DYNAMICAL EVOLUTION OF MULTI-COMPONENT CLUSTERS

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ABSTRACT. Presence of stars with disparate masses causes great differences in the dynamical evolution of star clusters from the evolution of single component clusters. One remarkable effect is acceleration of the evolution. Another effect is destabilization or stabilization. In two-component clusters equipartition at the cluster centre is nearly achieved if Spitzer's (1969) condition is satisfied. In multi-component clusters equipartition at the cluster centre is achieved if either the range of stellar mass is very narrow or the mass spectrum is very steep. Global equipartition is never achieved. Post-collapse evolution of multi-component clusters is discussed briefly and some remained problems are presented.

1. INTRODUCTION

Several pioneering works on multi-component clusters have been done. Michie (1963) calculated the escape rate from multi-component clusters. Spitzer (1969) obtained the condition for equipartition in two-component clusters. Hénon (1969) estimated the escape rate due to close encounters. Spitzer and Hart (1971) and Hénon (1971) made Monte Carlo simulations of multi-component clusters and revealed some basic properties of the dynamical evolution of multi-component clusters. Saslaw and De Young (1971) considered the condition for equipartition in clusters with continuous mass spectra. Aarseth (1974) made N-body simulations of isolated clusters and emphasized the importance of binaries. Angeletti and Giannone (1977b) found the importance of mass loss from stars. Recently Larson (1984) constructed a model of two-component clusters in post-collapse stage.

In this paper I do not mention every aspect of multi-component clusters discussed by the above authors but I would like to clarify the physical processes of gravitational encounters by focusing attention to the late evolution of cores of globular clusters. Therefore I do not mention on the effects of stellar mass loss or escape of stars from clusters, which are important in the early stages of dynamical evolution.

This paper comprises following subjects. Section 2 discusses

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dynamical evolution of two-component clusters, which are the simplest multi-component clusters. Section 3 discusses the evolution of clusters with components more than two. Section 4 discusses briefly the post-collapse evolution of multi-component clusters. Finally several remained problems are presented in section 5.

2. TWO-COMPONENT CLUSTERS

2.1. Basic Properties of Two-Component Clusters

In single component clusters there is only one motive force of evolution, that is the gravothermal instability (see e.g. Spitzer 1985). In the gravothermal instability of single component clusters, the energy is transported from the cluster core to the halo. On the other hand, in multi-component clusters, in addition to the gravothermal instability, energy exchange between components plays an important role.

One remarkable effect of presence of disparate masses is the acceleration of evolution. It takes about fifteen half mass relaxation times to reach the infinite central density in an isolated single component cluster (Cohn 1980, Marchant and Shapiro 1980). Here the half-mass relaxation time, t_{rh}, is defined by

$$t_{rh} = \frac{0.06 M^{1/2} R_h^{3/2}}{\langle m \rangle \, G^{1/2} \log \Lambda} \tag{1}$$

(Spitzer and Hart 1971), where R_h is the half mass radius, G is the gravitational constant and $\log\Lambda$ is the Coulomb logarithm.

The time needed for complete collapse becomes significantly shorter in multi-component clusters (e.g. Spitzer and Hart 1971). In a two-component cluster with $m_2/m_1 = 2$ and $M_2/M_1 = 0.11$, the collapse time is about one half of the single component cluster (see table 1a), where m_i is the mass of a star in the i-th component and M_i is the total mass of the i-th component. In two-component clusters with $m_2/m_1 = 5$, the collapse time is much shorter: In a cluster with $m_2/m_1 = 5$ and $M_2/M_1 = 0.072$, the collapse time is about one tenth of the single component cluster. For detail see tables Ia and Ib.

TABLE Ia. The time (in the unit of t_{rh}) required for the complete collapse in two-component clusters with $m_2/m_1 = 2$.

M_2/M_1...	0.0	0.001	0.01	0.05	0.11	1.0	9.0
t_{cc}...	15.4	15.3	13.6	10.2	8.5	9.6	13.6

TABLE Ib. The same as table 1a but for $m_2/m_1 = 5$.

M_2/M_1...	0.001	0.005	0.014	0.072	0.30
t_{cc}...	14.8	10.8	4.2	1.7	1.9

The acceleration of evolution in two-component clusters can be understood as follows. In order that a single component cluster evolves, the energy must be transferred from its core to its halo. Since the relaxation time at the halo is very long, it takes considerable time for the single-component cluster to evolve. On the other hand, a two-component cluster can evolve if the energy is transferred from the massive component to the less massive component. The relaxation time at the core is relatively short so that the two-component cluster can evolve much faster than the single-component cluster.

