52

The QCD condensates

We anticipated this discussion in Chapter 27 when we discussed the anatomy of the SVZ expansion. Here, we shall review the different determinations of the QCD condensates from QSSR.

Indeed, a good control of the values of the QCD condensates is necessary in the phenomenological applications of QSSR. The non-vanishing value of the light quark condensate, which we shall discuss in the next section, is intimately related to the GMOR realization of chiral symmetry, as can be inferred from the PCAC relation. SVZ [1,654] have also postulated that QCD is spontaneously broken by the gluon condensate, which they confirm from their analysis of the charmonium sum rule. The non-vanishing value of the gluon condensate and the gluon correlation length has been also checked on the lattice [402]. Since the pioneering work of SVZ, [1] a lot of effort has been devoted to this issue as can be found in the long list of published papers in this subject (for reviews see e.g. [3],[51,46], [356–363]). The condensates have been extracted from the light mesons [403–409], [325], [33,328], [341,387], and in [329] (Section 52.10), from the baryons [424–430], from the heavy quarkonia [433,434], and [313] (Section 51.3), and from the heavy-light mesons [401].

The $e^+e^- \to$ hadrons and τ-decays data have been always used as a laboratory for testing the perturbative and non-perturbative structure of QCD [1,3,325], [403–409], [346,338,341] and [329,161] (Sections 19.4 and 52.10). As already mentioned, these channels have the great advantage that the spectral functions are measured in a region where pQCD is applicable and therefore the analysis does not suffer from any model dependence in the parametrization of the spectral functions. Therefore, one expects that the determinations from these channels are model-independent.

52.1 Dimension-two tachyonic gluon mass

- $e^+e^- \to I = 1$ hadrons below 2 GeV has been also used for extracting the hypothetical dimension-two term beyond the SVZ expansion which has been interpreted in [161] as the effect due to a tachyonic gluon mass. In [341,329], ratios of Laplace sum rules and τ-like sum rules which can disentangle the leading radiative perturbative corrections from the non-perturbative contributions have been used. As a result, one is able to extract the tiny contribution due to the dimension-two

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terms. The outcome of the analysis is:

$$d_2 \equiv -1.05\left(\frac{\alpha_s}{\pi}\right)\lambda^2 \simeq (0.03 \sim 0.07)\ \mathrm{GeV}^2\ . \tag{52.1}$$

This is not the case of other attempts (see e.g. [653]), where in these approaches the α_s contribution masks the one of the tachyonic gluon mass.

- In [161], an alternative estimate of this quantity has been produced in the pseudoscalar channel where one can notice that the size of this contribution is about four times the one in the ρ-meson channel, such that its effect is more sizeable. At the optimization scale of the sum rule, one obtains:

$$\left(\frac{\alpha_s}{\pi}\right)\lambda^2 \simeq -(0.12 \pm 0.06)\ \mathrm{GeV}^2\ , \tag{52.2}$$

which is more precise than the etimate using e^+e^- data but still inaccurate.

- The value quoted in Table 48.2 corresponds to the intersection of these two estimates which we take to be:

$$\left(\frac{\alpha_s}{\pi}\right)\lambda^2 \simeq -(0.06 \sim 0.07)\ \mathrm{GeV}^2\ . \tag{52.3}$$

52.2 Dimension-three quark condensate

The derivation of the $\langle\bar\psi\psi\rangle$ condensate from the sum rules will be discussed in the chapter on light quark masses and light baryons.

52.3 Dimension-four gluon condensate

- The estimate of the gluon condensate from τ-decays [328,33] is not very conclusive, which one can understand from [325] because its contribution has an extra α_s correction in the QCD expression of the τ width. In this case, the analysis of e^+e^- data from the usual QSSR (in particular the Laplace sum rule) is superior. Most recent results using e^+e^- data have been obtained in the Section 52.10. It reads:

$$\langle\alpha_s G^2\rangle \simeq (7.1 \pm 0.7) \times 10^{-2}\ \mathrm{GeV}^4\ , \tag{52.4}$$

showing that the original SVZ result has been underestimated by a factor of about 2. An analogous result has been already obtained in the past by Bell–Bertlmann [91–93] using Laplace sum rules for heavy quark systems and adding a quantum mechanics argument for supporting their result. Similar conclusions have been reached in [655–658], while the validity of the SVZ value has been also questioned in [659]. Analogously [405,406] use high moments in n of FESR in $e^+e^- \to I = 1$ hadrons and have found larger values of the different condensates but the results were not very convincing due to the large sensitivity of the moments on the high-energy tails of the spectral functions.

