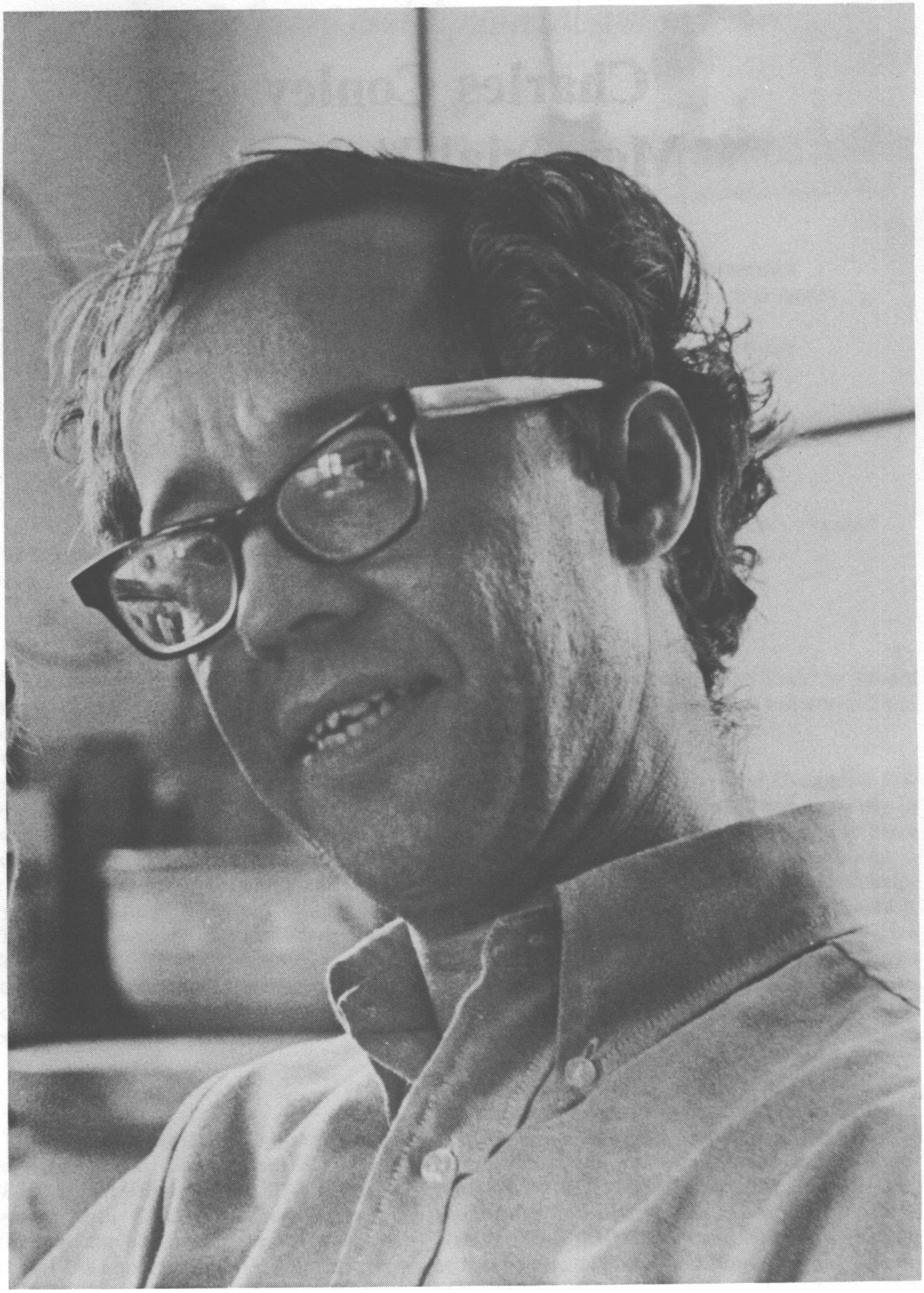

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Charles C. Conley, 1933–1984

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After two years as a graduate student in Madison, I was ready to quit mathematics to pursue something more interesting and useful. I felt that physics was a possibility. Upon discovering my intention to take a course in mechanics from the Physics Department, one of my professors suggested that I should learn mechanics 'the right way' from 'that young fellow Conley'. I was skeptical, but I enrolled in a course called Dynamical Systems, taught by Charles Conley. There I had an experience I later found out was shared by most of Charlie's Ph.D. students: although the lectures seemed disorganized and fragmented, Charlie's enthusiasm for the material was so contagious that I quickly discarded other aspirations and devoted myself to studying dynamical systems.

