ON THE ELECTRO-MAGNETIC COUPLING BETWEEN THE CORE AND THE MANTLE OF THE EARTH

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1. Introduction

Recently theories of the Earth's rotation based on models with an elastic mantle and fluid core have achieved considerable success. It was assumed in these models that the dissipative (electro-magnetic and viscous) couplings between the fluid core and the mantle have negligible effects. The discrepancies between the theories and the observations were much reduced. It seems, however, that these theories are not sufficiently successful in explaining the observed phenomena concerning the Chandler wobble. For example, we had a very small polar radius in the decade around 1930, and the phase of the polar wobble was much advanced in this decade compared with the mean period of revolution. The apparent period of the polar wobble maintained an almost constant value of 1.16 yr during 1930-1942 (Sugawa 1969; Guinot 1972; Stoyko 1972). The present author (Sekiguchi 1975) found from the latitude observations between 1830 and 1860 that the amplitude was also diminished and the period had an almost constant value of 1.16 yr. The common behaviour of the polar wobble in both periods is difficult to explain as an accidental happening.

The bifurcation of the spectral peak near the Chandlerian frequency in the spectrum of the polar wobble is well known (Yatskiv, Korsun' and Rykhlova 1972). The present author (Sekiguchi 1976) tired to interpret it phenomenologically, but did not obtain an adequate explanation.

Considering these circumstances, it is possible to doubt whether the existing theories retain their validity for the explanation of the observed phenomena concerning the Chandler wobble. Indeed, the Chandler wobble is a very peculiar oscillatory mode, compared with the other motions of the rotational axis. The periods of the relative motion between the core and the mantle are exactly or nearly equal to one sidereal day for the precession, the nutations and the nearly diurnal sway. However, for the Chandler wobble the period is about 430 days, distinctly longer than for the other modes.

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The effects of electro-magnetic coupling were studied by several writers (MacDonald and Ness 1961, Rochester and Smylie 1965), and their conclusions favored the insignificance of this effect on the Earth's rotation. But they assumed the diurnal frequency for the relative motions between the core and the mantle. Therefore, it is worthwhile to examine whether or not the electro-magnetic coupling is still insignificant for the Chandler wobble.

2. Hydro-Magnetic Motions of the Viscous Fluid

We assume that the Earth's core is a uniform liquid, having a finite viscosity η and an electrical conductivity σ. The core boundary is assumed to be the plane $z = 0$, and the core material fills the region $z > 0$. The magnetic field H_0 is assumed to be uniform and perpendicular to the core boundary. Oscillation of the core material is allowed only in the y-direction.

The system of the magneto-hydrodynamical and Maxwellian equations permits solutions of the form

$$\tilde{v}_y = \tilde{v} \exp(\tilde{\alpha} z + j\omega t) \quad , \tag{1}$$

where the tilde signifies a complex number, ω is the circular frequency of the oscillation, j is the imaginary unit, $\tilde{v}_y$ is the y-component of the velocity of the core material, and t is the time. As the solution must remain finite for $z > 0$, the real parts of $\tilde{\alpha}$ must be non-positive. Under these conditions, only two oscillatory modes are permitted, where we have

$$\begin{aligned} \tilde{\alpha}_1^2 &= -4\pi\omega^2/(\mu H_0^2/\rho + j\omega/\mu\sigma) \quad , \\ \tilde{\alpha}_2^2 &= (\mu^2 H_0^2 \sigma + j\sigma\omega)/\eta \quad , \end{aligned} \tag{2}$$

where μ is the magnetic permeability of the core material. The damping distances Δz_i and the wavelengths L_i $(i = 1,2)$ can be calculated from these values, assuming $H_0 = 10$ gauss, $\mu = 1$, $\sigma = 3 \times 10^{-6}$ e.m.u., and $\eta = 10$ cp (Gans 1972). The results for the diurnal and the Chandlerian frequencies are shown in Table 1.

The first mode is identical to the Alfvén wave and the second one is the damping wave, which reduces to the oscillation of the viscous fluid when the magnetic field vanishes. We call the first one the Alfvén mode, and the second the viscous mode.

Table 1. The values of the damping distances and wavelengths

	ω	Diurnal Frequency $7.292 \times 10^{-5}\ s^{-1}$	Chandlerian Frequency $1.696 \times 10^{-7}\ s^{-1}$
$\tilde{\alpha}_1$	Δz_1	0.36 km	18500 km
(Alfvén Mode)	L_1	1.5 km	330 km
$\tilde{\alpha}_2$	Δz_2	14 cm	18 cm
(viscous Mode)	L_2	127 cm	410 cm

3. The Coupling Between the Core and the Mantle

The motion of the mantle is transferred to the core by these two modes. The motion in the core consists of these two parts, for which the ratio is determined by the boundary conditions at the surface of the core.

We assume that the core material is at rest for $z = +\infty$, and the motion of the mantle is

$$\tilde{v}_y = \tilde{v}_0 \exp(j\omega t) \quad . \tag{3}$$

At the core boundary, the electric and magnetic field, and the motion of the material are continuous. Under these conditions, we obtain

$$\tilde{v}_1/\tilde{v}_0 = 0.387 \exp(-67.^\circ 3j) \quad ,$$

$$\tilde{v}_2/\tilde{v}_0 = 0.923 \exp(22.^\circ 8j) \quad ,$$

for the Chandlerian frequency, and

$$\tilde{v}_1/\tilde{v}_0 = 0.012 \exp(238.^\circ 6j) \quad ,$$

$$\tilde{v}_2/\tilde{v}_0 = 1.007 \exp(0.^\circ 6j) \quad ,$$

for the diurnal frequency. Here the arguments in the exponential functions are expressed in degrees, so that $\exp(90.^\circ 0j) = j$.

These results show that, for the diurnal frequency, the main part of the motion of the mantle is transferred to the core in the viscous mode, which fades out within a very short distance. Only 1.2% of the mantle's motion is transferred in the Alfvén wave, and it still fades out immediately. For the Chandlerian frequency, on the contrary, the

Alfvén mode is transferred with a considerable amplitude. This motion has a phase lag of 67°.3 at the top of the core, but can propagate without any damping. The part of the motion which is transferred in the viscous mode fades out in a very short distance.

4. Influences on the Rotation of the Earth

Though there is a possibility that the core can receive a considerable influence for such a low frequency, it is difficult to construct an exact theory of the Earth's rotation based on this principle. We do not have sufficient knowledge about the shape and the intensity of the magnetic field in the core. It may be neither uniform nor invariant. Because of this, it is natural to suppose that the Alfvén wave in the core has neither a constant direction nor a constant velocity.

In the Earth's core, the toroidal magnetic field may be dominant toward the interior, and the magnetic lines of force may be nearly horizontal in a comparatively shallow layer of the core. As the Alfvén wave propagates along the magnetic lines of force, its motion would stay in this thin superficial layer of the core. Below this layer, the material would behave according to the existing theory.

The thickness of the layer may not be uniform, and is ruled by the shape of the magnetic field in the core. Moreover, it must be variable as the magnetic field changes. This may cause the change in the effective ellipticity of the core, and consequently the change in the Chandlerian frequency. The idea of the "Topographic Coupling" proposed by Hide (1969) still has room to be kept alive for the case of the Chandlerian frequency.

Based on some appropriate assumptions, we can expect 2% fluctuations of the Chandlerian frequency, and 5% increments of the damping coefficient of the polar wobble. We believe that these are the causes of the phenomena mentioned in the introduction.

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