- In [3], we have reworked in detail the different estimates of the gluon condensate from charmonium systems and come to the conclusions that using the standard sum rules of SVZ, one cannot extract an accurate value of the gluon condensate from the charmonium sum rules because of the uncertainties induced by the correlated value and definition of the charm quark mass. The emerging value from different heavy quark sum rules analysis is [3]:

$$\langle\alpha_s G^2\rangle \simeq (4 \sim 6) \times 10^{-2}\ \mathrm{GeV}^4\ . \tag{52.5}$$

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- However, working with sum rules which can disentangle these different contributions, one can extract a more precise result. In Section 51.3, one has observed after examining the different QCD contributions in various quarkonia sum rules that the ones for the $J/\Psi-\eta_c$ and $\Upsilon-\eta_b$ mass splittings is quite sensitive to the value of the gluon condensate, which one can disentangle from the quark mass and leading α_s corrections. In the case of the J/ψ-η_c, the sum rule reads:

$$\mathcal{R}_{VP} \equiv \frac{M^2_{J/\psi}}{M^2_{\eta_c}} = \Delta_0^{VP}\left[1+\alpha_s(\sigma)\Delta^{VP}_{\alpha_s}+\frac{4\pi}{9}\langle\alpha_s G^2\rangle\sigma^2x^2\Delta^{VP}_G\right] , \tag{52.6}$$

where $\sigma \simeq (0.9\pm 0.1)$ GeV^{-2}; $1/x \equiv 4M_c^2\sigma \simeq (6.6\pm 1.8)$ if one uses the conservative value of the charm pole mass $M_c \simeq (1.2\sim 1.5)$ GeV, while numerically, the complete non-expanded expressions in the quark mass read:

$$\Delta_0^{VP} = 0.995^{+0.001}_{-0.004} \quad \Delta^{VP}_{\alpha_s} = 0.0233^{-0.009}_{+0.011} \quad \Delta^{VP}_G = 29.77^{+8.86}_{-10.23} , \tag{52.7}$$

which leads, respectively, to a correction of about 0.5, 2 and 7% of the leading order term for a typical value of the QCD parameters. It is informative to give the expression of these terms in the limit where the quark mass is large. In this way, one obtains to leading order in x from the table in Section 51.3

$$\Delta_0^{VP} = 1-\frac{x^2}{2} \qquad \Delta^{VP}_{\alpha_s} = \frac{\sqrt{\pi}}{9}x^{3/2}+1.539x^2 \qquad \Delta^{VP}_G = \frac{5}{x}\left(1-\frac{4}{5}x\right) , \tag{52.8}$$

which shows, in particular, that the x dependence appearing in the gluon condensate correction is partially compensated by the $1/x$ behaviour of Δ^{VP}_G. Using the experimental value $\mathcal{R}^{exp}_{VP}=1.082$ and $\alpha_s(\sigma)=0.48^{+0.17}_{-0.10}$ for four flavours, one obtains:

$$\langle\alpha_s G^2\rangle = (10\pm 4)10^{-2}\ \text{GeV}^4. \tag{52.9}$$

A similar analysis for the Υ-χ_b mass-splitting gives a much more accurate result as we work at higher scales. Numerically, the sum rule reads [313]:

$$\frac{M^{\text{c.o.m}}_{\chi_b}-M_\Upsilon}{M_\Upsilon} \simeq \left(1.53^{+0.26}_{-0.42}\right)\times 10^{-2}+\left(1.20^{+0.10}_{-0.20}\right)\times 10^{-2}+\left(0.28^{+0.08}_{-0.06}\right)\text{GeV}^{-4}\langle\alpha_s G^2\rangle , \tag{52.10}$$

where $M^{\text{c.o.m}}_{\chi_b}$ is the centre of mass energy:

$$M^{\text{c.o.m}} = \frac{1}{9}\left[5M_{P_2^3}+3M_{P_0^3}+M_{P_0^3}\right] , \tag{52.11}$$