The year was 1966, and Charlie had been in Madison for three years. He was thinking a lot about celestial mechanics and was still consulting for the National Aeronautics and Space Administration, as he had since 1963. He had recently bought a farm outside of Madison with the money he had earned consulting. He lived in the city, but he visited the farm frequently. He was still playing shortstop for the intramural softball team. That was before his dislocated shoulder got so bad he had to quit.

Charlie came to Madison in the fall of 1963 from the Courant Institute, where he was a postdoctoral fellow for two years. Although he received his Ph.D. from MIT officially in 1961, he moved to New York in 1960, at the same time that Jürgen Moser moved from MIT to the Courant Institute. Jürgen was Charlie's thesis advisor.

While in New York, Charlie met his wife-to-be, Catharine Smith, known to everyone as Kit. They were married on 28 December 1963. They had three children: Charles Henry, born 6 October 1964, Catharine Anastasia, born 9 October 1966, and John Alan, born 7 October 1968. Charlie once told me that he had wanted to keep on going, but that Kit had vetoed the idea.

Charlie himself was born on 26 September 1933 in Royal Oak, Michigan. His full name was Charles Cameron Conley; his parents were Charles Andrew Conley and Bertha Cameron. He was the only boy in a family of six children. He graduated from Royal Oak High School in 1949. He attended Wayne State University in Detroit for one year before he joined the Air Force. He spent four and a half years in the Air Force, most of the time stationed in England. After his discharge, he returned to Wayne State, where he earned a B.S. degree in 1957 and an M.S. degree in 1958. He then moved to Boston to attend the Massachusetts Institute of Technology.

Charlie's colleagues in Madison gave him tenure in 1965 after only two years as an Assistant Professor. He was promoted to Professor only three years later in 1968. Being a graduate student at the time, I was blissfully unaware of the promotion processes going on around me. I did notice that Charlie had abruptly stopped wearing his old sweatshirt to the University and had started wearing a coat and tie. When pressed, he admitted that it had something to do with his promotion. He said that he no longer had to impress his colleagues, so he now could dress as he pleased. Charlie had a way of making a joke of the truth so that you would think he was kidding.

Charlie had several eccentricities. They came naturally to him, but he was aware of them and, to some extent, cultivated them. He once told me that it was good to have some peculiarities because they provided people with a way to remember you. Everyone who worked with Charlie has a story to tell about the time when he was talking so enthusiastically about mathematics that he missed a turn and drove miles out of his way before he realized it, or about the time when he boarded the train to Paris while trying to get to Bochum. He once revealed to me his secret about avoiding administrative work: 'If you screw something up badly enough, they won't ask you to do it again'. I am certain that this concept did not originate with Charlie, but I know that he took it to heart.

The time he saved by avoiding administrative work he spent doing what he did best: advising his Ph.D. thesis students. He had a natural gift for transmitting his ideas and enthusiasm. He did not relegate his students to cleaning up minor details or to illuminating amusing sidetracks; his students were an integral part of his overall research program. Indeed, milestones in his program often appeared as the Ph.D. dissertations of his students. For example, the fundamental ideas on isolating blocks first appeared in Easton's thesis, where the blocks were called 'submanifolds convex to a flow'. Another example is that the basic theorems on connection matrices appeared in Franzosa's thesis.

In retrospect, one can see that Conley's work constitutes a unified and coherent program. His goal was to develop what he termed 'crude' methods to detect the macroscopic behaviour of dynamical systems. By 'crude' he meant topological. To him, fundamental phenomena should exist for topological reasons. He considered analysis as important for two purposes. In a particular problem, verification of the topological conditions may require analytic estimates. However, more subtle and, he admitted, sometimes more interesting phenomena were fundamentally analytic in nature. He decried the use of analysis to prove a result which could be proved using topology. On the other hand, he was fascinated by results which would not yield to his topological approach.

