Polar Motion: Historical and Scientific Problems
ASP Conference Series, Vol. 208, 2000
S. Dick, D. McCarthy, and B. Luzum, eds.

Study of the Regime of the Polar Motion by Means of Numerical Method

L. Rykhlova

Institute for Astronomy, Russian Academy of Sciences, 109017 Pyatnitskaya Str. 48, Moscow, Russia. e-mail: rykhlova@inasan.rssi.ru

G. Kurbasova

Crimea Astrophysical Observatory, Ukrainian Academy of Sciences, p/o Nauchnyi, Crimea, 334413 Ukraine, e-mail: gsk@crao.crimea.ua

Abstract. Analysis of the spectral power-density function and the Poincaré method are used to identify the type of motion exhibited by the Earth's pole from 1900–1990. The former method allows identification of a quasi-periodic regime with two main frequencies — annual and Chandler. The existence of combinational frequencies in this spectrum and the broadening of the spectral peaks suggest a quasi-periodic regime making a transition to chaos. It is concluded that the identification of the polar motion regime over the time interval considered is insufficient to predict the further evolution of the system over arbitrarily long times.

1. Introduction

The first apparent evidence for the motion of the real pole of the Earth relative to some conventional pole was found using observations performed in the second half of the 19th century. Accurate data on variations of the pole's coordinates are available only for the last hundred years. Information on variations of the pole's position is of scientific interest in characterizing oscillations of the rotational axis of the Earth. Description of this process over long time intervals is of great importance for our understanding of the future evolution of the Earth and the Earth-Moon system. Moreover, data on the coordinates of the Earth's pole are necessary to derive corrections of observational data in order to reduce them in a unified system.

The main difficulty in obtaining the information about polar motion that is required for various scientific and practical purposes over arbitrary time intervals is that a mathematically rigorous description is only possible for the forced (lunisolar) motion and nutation. Free motion of the Earth's axis is highly irregular, and is subject to a number of effects that are either poorly understood or not easy to account for. This motion can, thus, only be monitored using direct observations. Information on secular irregularities of the Earth's rotation is of special interest. Studies of the secular polar motion are hindered by the fact that precise astronomical data cannot provide a solid basis for accurate extrapolation of the polar motion. In addition, the trajectory of the secular mean polar motion over this interval forms a complex curve. Uncertainty about the character of the secular mean polar motion in the past increases if we must resort to indirect data, such as paleomagnetic, paleontologic, or paleoclimatic information. These data indicate that, many hundreds of years ago, the pole was very distant form its current position, and the Earth's rotation was appreciably more rapid than it is today. Despite the limited accuracy of such indirect data, their analysis provides evidence for possible large shifts of the pole in the past. This suggests that the polar motion regime varies on a multi-century time scale.

Identification of the polar motion regime using data on the pole's coordinates over a period of about a century provides evidence for some features typical of a transition regime. Our analysis here is essentially based on the modern theory of nonlinear systems or, more specifically, the qualitative theory of dynamic dissipative systems. In the framework of this theory, the main irregularities of polar motion are related to evolutionary phenomena originating from a unique combination of physical laws and specific initial conditions and boundary conditions.

We discuss here a possible new approach to the identification of the dynamic regime of the motion of the Earth's pole from 1900 until 1990 using the experimental data (Vondrák *et al.*, 1995). The initial data on the coordinates of the pole x_i, y_i were reduced to the form $Z = \sqrt{x_i^2 + y_i^2} i = 1, 2, ..., N$. We then removed a linear trend from the sequence Z. To obtain information on the regime of the process generating the sequence Z, we used spectral power density (SPD) function analysis and the Poincaré method.

2. Analysis of the SPD Function

Figure 1 presents plots of the variation of the relative SPD (in dB) as a function of frequency (in cycles per year). The smoothed values of the SPD were calculated using a segmenting method, in which the data sequence Z was divided into 44-year segments with a 22-year overlap.

The two main peaks on the plots of relative SPD (Fig.1) correspond to oscillations with the frequencies f_2= 0.8409 cycles/year (P_2 = 434.4 days) and f_1 = 1 cycle/year. To identify the power density spectrum, we restrict our analysis to the two main frequencies and neglect the finite resolution in the SPD calculations.