with P_0^3, P_1^3 and P_2^3 refer respectively to the scalar, axial vector and tensor χ_b states. Using the experimental value of these mass-splittings, the analysis leads to:

$$\langle\alpha_s G^2\rangle = (6.9\pm 2.5)10^{-2}\ \text{GeV}^4 , \tag{52.12}$$

The two channels give the average:

$$\langle\alpha_s G^2\rangle = (7.5\pm 2.5)10^{-2}\ \text{GeV}^4 , \tag{52.13}$$

in good agreement with the previous value from $e^+e^-\to I=1$ hadrons data, although less precise.
- The average of these results from different sources reads:

$$\langle\alpha_s G^2\rangle = (7.1\pm 0.9)10^{-2}\ \text{GeV}^4 , \tag{52.14}$$

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which we consider as a final estimate from the sum rules compiled in Table 48.2. Lattice calculations support the above phenomenological estimate [402].

We then conclude from the previous analysis that *the gluon condensate* also breaks spontaneously QCD in a similar way that the quark condensate does for chiral symmetry. A more physical intuitive picture of the non-vanishing value of the gluon condensate is given in [654].

52.4 Dimension-five mixed quark-gluon condensate

- As discussed previously in Chapter 27, the mixed condensate can be parametrized as:

$$\left\langle \bar{\psi}\frac{\lambda_a}{2}\sigma_{\mu\nu}G_a^{\mu\nu}\psi\right\rangle \equiv M_0^2\alpha_s^{\gamma_M/-\beta_1}\langle\bar{\psi}\psi\rangle \ , \tag{52.15}$$

  where $\gamma_M = 1/3$ is the anomalous dimension. Due to its important rôle in the baryon sum rules analysis of odd dimension F_2 part of the correlator, the size of the mixed condensate has been initially obtained from this channel [424–430], where the result also appears to be independent of the choice of the nucleon interpolating currents. It reads:

$$M_0^2 \simeq 0.8\ \text{GeV}^2\ . \tag{52.16}$$

- Alternatively, the mixed condensate has been also obtained from the heavy-light quark systems [401], as it has been noticed that the B and B^* masses are quite sensitive to this quantity, which acts in opposing directions. A priori, this latter method is more reliable than the previous baryon sum rules, due to the smaller complication of this meson channel. It gives a result that is consistent with the one from the baryon sum rules, and extraordinarily accurate:

$$M_0^2 = (0.80 \pm 0.01)\ \text{GeV}^2\ , \tag{52.17}$$

  to the order we have used.
- From these two completely independent analyses, we deduce the value given in Table 48.2:

$$M_0^2 = (0.8 \pm 0.1)\ \text{GeV}^2\ , \tag{52.18}$$

  where we have estimated the error to be about 10% typical of the sum rule analysis. This result is in agreement, with the quenched lattice estimate [660], and with the one from an effective quark interaction model [661]. The result from an instanton liquid model [662] appears to be too high. The analysis of [663] also indicates that the mixed condensate shows a $SU(3)_F$ breaking analogous to the one of the quark condensate. However, this result is opposite with the one from baryon sum rules [426]. The discrepancy of these two results needs clarification.

52.5 Dimension-six four-quark condensates

In Section 52.10 the $e^+e^- \to I = 1$ hadrons data have been used for estimating the non-perturbative condensates. The result obtained in [329] after using different forms of the sum rules and the last iteration of different steps is quoted in Table 48.2. It reads:

$$\delta_V^{(6)} \simeq = (3.7 \pm 0.6) \times 10^{-2} \tag{52.19}$$

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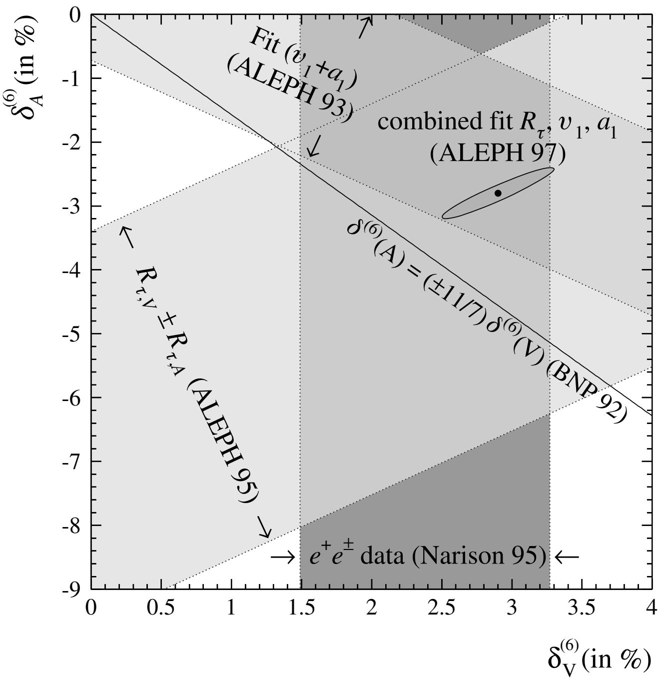

