rieger, R. Fraissé, Gr. C. Moisil, all in French.

P.S. Novikov, in Russian.

It is clearly impractical to review all these papers or even to list their titles. Instead, I shall make some remarks about three of the papers that caught my fancy on first browsing through the volume.

(1) S. MacLane, Locally small categories and the foundations of set theory. There are several tentative proposals, for avoiding the paradoxes of set theory, the one accepted by most mathematicians being due to Gödel. One distinguishes between sets and classes, all sets are classes, but only sets can be members of classes. Many recent constructions in homological algebra have violated this principle; for example, people have considered the category of all categories, in spite of the fact that categories are usually too large to be sets. MacLane proposes a uniform method for overcoming these difficulties, and incidentally gives a very readable and concise account of the major concepts and recent developments in homological algebra.

(2) L. Henkin, Some remarks on infinitely long formulas. The paper deals with three ways of creating formulas of infinite length: First by allowing infinitary predicates, secondly by allowing infinite