Another remarkable effect of presence of disparate masses is destabilization. An isothermal single component cluster is gravothermally unstable if the density contrast, ρ_c/ρ_b, is larger than 709 (Antonov 1962, Lynden-Bell and Wood 1968), where ρ_c is the central density and ρ_b is the density at the boundary. On the other hand, an isothermal two-component cluster with $m_2/m_1 = 10$ and $M_2/M = 0.3$ is gravothermally unstable with very small density contrast, i.e., $\rho_c/\rho_b > 19$ (Yoshizawa et at. 1978), where M is the total mass of the cluster.

The destabilization can be understood as follows. In order that a single component cluster causes the gravothermal instability, the energy must be transported from the core to the halo (Lynden-Bell and Wood 1968, Hachisu and Sugimoto 1978). In other words, the halo works as a heat reservoir. Therefore an extended halo is necessary for the single component cluster to cause the gravothermal instability. In a two-component cluster, the energy can be deposited in the less massive component so that no halo is necessary to cause the gravothermal instability.

The presence of disparate masses also causes stabilization (Katz and Taff 1983). The isothermal cluster with $m_2/m_1 = 10$ and $M_2/M = 0.003$ is stable if $\rho_c/\rho_b < 5012$ (Yoshizawa et al. 1978). This stabilization can be understood as follows. Figure A1 of Yoshizawa et al. shows that $\rho_2(0)/\rho_1(0)$ is constant (= about eight) along the marginally stable states near this model. This means that the development of the halo does not affect the stability or that the stability is determined by the state of the core. In other words, the instability is caused by the exchange of energy between components in the core. However, if the halo becomes too extended (for example, density contrast of the less massive component exceeds 709), the less massive component becomes unstable as a single component cluster. The density contrast at this stage is about $\rho_2(0)/\rho_1(0)$ times 709 so that it is quite larger than 709. In this sense, the stabilization in two-component clusters is deceptive.

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2.2. Dynamical Evolution of Two-Component Clusters

It has been well known that mass stratification occurs in two-component star clusters (e.g. Spitzer and Hart 1971, Saito and Yoshizawa 1976, Angeletti and Giannone 1977a). These authors employed either a Monte Carlo technique or a fluid dynamical approach. Recently Cohn (1979, 1980, 1985) devised a new technique to integrate the orbit-averaged Fokker-Planck equation (e.g. Lightman and Shapiro 1978) numerically. Cohn's method solves the orbit-averaged Fokker-Planck equation very accurately and numerical noises produced are very small. Therefore Cohn's method is the most suitable for the detailed study of slow evolution of large-N (say 10^6 stars) clusters due to two-body encounters. Inagaki and Wiyanto (1984, hereafter referred to as IW) made several simulations, using Cohn's method with the assumption of isotropy in velocity space. Assumption of isotropy will be enough for the study of evolution of the core because anisotropy will be small there. Moreover Cohn (1985) confirmed that in a single component cluster there is no differences in the density profile and in the collapse rate even if anisotropy of the velocity distribution is taken into account.

IW adopted Plummer's model as initial models. The density distribution of Plummer's model is given by

$$\rho_i(r) = \frac{3M_i}{4\pi r_0^3} \frac{1}{[1 + (r/r_0)^2]^{5/2}} \tag{2}$$

and its velocity dispersion is given by

$$\langle v_i^2 \rangle = \frac{GM}{2r_0} \frac{1}{[1 + (r/r_0)^2]^{1/2}} , \tag{3}$$

where r_0 is a scale length and G is the gravitational constant. Equations (2) and (3) show that the density profiles of both components are similar and the velocity dispersions are the same. In other words, the kinetic temperature of a component is proportional to the mass of a constituent star of the component. Figure 1 shows the evolution of the central density and temperature for the model with $m_2/m_1 = 2$ and $M_2/M_1 = 0.01$. The central potential is adopted as the time axis. In this model $\rho_2(0) = 0.01\rho_1(0)$ initially so that the initial overall evolution is governed by the less massive component. In other words, the less massive component collapses like a single component system. The massive component is considered to be floating in the potential well made by the less massive component. In such a case temperature of the massive component drops swiftly and the temperatures of the both components become nearly equal at about 8 t_{rh}. It should be noted that the temperature difference remains large at the halo (see figure 3 of IW) since the relaxation time is large there. After this epoch until the density of the massive component dominates the density of the less massive component, the difference in central temperatures between components remains negligible while the less massive component is collapsing. During this stage the density of the