The plots of the relative SPD in Fig. 1 indicate that the fundamental oscillations f_1 and f_2 generate, in addition to sidelobe frequencies, oscillations at combinational frequencies equal to multiples of $f = f_2 + f_1$. In other words, the SPD function is comprised of oscillations with frequencies $|m_1 f_1 + m_2 f_2|$, where m_1 and m_2 are small integers: $0, \pm 1, \pm 2, ...$. This is possible if Z(t) is a quasi-periodic function made up of products of common trigonometric functions, *e.g.*, $\sin(f_1 t) \sin(f_2 t)$. In this case, the main frequencies $f_1, f_2, |f_1 - f_2|, |f_1 + f_2|$ and their harmonics can be distinguished in the SPD plots (Fig. 1). The phase trajectory corresponding to a quasiperiodic process is defined on a two-dimensional torus, so that the relevant attractor is a T^2 torus. The ratio of the frequencies f_1 and f_2 can be either rational or irrational.

It is not possible to draw conclusions about the commensurability of the fundamental frequencies for the actual motion of the Earth's pole, since the

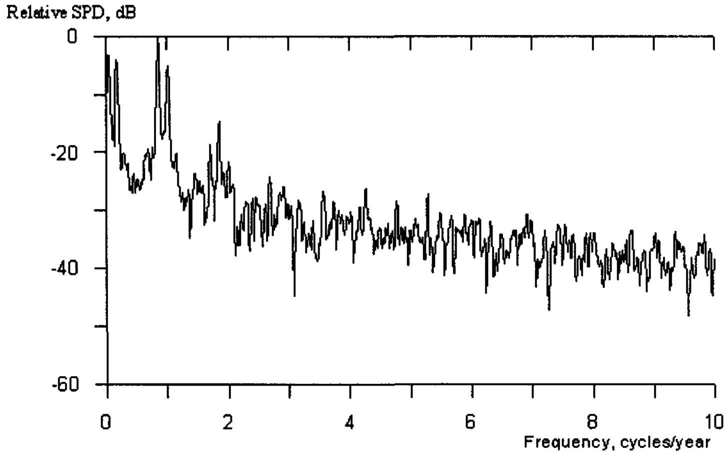

Figure 1. Smoothed periodogram of estimates of the spectral power density (SPD) for the data on the pole's coordinates.

value of the ratio f_1 / f_2 is not known accurately. Nonetheless, we can see that, given the above values of f_1 and f_2 , f_1^2/f_2^2 is equal to $\sqrt{2}$ with a relative error of 0.0009%.

Let us now consider the two possible cases. In the case of rational f_1/f_2, the stable trajectory of the polar motion would intersect itself after q rotations, thus describing a closed curve. This corresponds to so-called stable synchronization or frequency pulling. Such motion is predictable even at arbitrary large q. In general, this regime is not realized in the time period considered: observations of Z(t) over any finite interval of time are not sufficient to predict the future behavior of the process.

In the case of irrational f_1/f_2 the corresponding trajectory densely winds over the entire torus surface without intersecting itself; *i.e.*, when the fundamental frequencies are not commensurate, the phase trajectory is an unclosed curve on the torus T^2. The spectrum of such a quasi-periodic process is made up of an infinite, everywhere-dense set at all frequencies $|m_1 f_1 \pm m_2 f_2|$. It is not always the case that a plot of the SPD is continuous, since neighboring frequencies do not always have similar amplitudes. The amplitudes of high-order lines (m_1 and m_2 exceeding several units) are too small to be distinguished against the background noise. Nevertheless, the appearance of sub-harmonics and low-frequency broadening of the spectrum indicate that the oscillations are complex, possibly with many hidden degrees of freedom. Such oscillations can be precursors of a chaotic process, even if the number of degrees of freedom is limited.

Thus, the plot of the relative SPD is quite complicated and indicates the presence of a quasi-periodic regime for the temporal evolution of Z(t). We obtained more detailed information about the features of this regime over the time considered using the Poincaré method.

