BULL. AUSTRAL. MATH. SOC. 06A55, 20E99
VOL. 17 (1977), 479-480.

An embedding theorem for ordered groups: Addendum

Colin D. Fox

A simple argument yields the following generalization of Theorem 6 of [1] (whose notation is retained without further explanation).

THEOREM. *Let* G *be an O-group and* $\underline{\underline{V}}$ *a variety of groups. Then* $G \in \underline{\underline{V}}$ *implies* $G^{\#} \in \underline{\underline{V}}$.

Proof. Suppose $W(x_1, \ldots, x_n)$ is a law of $\underline{\underline{V}}$ and take $(g_1, a_1), \ldots, (g_n, a_n)$ in $G^{\#}$. So

$$\begin{aligned} W((g_1, a_1), \ldots, (g_n, a_n)) &= \left(W(g_1, \ldots, g_n), W'\left(b_1\phi^{\#}_{h_1}, \ldots, b_r\phi^{\#}_{h_r}\right)\right) \\ &= \left(1, W'\left(b_1\phi^{\#}_{h_1}, \ldots, b_r\phi^{\#}_{h_r}\right)\right) \end{aligned}$$

for some word $W'(x_1, \ldots, x_r)$ and where each $b_i \in \{a_1, \ldots, a_n\}$ and $h_i \in G$.

Setting $m = m(a_i)$, $i = 1, 2, \ldots, n$, we have, in G ,

$$1 = W\left(g_1a_1^m, \ldots, g_na_n^m\right) = W'\left(b_1^m\phi_{h_1}, \ldots, b_r^m\phi_{h_r}\right) .$$

Hence in $G^{\#}$,

$$1 = \left(1, W'\left(b_1^m\phi_{h_1}, \ldots, b_r^m\phi_{h_r}\right)\right) = \left(1, W'\left(b_1\phi^{\#}_{h_1}, \ldots, b_r\phi^{\#}_{h_r}\right)\right)^m .$$

Since $G^{\#}$ is torsion-free, $W((g_1, a_1), \ldots, (g_n, a_n)) = 1$ in $G^{\#}$.

Received 27 September 1977.

https://doi.org/10.1017/S0004972700010765 Published online by Cambridge University Press

Reference

[1] Colin D. Fox, "An embedding theorem for ordered groups", *Bull. Austral. Math. Soc.* 12 (1975), 321-335.

Department of Pure Mathematics,
La Trobe University,
Bundoora,
Victoria.

https://doi.org/10.1017/S0004972700010765 Published online by Cambridge University Press