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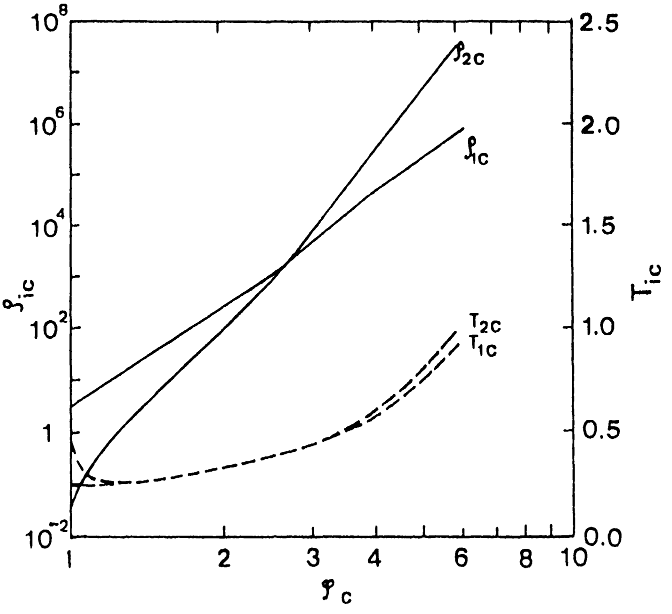

Figure 1. Time evolution of the central density and temperature for each component as a function of the central potential for the model with $m_2/m_1 = 2$ and $M_2/M_1 = 0.01$.

massive component increases more rapidly than that of the less massive component. The evolution at this stage is explained well by Lightman and Fall's (1978) model (see IW).

After the density of the massive component dominates that of the less massive component, the massive component causes the gravothermal instability and collapses independently from the less massive component. At this stage the temperature difference between components becomes large again. It should be noticed that even though the fraction of the massive stars is very small, mass stratification occurs finally and the temperature difference becomes large. In this sense equipartition holds only temporally.

If the fraction of the massive stars is large, for example, $M_2/M_1 = 1.0$, there is no initial stage such that the density of the less massive component dominates that of the massive component. Figure 2 shows the evolution of the central density and temperature of the model with $m_2/m_1 = 2$ and $M_2/M_1 = 1.0$. The temperature difference decreases initially and reaches the minimum value at about $t = 5.6t_{rh}$. At this epoch $(T_{2c} - T_{1c})/T_{1c} = 0.17$, where T_{ic} is the central temperature of the i-th component. The temperature difference grows monotonically after this epoch.

Figure 3a shows how the minimum of the relative temperature difference along the evolutional sequence, $[(T_{2c} - T_{1c})/T_{1c}]_{min}$, depends on M_2/M_1 for the models of $m_2/m_1 = 2$. It is seen that the temperature difference is larger as M_2/M_1 is larger. This value is larger for the

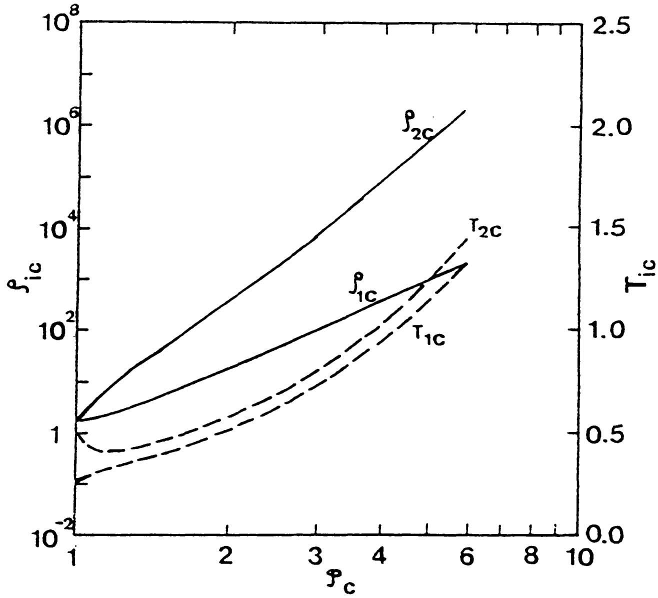

Figure 2. The same as figure 1 but for $M_2/M_1 = 1.0$.

models of $m_2/m_1 = 5$: $[(T_{2c} - T_{1c})/T_{1c}]_{min}$ is about 50% for the model of $M_2/M_1 = 0.1$ (figure 3b). Figures 3a and 3b show that the relative temperature difference is constant for both large M_2/M_1 and small M_2/M_1. The dependence of the relative temperature difference on M_2/M_1 is the