Fig. 52.1. Dimension-six condensate contributions to $R_{\tau,V/A}$.

which is normalized as in Eq. (25.49):

$$M_\tau^6 \delta_{V/A}^{(6)} = \begin{pmatrix} 7 \\ -11 \end{pmatrix} \frac{256\pi^3}{27} \rho\alpha_s \langle\bar\psi\psi\rangle^2 \,, \tag{52.20}$$

where $\rho = 1$ if one uses vacuum saturation for the estimate of the four-quark operators. For a comparison, Fig. 52.1 (figure taken from ALEPH) shows the different estimates from τ-decay in the vector (V) and axial-vector (A) channels. The ALEPH result is [33]:

$$\delta_V^{(6)} \simeq -\delta_A^{(6)} = (2.9 \pm 0.4) \times 10^{-2} \,, \qquad \delta_{V+A}^{(6)} = -(0.1 \pm 0.4) \times 10^{-2} \,. \tag{52.21}$$

The BNP [325] result shown in Fig. 52.1 is based on the vacuum saturation assumption for the ratio of the axial-vector over the vector channel contributions. One should, however, notice that the result of Section 52.10 quoted by ALEPH in this figure corresponds to the first iteration result in the section. The improved result obtained in Section 52.10 and quoted in Table 48.2 has a central value slightly higher and more precise than the one quoted in Fig. 52.1. It corresponds to the value quoted in Eq. (52.19) which is more precise and in excellent agreement with the ALEPH result.

We conclude from the previous analysis that the vacuum saturation or equivalently the leading $1/N_c$ is a very crude estimate of the four-quark condensate values. In most cases, the real value is two to three times the vacuum saturation value. These recent results confirm

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earlier estimates from e^+e^- data using the ratio of moments [404] and from baryon sum rules [424].

52.6 Dimension-six gluon condensates

These condensates have already been discussed in Chapter 27. To my knowledge, there are no sum rules estimates of these quantities. Therefore, we have nothing to add to the discussions in the previous chapter. We shall use the value given in Table 48.2 coming from a conservative range from lattice [402] and DIGA model.

52.7 Dimension-eight condensates

In Section 52.10, the analysis has been pursued for fixing the size of the dimension-eight condensates appearing in the OPE of the vector channel. The final result quoted there is:

$$d_{8,V} \equiv M_\tau^8 \delta_V^{(8)} \simeq -(1.5 \pm 0.6)\ \mathrm{GeV}^8\ , \tag{52.22}$$

normalized in the same way as in Eq. (25.49), although a value of $-(0.85 \pm 0.18)$ GeV^8 has also been obtained in the first stage of the iteration. This result is consistent with the one about -0.95 GeV^8 in [346] and the one from ALEPH [33]:

$$d_{8,V} = -(0.9 \pm 0.1)\ \mathrm{GeV}^8 \tag{52.23}$$

and of OPAL [33]:

$$d_{8,V} = -(0.8 \pm 0.1)\ \mathrm{GeV}^8 \tag{52.24}$$

from τ-decay data. We can consider as a final result from the different estimates the (arithmetic) average value:

$$d_{8,V} \equiv M_\tau^8 \delta_V^{(8)} \simeq -(1.1 \pm 0.3)\ \mathrm{GeV}^8 \tag{52.25}$$