The fundamental elements of a dynamical system are, in the Conley view, 'isolated invariant sets'. An invariant set is 'isolated' if it is the maximal invariant set in some neighborhood of itself. From the macroscopic view, the isolated invariant set itself is not as important as the 'isolating block' which surrounds it. In its most easily articulated form, an 'isolating block' is a set whose boundary has no internal tangencies to the flow. That is, if the flow is tangential to the boundary of the block, then the orbits leave the block both forward and backward in time. A fundamental result is that every isolating block contains an isolated invariant set in its interior and, conversely, every isolated invariant set can be surrounded by an isolating block.

An important property of the isolating block is that it is 'structurally stable' in the sense that it persists under perturbations of the flow. While the isolated invariant set itself may change dramatically under perturbation, the isolating block changes only slightly or, depending on the definitions and the topologies, not at all. Thus those macroscopic properties of the invariant set which can be determined from the block remain under perturbation. Furthermore, properties which cannot be deduced from the block cannot be expected to persist under perturbation.

The basic property that can be deduced from the block is now called the 'Conley index'. Loosely speaking, this index is the homotopy type of the isolating block with the 'exit set' identified to a point. The exit set is the set of points on the boundary where the flow leaves the block. A good way to think of this index is as a generalization of the Morse index. Indeed, Charlie always referred to his index as 'the Morse index'. If the isolated invariant set is a non-degenerate rest point for a smooth flow, then the Conley index and the Morse index coincide in the sense that the Conley index is the homotopy type of an n-dimensional sphere, where n is the dimension of the unstable manifold. The information contained in the Conley index includes not only the dimension of the 'unstable manifold', even in the case when it is not a manifold, but it includes also topological properties of the isolated invariant set.

Conley's view of the importance of isolating blocks did not stem from the mathematical elegance of the theory alone, but also from his profound belief that isolating blocks are fundamental to an understanding of natural phenomena. He had an interest in all branches of science and he eagerly learned new areas, always on the alert for applications of his theories. Simply put, he believed that the above-described 'structural stability' of isolating blocks meant that they were the only dynamical objects which could be detected in nature and that their properties reflected the important properties of natural systems. I remember several conversations in which he stated something like this: 'See this teacup? There must be some reason why, in the physical world of elementary particles and electromagnetic fields, this teacup exists as a teacup. Somewhere there must be an isolated invariant set corresponding to this teacup'. He was half serious and half joking.

A milestone of Conley's work which did not appear in the thesis of one of his students appeared as a preprint from the time he spent at the IBM Research Center in New York during the 1971–1972 academic year. There he wrote a technical report entitled 'The gradient structure of a flow: I' [**53**], published for the first time in this volume. I asked him several times between 1972 and 1978 why he didn't publish the report. His answer was always that it would be incorporated into his 'notes'. These 'notes' did eventually appear as the notes from the series of lectures he gave in Boulder, Colorado in the summer of 1976 [**36**].

The IBM report contains Charlie's basic ideas on 'chain recurrence' and 'attractors'. In it he proved the fundamental theorem that any flow on a compact metric space decomposes into the chain recurrent part and the 'gradient-like' part. More precisely, if each of the components of the chain recurrent set is identified to a point, then the resulting flow has a 'Lyapunov function' which decreases along all orbits except the fixed points. This result provides the foundation for what he termed the 'Morse decomposition' of a flow. The components of the chain recurrent set were called 'Morse sets' by Charlie. The orientation given by the flow provides a partial ordering for the Morse sets.

The Morse decomposition in turn provided the basis for Conley's later work on 'connection matrices'. The basic question he asked was this: what connecting orbits between the Morse sets are forced by topological considerations alone? The answer is given by the 'connection matrix'. Details can be found in the article by Moeckel in this volume. Applications for these ideas abound in the many papers that Charlie wrote jointly with Joel Smoller.

One of the impressive achievements of Conley's approach to dynamical systems came near the end of his career in his joint work with Edi Zehnder. They were able to prove conjectures about the number of fixed points of symplectic maps. The key ingredient in the proof is the use of the Conley index on a finite-dimensional approximation to the flow on the infinite-dimensional space of loops. In typical fashion, Charlie viewed the work as important only in that it pointed out the need for an infinite-dimensional version of his index.

Charlie's career came tragically and prematurely to an end on 20 November 1984. His death has left a mathematical and personal void in the lives of those of us who were fortunate enough to have worked closely with him. His unbounded enthusiasm, his driving energy and his profound influence are sorely missed.

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