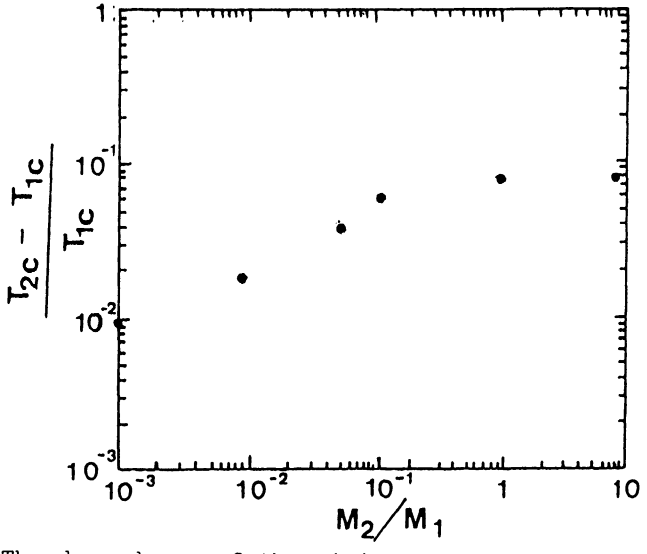

Figure 3a. The dependence of the minimum of the temperature difference on the fraction of the total mass of the massive component for the models of $m_2/m_1 = 2$.

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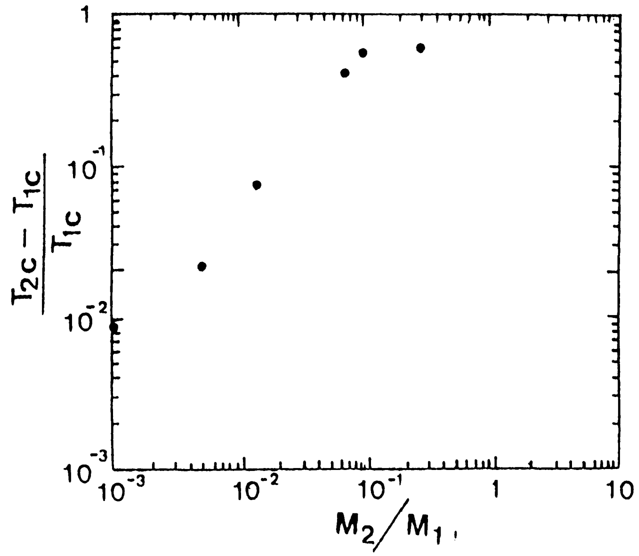

Figure 3b. The same as figure 3a but for $m_2/m_1 = 5$.

largest near the model with $M_2/M_1 = 0.16(m_2/m_1)^{-3/2}$, the value obtained by Spitzer (1969) as the critical value for existence of equipartition.

The evolution of the massive component after the central density of the massive component dominates the central density of the less massive component is nearly the same as that of the single component clusters. The collapse rate defined by $\xi_2 = t_{r2,c}\, d\ln \rho_{2c}/dt$ is the same as that of the single component clusters and $\langle v_{2c}^2\rangle \propto \rho_{2c}^{0.1}$, which is also the same relation as the single component clusters, where $t_{r2,c}$, ρ_{2c} and $\langle v_{2c}^2\rangle$ are the central relaxation time, density, and velocity dispersion, respectively, of the massive component. These relations holds even in the models of $m_2/m_1 = 5$ where the temperature difference is quite large.

IW started their simulations from Plummer's model, where equipartition does not hold initially. The evolution from models where the equipartition holds initially can be conjectured. Let us consider how clusters evolve if we start simulations from two-component Wilson's model, for example. The isotropized-orbit-averaged Fokker-Planck equation can be written in the form

$$\frac{\partial q}{\partial E}\frac{\partial f_i}{\partial t} - \frac{\partial q}{\partial t}\frac{\partial f_i}{\partial E} = 16\pi^2 G^2 \ln\Lambda \sum_j m_j^2 \frac{\partial}{\partial E}\Big\{f_i\Big[\int_{\varphi(0,t)}^{E} f_i f_j' q' \Big(\frac{\partial \ln f_i}{\partial E} - \frac{m_i}{m_j}\frac{\partial \ln f_j'}{\partial E'}\Big)dE' + q\int_E^0 f_i f_j' \Big(\frac{\partial \ln f_i}{\partial E} - \frac{m_i}{m_j}\frac{\partial \ln f_j'}{\partial E'}\Big)dE'\Big]\Big\} \quad (4)$$

(Inagaki 1980), where

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$$q = \frac{1}{3}\int_0^{\varphi^{-1}(E)} (2E - 2\varphi)^{3/2} r^2 dr. \tag{5}$$

In Wilson's model ln $f_i(E)$ is convex upwards because of the cut-off of the distribution function. In this case, equation (4) predicts that the energy flows outwards in the core region. This means that the core contracts (Inagaki 1980). As the core contracts, keeping equipartition in the core, the density of the massive component grows more rapidly than does the density of the less massive component (see IW). It should be reminded that the distribution function keeps convex upwards because the evolution of the distribution function is characterized by the extention of isothermal part into low energy region (see figure 1 of Cohn 1980). If the density of the massive component becomes high enough to cause the gravothermal instability, the gravothermal instability of the massive component takes place and the temperature difference between components becomes larger. Thus even if we start from models with equipartition initially, the final state is the gravothermal instability of the massive component and the temperature difference emerges.