The axial-vector channel is only accessible from τ-decay data. The results from ALEPH is:

$$d_{8,A} = (0.8 \pm 0.1)\ \mathrm{GeV}^8\ , \tag{52.26}$$

and from OPAL:

$$d_{8,A} = (0.4 \pm 0.2)\ \mathrm{GeV}^8\ , \tag{52.27}$$

where the two central values differ almost by a factor 2. We adopt the average of the two results as a final estimate:

$$d_{8,A} \equiv M_\tau^8 \delta_V^{(8)} \simeq (0.6 \pm 0.2)\ \mathrm{GeV}^8\ . \tag{52.28}$$

These estimates are about one order of magnitude bigger than the rough estimate coming from the vacuum saturation [325]:[1]

$$d_{8,V} \approx d_{8,A} \approx -\frac{39}{162}\pi^2 \langle \alpha_s G^2 \rangle^2 \approx 0.01 \text{ GeV}^8 . \tag{52.29}$$

A fit of the (pseudo)scalar channel [394] also shows that the size of the $D = 8$ condensates needed to reproduce the lattice data [393] at large x is also much bigger than the one from the vacuum saturation estimate of these operators.

52.8 Instanton like-contributions

In Section 52.10 an attempt has been made to estimate the contributions of the dimension-nine operators which can mimic the instanton-like effects to the vector correlator [382–387]. The result of the analysis is:

$$\delta_V^{(9)} \equiv \delta_V^{\text{inst}} = -(7.0 \pm 26.5)10^{-4} \left(\frac{1.78}{M_\tau}\right)^9 , \tag{52.30}$$

which is completely negligible in the sum rule working region. This result confirms the alternative phenomenological estimate [387]:

$$\delta_V^{\text{inst}} \approx 3 \times 10^{-3} , \tag{52.31}$$

and theoretical estimate [385]:

$$\delta_V^{\text{inst}} \approx 2 \times 10^{-5} \tag{52.32}$$

at the τ mass. In [384], one also expects a further cancellation for the sum of the vector and axial-vector channels:

$$\delta_{V+A}^{\text{inst}} \approx \frac{1}{20}\delta_V^{\text{inst}} . \tag{52.33}$$

Although there is a concensus over the negligible effect of small size instantons in the V/A channels, the situation in the (pseudo)scalar channel is more controversial. In [385], one also expects that the instanton effect is negligible at the sum rule working scale of about 1 GeV, while in [383] one expects that it breaks completely the OPE in this channel.

In the remaining part of the discussions of this book, we shall adopt the pragmatic attitude that the usual OPE describes the (pseudo)scalar channel and the inclusion of the quadratic term restores the discrepancy between the scales in the ρ and π meson sum rules [161]. This attitude is supported by the lattice result for the (pseudo)scalar two-point function [393], which can be fitted quite well until large x by the OPE including quadratic λ^2 term and the dimension-eight condensate.

[1] A missprint of a factor $1/\pi^2$ has been corrected in the BNP formula.

52.9 Sum of non-perturbative contributions to $e^+e^- \to I = 1$ hadrons and τ decays

A fit of the sum of the different non-perturbative terms entering in the QCD expression of τ decays has been also done by ALEPH and OPAL [33,328]. Using the normalization in Section 25.5, ALEPH obtains:

$$\delta_{NP,V} = 0.020 \pm 0.017\,, \qquad \delta_{NP,A} = -0.027 \pm 0.009\,, \qquad \delta_{NP,V+A} = -0.003 \pm 0.005\,, \tag{52.34}$$

while OPAL obtains for different structures of the perturbative QCD series:

$$\delta_{NP,V} = 0.016 \pm 0.004\,, \quad \delta_{NP,A} = -0.023 \pm 0.004\,, \quad \delta_{NP,V+A} = -0.0035 \pm 0.0035\,. \tag{52.35}$$

In Section 52.10, the sum of the different non-perturbative contributions including the dimension-nine condensates from e^+e^- data is found to be:

$$\delta_{NP,V} = 0.024 \pm 0.009 \tag{52.36}$$

in good agreement with the former results from τ-decays.

52.10 Reprinted paper

QCD tests from $e^+e^- \to I = 1$ hadrons data and implication on the value of α_s from τ-decays

S. Narison

1. Introduction

Measurements of the QCD scale Λ and of the q^2-evolution of the QCD coupling are one of the most important tests of perturbative QCD. At present LEP and τ-decay data [1–7] indicate that the value of α_s is systematically higher than the one extracted from deep-inelastic low-energy data[1]. The existing estimate of α_s from QCD spectral sum rules [9] à la SVZ [10] in e^+e^- data [11,12] apparently favours a low value of α_s, a result, which is, however, in contradiction with the recent CVC-test performed by [13] using e^+e^- data. It is therefore essential to test the reliability of the low-energy predictions before speculating on the phenomenological consequences implied by the previous discrepancy.

