Proceedings of the Edinburgh Mathematical Society (1993) 37, 191–192 ©

BOOK REVIEWS

DRAZIN, P. G., *Nonlinear systems* (Cambridge University Press, Cambridge 1992) 330 pp., hardback 0 521 40489, £40; paper 0 521 40668 4, £17.95.

Why is chaos like a beard? The answer will be found as early as p. 3 of this textbook, in the introductory chapter, but chaos itself is treated at length only in the last chapter. This chapter contains interesting and important topics not usually seen in books at this level (see below), including a summary of routes to chaos, and the characterization of chaos in time series. Similarly the fourth chapter provides introductions to Cantor sets, fractals, and Feigenbaum theory. The remaining chapters are more routine: bifurcations, difference equations, and free and forced oscillations. These chapters have much in common with other applied mathematics texts in this general area.

The book is aimed at final-year undergraduates or first-year postgraduates, and is said to be suitable not only for mathematics students but also for eager students in physics, engineering, economics, etc. Indeed the treatment is refreshingly informal, with no theorems, few abstractions, and plenty of derivations and worked examples integrated seamlessly into the text, though unfortunately applications rarely get more than a passing mention.

There are very large numbers of problems for the reader. The answers and hints given at the back seem adequate, though the outline answer to Question 5.2 seems to me to lead in quite the wrong direction, and the calculations in 8.1 are sufficiently tricky that a statement of the answer would have been reassuring; perhaps it was considered too long to print.

There are listings of a number of short, useful, BASIC computer programs, and the bibliography includes movies and videos. The index is detailed enough to be useful, and I have found few misprints, my favourite being a quote from Gentnude Stein [*sic*].

D. C. HEGGIE

HOGGAR, S. G. *Mathematics for computer graphics* (Cambridge University Press, Cambridge 1993), xviii+472 pp., 0 521 37574 6, £25.

This is an unusual book, its fractal-festooned dustjacket giving a clue to the pictorial riches contained inside. For this is not a service textbook on the mathematics that a computer graphics enthusiast needs to know. (It contains no projective geometry.) Rather, it is primarily a book about *Pictures with Symmetry*.

The idea of writing a text centred around pictures with symmetry is an inspired one. For, if you start by admiring the pictures, they raise questions which lead to many different branches of mathematics. On the other hand, the mathematics is very much enlivened by being applied to understanding the pictures.

Let us discuss the pictures separately from the mathematics. To see how they fit together, I refer you to the book! First the pictures. The first major class of pictures (i.e. subsets of $\mathbb{R}^2$ with some symmetry) considered is braid patterns—pictures all of whose translational symmetries form a discrete set of parallel vectors. Such pictures may have other symmetries (e.g. reflections), resulting in braid patterns being classified into seven distinct types, depending on the totality of their symmetries.

The next major class of pictures is the *net*, the orbit of a single point under some finite

symmetry group of the plane. These are classified into five types (parallelogram, rectangular, centred rectangular, square and hexagonal—unfortunately there is no adjectival form of parallelogram!). Joining the vertices of a net leads to the notion of *cell*. This could contain a picture which is repeated across the plane, leading to Pólya's famous classification of the seventeen plane patterns (symmetries, wallpaper types). Next come tilings, including the classification of the twelve Archimedean tilings of the plane. (These tilings are the ones where all vertices of the tiling are isomorphic).

The first fractal pictures which appear are the now well-known ones such as the Cantor set and the Sierpiński gasket. Here, for the first time, the symmetry of the picture comes from the sets being unions of affine transformations of themselves ("self-similarity"). This leads in due course to the full glory of the Mandelbrot set, and an array of Julia sets (on full colour plates).

A major feature of the book is the DIY section, where, with the help of Barnsley's iterated function systems, one can design one's own pictures. The pictures produced are self-similar. A more efficient method, the random iteration algorithm, is also described.

The pictures and diagrams are a notable feature of the book, and add greatly to its attractiveness and readability. A few of them, however, show the limitations of the computer drawing package used (e.g. Figs. 11.8 and 12.10 have non-smooth "smooth" curves).

And now to the mathematics. This is spread in large and small parcels throughout the book. Some of it is elementary and very familiar: complex numbers, vector and matrix algebra, elementary topology. Groups are there, of course, but without the prominence one might perhaps expect in a book where symmetry is so important. Much of the mathematics is more advanced. The section on topology is surprisingly extensive, and there is even some measure theory. There is a nice section on quaternions and their connection with rotations of $\mathbb{R}^3$. This material is applied to the problem of interpolating between rotations. This is a topic useful both for computer animation and robotics, and on its own justifies the title of the book.

The final section is an account of the dynamics of iterated complex functions à la Julia, Fatou and the many authors of the last decade. As is very well known, this subject was truly picture-driven, only taking off when it became possible to draw Julia sets and the like by computer.

The text is composed on a word-processor which, while fine for plain text, has not produced high quality mathematical formulae. There are, throughout, continual problems with point size and with positioning of symbols, particularly subscripts and superscripts. I feel that the publishers should bear the main responsibility for this lapse. However, presumably the use of word-processing rather than typesetting did help to keep the cost of the text to the very reasonable price of £25. But overall, this is an attractive, readable text containing much that is of interest, both for a casual browse and for more extensive study. Parts of it would be highly suitable for a final year undergraduate or first year graduate course. I highly recommend it.

C. J. Smyth