Global equipartition in the sense that the average temperatures throughout a cluster do not depend on components is not achieved in any case. If the initial state is far from equipartition, the temperature at the halo does not change so much because the relaxation time is very long there. Therefore equipartition is not achieved in this case (see figure 8 of IW). Even if global equipartition holds initially, the temperature difference becomes large finally because of the final collapse of the massive component.

3. MULTI-COMPONENT CLUSTERS

Spitzer and Hart (1971) simulated the evolution of three-component clusters and found that the most massive stars condense strongly in the central region and the least massive stars concentrate in the halo. Spitzer and Shull (1975) made more detailed study of the evolution of three-component clusters and found that the final collapse of the most massive component is in the same way as do the cores of single component clusters. Hénon (1971) simulated evolution of a five component cluster with a flat mass spectrum and showed that evolution proceeds in the direction away from equipartition. Recently Stodółkiewicz (1985) made Monte Carlo simulations of globular clusters, using a realistic mass function.

In spite of the effort of these authors, some of the basic properties of the evolution of multi-component clusters had been unclear until Inagaki and Saslaw (1985, hereafter referred to as IS) made systematic simulations of multi-component clusters. IS integrated the isotropized-orbit-averaged Fokker-Planck equation, using Cohn's (1980) scheme. They assumed simple power law mass spectra of the form,

$$dM \propto m^{-\alpha} dm \qquad \text{for} \quad m_{min} \le m \le m_{max}. \tag{6}$$

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They made several simulations with different m_{max}/m_{min} and α (Table II).

TABLE II. The time (in the unit of t_{rh}) required for complete core collapse in multi-component clusters. In the table n is the number of components.

	$\frac{m_{max}}{m_{min}} = 10$ (n = 15)	$\frac{m_{max}}{m_{min}} = 10$ (n = 5)	$\frac{m_{max}}{m_{min}} = 4$ (n = 5)	$\frac{m_{max}}{m_{min}} = 2.8$ (n = 5)	$\frac{m_{max}}{m_{min}} = 2$ (n = 5)
$\alpha = 1.0$	2.6	4.0	---	7.5	10.4
$\alpha = 2.5$	1.9	2.0	4.3	6.6	10.0
$\alpha = 4.0$	4.3	4.9	5.7	7.3	----
$\alpha = 6.0$	9.0	10.4	9.6	9.7	11.1

The time for the complete core collapse in multi-component clusters is shown in Table II. One important conclusion drawn from tables I and II is that the collapse time is mainly determined by m_{max}/m_{min} and α not by the number of components. In multi-component clusters, the fastest collapse time in the unit of the half mass relaxation time is attained when α is about 2.5. The shortest collapse time is about 10 t_{rh} for five-component clusters with $m_{max}/m_{min} = 2$. This collapse time is comparable with that of the two-component cluster with $m_2/m_1 = 2$ and $M_2/M_1 = 0.05$. The collapse time of the fifteen-component cluster with $m_{max}/m_{min} = 10$ and $\alpha = 2.5$ is 1.9 t_{rh}, which is nearly the same as the collapse time of the two-component cluster with $m_2/m_1 = 5$ and $M_2/M_1 = 0.072$. Thus the shortest collapse time for given m_{max}/m_{min} can be obtained even with a two-component cluster with a certain value of M_2/M_1. The collapse time does not become shorter indefinitely even if very large m_{max}/m_{min} is adopted. The shortest collapse time is about the same for $m_{max}/m_{min} \gtrsim 5$.

Another important question is what is the condition for equipartition in multi-component clusters. Saslaw and De Young (1971) claimed that 'stable equipartition' could not be achieved for any reasonable mass spectrum. 'Stable equipartition' is the state such that equipartition is unaffected by small changes in the mass spectrum. Vishniac (1978), however, suggested that equipartition is possible if the mass spectrum of a cluster is sufficiently steep. Unfortunately, his analysis was based on the assumption that the density profiles of different components are homologous. This assumption is generally not satisfied because the density profiles of non-dominant components are affected by the gravitational attraction of dominant components.