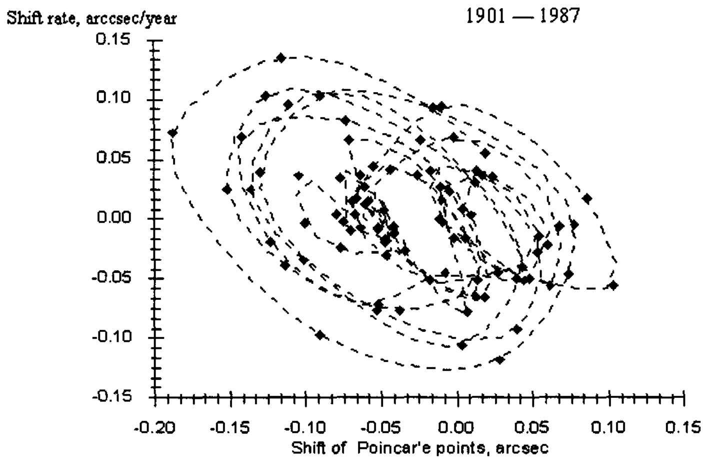

Figure 2. The Poincaré mapping.

3. Evolution of Poincaré Sections

The Poincaré method makes it possible to reduce a phase trajectory in three-dimensional phase space to a mapping of the evolution of the process in a phase plane whose coordinate axes plot displacement and velocity. Thus, this method simplifies our investigation of the regime of dynamic process of polar motion. To obtain the Poincaré mapping, we extract a sample from the data set Z, corresponding to the times $t_n = iP + t_0$ $(i = 0, I, ..., N)$, where t_0 corresponds to January 0, 1900 and $N = 90$.

Figure 2 presents a schematic diagram of the Poincaré mapping on the phase plane for the data Z_1. This image is neither a collection of finite points (a sub-harmonic regime) nor a closed curve (a quasi-periodic regime). It is known that Poincaré mappings of conservative or weakly-dissipative systems often resemble an unordered cluster of points in the phase plane. Such motions are sometimes called stochastic. Analysis of the Poincaré mapping (Fig. 2) reveals a six-year cycle. The power density spectra of the initial data grouped into sequential six-year intervals exhibit a single fundamental frequency that is close to either f_1 or f_2 depending on the superposition of the oscillations in a given time interval under the action of external forces. When the duration of the interval is increased, the periodic regime seen in the six-year intervals becomes unstable, and is replaced by a quasiperiodic regime with two fundamental frequencies (Fig. 1).

This process is accompanied by shifts of the centers of equilibrium of the Poincaré sections (Fig. 2) and of the corresponding phase-space orbits relative to the common origin. These shifts occur in the sequence LRRLLLLLRRLLLLL, where L and R denote positions to the left and to the right, respectively. As a result, the set of Poincaré mapping points in the phase plane (Fig. 2) has a structure resembling the trajectory of a particle in a double potential well under the action of an external driving force.

https://doi.org/10.1017/S0252921100061649 Published online by Cambridge University Press

4. Conclusions

(a) Analysis of the plot of the spectral power density (Fig. 1) enables identification of a quasi-periodic regime with two fundamental frequencies f_1 and f_2, in the time interval considered. The combinational frequencies and spectral peak broadening seen in this spectrum resemble a quasi-periodic regime leading to chaos. However, a necessary condition for such a transition is the appearance of a third independent frequency in the spectrum.

(b) Analysis of the evolution of six-year Poincaré sections shows that the system dynamics are not invariant with respect to time reversal.

(c) The properties of the area variations and periodic shifts of the phase orbits form a structure in the phase plane that is similar to the trajectory of a particle in a double potential well under the action of an external driving force. This analogy may indicate the existence of a complex evolutionary process, whose dynamics include both the effects of damping and of mechanisms sustaining the motion. In this case, the spectrum may become broadened at low frequencies due to the periodic external force.

References

Vondrák, J., Ron, C., Pešek, I., and Čepek, A., 1995, *Astron. Astrophys.*, **297**, p. 899.