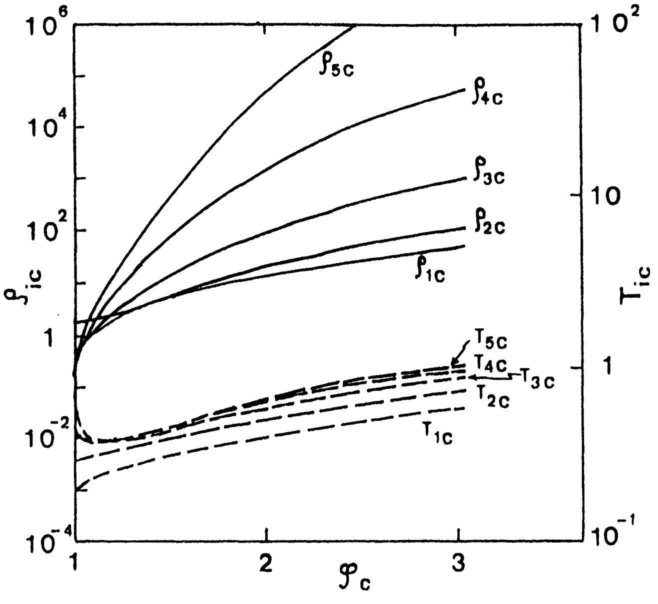

Figure 4. Time evolution of the central density and temperature for each component as a function of the central potential for the model with m_{max}/m_{min} = 10 and α = 2.5.

Thus it was necessary to examine the condition for equipartition in evolving models of globular clusters. IS adopted Plummer's models characterized by equations (2) and (3) as initial conditions. Figure 4 shows time evolution of the central density and temperature of a five component cluster with m_{max}/m_{min} = 10 and α = 2.5. It is seen that initially the temperatures of the massive components decrease rapidly and the forth and the fifth components become in equipartition. This equipartition, however, does not last long. When the density of the most massive component dominates significantly at later stages, the central temperature of the most massive component becomes higher than the central temperatures of the other components. Vishniac (1978) found that $\alpha \gtrsim 2.5$ is the necessary condition for equipartition. Figure 4, however, shows that $[(T_{5c} - T_{1c})/T_{1c}]_{min}$ is as large as 47%.

The steeper is the mass spectrum, the smaller is the temperature difference. For example, $[(T_{5c} - T_{1c})/T_{1c}]_{min}$ is about 9% for the model of m_{max}/m_{min} = 10 and α = 4. In this model the central temperatures of the second to the fifth components become nearly equal at intermediate stage of the evolution. Figure 5 shows time evolution of the central density and temperature of the model with m_{max}/m_{min} = 10 and α = 6.0. It is noticed that at this very steep mass spectrum equipartition at the cluster centre is nearly achieved. Thus it is seen that the mass spectrum

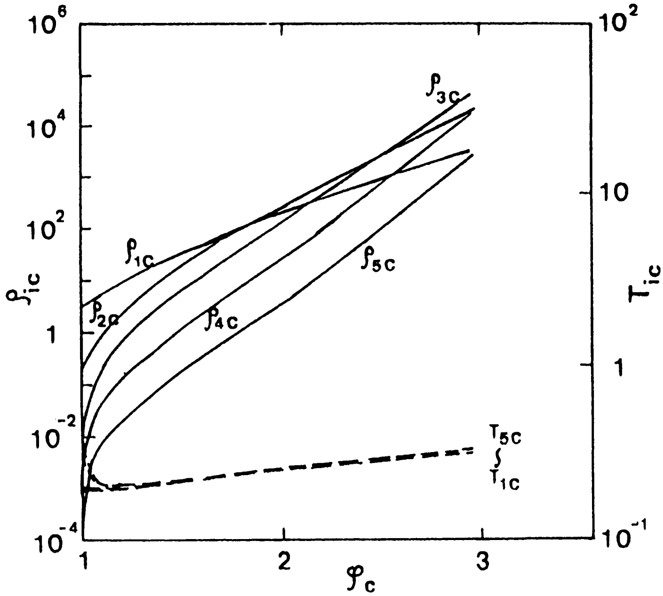

Figure 5. The same as figure 4 but for $\alpha = 6.0$.

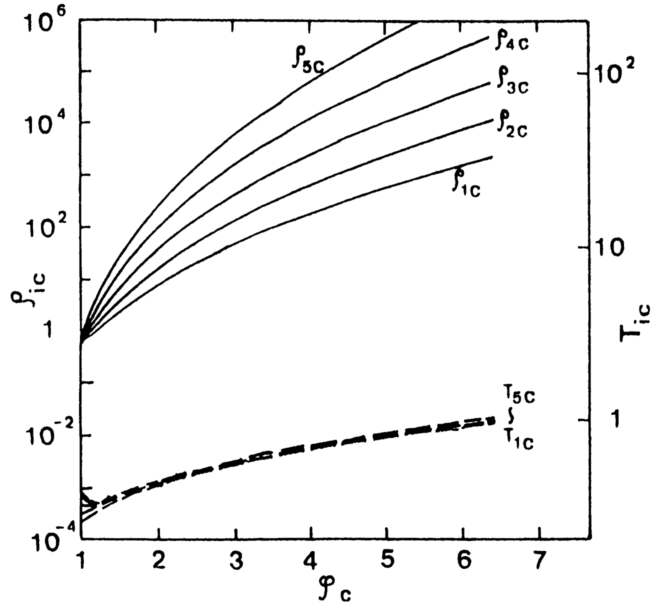

Figure 6. The same as figure 4 but for $m_{max}/m_{min} = 2$ and $\alpha = 1.0$.

must be as steep as $\alpha \gtrsim 6$ in order that a cluster with $m_{max}/m_{min} = 10$ achieves equipartition at the centre. These tendencies are unchanged

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even though the number of components are as large as fifteen (see table 3 and IS for detail).

When the ratio of the maximum mass to the minimum mass of a star is 2, the results are quite different. Even for a very flat spectrum with $\alpha = 1$, $[(T_{5c} - T_{1c})/T_{1c}]_{min}$ is only 7% (figure 6). Table III shows the minimum of the relative central temperature difference between the most massive component and the least massive component, $[(T_{nc} - T_{1c})/T_{1c}]_{min}$, where n is the number of components. From table III we see that either $\alpha \gtrsim 6$ or $m_{max}/m_{min} \lesssim 2.8$ is necessary for the achievement of equipartition at the cluster centre.

TABLE III. The minimum of the relative central temperature difference, $[(T_{nc} - T_{1c})/T_{1c}]_{min}$, along the evolutional sequence.

	$\frac{m_{max}}{m_{min}} = 10$ (n = 15)	$\frac{m_{max}}{m_{min}} = 10$ (n = 5)	$\frac{m_{max}}{m_{min}} = 4$ (n = 5)	$\frac{m_{max}}{m_{min}} = 2.8$ (n = 5)	$\frac{m_{max}}{m_{min}} = 2$ (n = 5)
$\alpha = 1.0$	1.20	1.40	----	0.14	0.07
$\alpha = 2.5$	0.68	0.47	0.21	0.12	0.06
$\alpha = 4.0$	0.20	0.09	0.10	0.07	----
$\alpha = 6.0$	0.06	0.03	0.04	0.03	0.03

As the number of components increases, the fraction of the mass contained in a component decreases. Therefore it is rather difficult to see whether the final collapse of the core occurs in the same way as do the cores of single component clusters. However, if we define the rate of core collapse by $\xi \equiv t_{rc} d\ln\rho_c/dt$, we see that this quantity has the same value as the single component clusters. Here the central relaxation time is defined by

$$t_{rc} = \frac{(2\langle\bar{v}_c^2\rangle/3)^{3/2}}{2\pi G^2 \bar{m}\rho_c \ln \Lambda} , \tag{7}$$

where $\langle\bar{v}_c^2\rangle$ and $\bar{m}$ are the average values of the central velocity dispersion and the mass, respectively. The average values are taken with the weights of the central number densities, i.e., e.g.,

$$\bar{m} = \frac{\sum_i m_i n_{ic}}{\sum_i n_{ic}} \tag{8}$$

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where n_{ic} is the central number density of the i-th component. The relation between the central velocity dispersion of the most massive component and its central density is also the same as in single-component clusters, i.e. $\langle v_{nc}^2 \rangle \propto \rho_{nc}^{0.1}$.

4. POST-COLLAPSE EVOLUTION OF MULTI-COMPONENT CLUSTERS

Recently several basic properties of post-collapse evolution have been revealed (see Heggie 1985). However, most studies were made under the assumption that the cluster consists of stars with the same mass. The only exception is the work by Stodółkiewicz (1982, 1985) who allows disparate masses.

As is mentioned in sections 2 and 3, the time required for the complete core collapse is shorter in multi-component clusters than in single component clusters. Therefore multi-component clusters set in the post-collapse phase earlier than single component clusters. Moreover, if we take into account that post-collapse evolution is powered by hard binaries, the post-collapse phase starts much earlier in multi-component clusters than in single component clusters. According to Heggie (1975), the formation rate of hard binaries is proportional to

$$Q(x) = \frac{14\sqrt{2}\pi^2 G^2}{3} \frac{(m_1 m_2)^4 m_3^{5/2}}{(m_1+m_2)^{1/2}(m_1+m_2+m_3)^{1/2}} \beta x^{-9/2} , \qquad (9)$$

where stars with the masses m_1 and m_2 form a binary and the star with the mass m_3 is recoiled from the binary. For the meaning of other symbols, see Heggie (1975). Equation (9) shows that if m_1 and m_2 are larger than m_3, the formation rate of hard binaries is larger than in the same mass case. Therefore in multi-component clusters massive stars form binaries more efficiently than in single component clusters.

TABLE IV. The time (in the unit of t_{rh}) when persistent hard binaries are formed in N-body simulations.

	N = 250	N = 1000
single component cluster	14.1	21.0
α = 2.5	3.7	3.1
α = 4.0	8.9	3.3
α = 6.0	3.8	5.9

Table IV shows the time of the onset of the post-collapse stage when persistent hard binaries are formed in N-body simulations. These simulations were performed with NBODY5 code developed by Aarseth (1984, 1985). The code can regularize not only strong two-body encounters but also violent three-body encounters. Therefore the code is suitable for the study of post-collapse evolution of small-N systems. From table IV we see that the onset of the post-collapse phase is much earlier in multi-component clusters than in single component clusters. Dependence on mass spectra is not clear from table IV. More detailed studies will be necessary.

Larson (1984) constructed a model of two-component clusters in post-collapse phase. He assumed that $\rho_2 \propto r^{-2.5}$, $m_2\langle v_2{}^2\rangle/m_1\langle v_1{}^2\rangle = 1.16$, and M_2/M is a few percent. From these assumptions he derived that $m_1 = 0.8\ M_\odot$ and $m_2 = 2.8\ M_\odot$. He thus considered that the massive stars are unseen black holes and the less massive stars are visible stars. He found that the calculated profiles of the light and velocity dispersion of visible stars are in agreement with observed profiles of the light and velocity dispersion, respectively, of globular clusters. It is interesting to examine whether his model is reproduced in an evolutional model of a two-component cluster.

We can make some predictions on the post-collapse evolution of two-component clusters. From the study of pre-collapse evolution by IW, we know that the density profile of the massive component is proportional to $r^{-2.2}$ at the time of complete core collapse. The density profile of the less massive component is much flatter (see figure 4 of IW). If we take account of the binary formation, hard binaries of massive stars will form when the central density of the massive stars becomes high enough. Thereafter expansion of the massive component begins. As the density of the massive component becomes smaller, the density of the less massive component also becomes smaller. In single component clusters, the post-collapse evolution is characterized by the expansion of isothermal region (Inagaki and Lynden-Bell 1983). From this analogy we expect that the region where equipartition is achieved expands. The radius of this region, r_*, is characterized by the equation, $t_{eq}(r_*) \simeq t - t_{cc}$, where t_{eq} is the time required for equipartition and t_{cc} is the time of the complete core collapse.

5. FUTURE PROBLEMS

(1) Construction of models of real globular clusters: In section 3 we see that $m_{max}/m_{min} \lesssim 2.8$ or $\alpha \gtrsim 6$ is necessary for the achievement of equipartition at the cluster centre. In real globular clusters the mass of the most massive stars will be larger than 1 $M_\odot$ though they are unluminous. On the other hand, the mass of the least massive stars will be about 0.1 $M_\odot$. The exponent in the mass spectrum is $1.3 < \alpha < 3$ for M3 (Gunn 1980). Therefore deviation from equipartition is expected in real globular clusters. However, most models of globular clusters (e.g. Da Costa and Freeman 1976, Illingworth and King 1977) were constructed under the assumption of equipartition at the cluster centre. Therefore

it is necessary to construct models of real globular clusters, taking account of deviation from equipartition.

(2) Post-collapse evolution of multi-component clusters: As is seen in section 4 we know only some qualitative features of post collapse evolution of multi-component clusters. It is, therefore, necessary to study post-collapse evolution of multi-component clusters quantitatively, using detailed evolutional models.

(3) Effects of tidal dissipational encounters: In single component clusters, two-body binaries formed by tidal dissipational encounters govern the evolution in late stages (Inagaki 1984, Ostriker 1985, Hut and Inagaki 1985). It is necessary to consider the role of two-body binaries in a cluster with realistic mass spectra.

(4) Construction of realistic evolutional models of globular clusters: Some basic properties of gravitational encounters in multi-component clusters are discussed in section 3. To understand the evolution of real globular clusters, it is necessary to construct more realistic evolutional models, taking account of stellar evolution and of tidal effects due to the Galaxy, etc.

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DISCUSSION

KING: Can you give us some simple physical feeling for why a multi-component model evolves faster than a single-component model?

INAGAKI: In order that a single-component cluster evolves, heat must be transported from the core to the halo. It takes a few half-mass relaxation times for such a transportation. On the other hand, a multi-component cluster can evolve if energy is transported from the massive component to the less massive component *in the core*. About one relaxation time is enough for the exchange of energy between components.

APPLEGATE: I have calculated the evolution of young globular clusters, such as those found by Ken Freeman in the LMC, and find that the energy exchange between the different mass components is very important. One must, however, include stellar evolution in these systems because the massive stars evolve off the main sequence and loose mass on timescales comparable with the relaxation time.

INAGAKI: I agree with you.