Deep-inelastic scattering processes need a better control of the parton distributions and of the power corrections in order to be competitive with the LEP and tau-decay measurements. In addition, perturbative corrections in these processes should be pushed so far such that the remaining uncertainties will only be due to the re-summation of the perturbative series at large order. Indeed, the τ-decay rate has been calculated including the α_s^3-term [3], while an

[1] However, new results of jet studies in deep-inelastic ep-scattering at HERA for photon momentum transfer $10 \leq Q^2$ [GeV^2] ≤ 4000 give a value of α_s [8] compatible with the LEP-average.

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estimate [14] and a measurement [15] of the α_s^4 coefficient is done. Moreover, a resummation of the $(\beta_1\alpha_s)^n$ of the perturbative series is now available [16].

The QCD spectral sum rule (QSSR) [9] à la SVZ [10] applied to the $I = 1$ part of the $e^+e^- \to$ hadrons total cross-section has a QCD expression very similar to the τ-decay inclusive width, such that on a theoretical basis, one can also have a good control of it.

In a previous paper [17], we have derived in a model-independent way the running mass of the strange quark from the difference between the $I = 1$ and $I = 0$ parts of the $e^+e^- \to$ hadrons total cross-section. In this paper, we pursue this analysis by re-examining the estimate of α_s and of the condensates including the instanton-like and the *marginal* $D = 2$-like operators obtained from the $I = 1$ channel of the e^+e^- data. In so doing, we re-examine the exponential Laplace sum rule used by [11] in e^+e^-, which is a generalization of the ρ-meson sum rule studied originally by SVZ [10]. We also expect that the Laplace sum rule gives a more reliable result than the FESR due to the presence of the exponential weight factor which suppresses the effects of higher meson masses in the sum rule. This is important in the particular channel studied here as the data are very inaccurate above 1.4–1.8 GeV, where, at this energy, the optimal result from FESR satisfies the so-called heat evolution test [12,18,19]. That makes the FESR prediction strongly dependent on the way the data in this region are parametrized, a feature which we have examined [13,20] for criticizing the work of [21]. We also test the existing and controversial estimates [18,19] of the $D = 2$-type operator obtained from QSSR. Combining our different non-perturbative results with the recent resummed perturbative series [16], we re-estimate and confirm the value of α_s from τ-decays.

2. α_s from $e^+e^- \to I = 1$ hadrons data

Existing estimates of α_s or Λ from different aspects of QSSR for $e^+e^- \to I = 1$ hadrons data [11,12] lead to values much smaller than the present LEP and τ-decay measurements [3–7]. However, such results contradict the stability-test on the extraction of α_s from τ-like inclusive decay [13] obtained using CVC in e^+e^- [22] for different values of the τ-mass. In the following, we shall re-examine the reliability of these sum rule results.

We shall not reconsider the result from FESR [12] due to the drawbacks of this method mentioned previously, and also, because the FESR-analysis has been re-used recently [18,19] in the determination of the $D = 2$-type operator, which we shall come back later on.

Λ_3 and the condensates have been extracted in [11] from the Laplace sum rule:

$$\mathcal{L}_1 \equiv \frac{2}{3}\tau \int_{4m_\pi^2}^{\infty} ds\, e^{-st} R^{I=1}(s) \tag{1}$$

and from its $\tau \equiv 1/M^2$ derivative:

$$\mathcal{L}_2 \equiv \frac{2}{3}\tau^2 \int_{4m_\pi^2}^{\infty} ds\, s\, e^{-s\tau} R^{I=1}(s)\ , \tag{2}$$

where:

$$R^I \equiv \frac{\sigma(e^+e^- \to I \text{ hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}\ . \tag{3}$$

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In the chiral limit $m_u = m_d = 0$, the QCD expressions of the sum rule can be written as:

$$\mathcal{L}_i = 1 + \sum_{D=0,2,4,\ldots} \Delta_i^{(D)}. \tag{4}$$

The perturbative corrections can be deduced from the ones of $R^{I=1}$ obtained to order α_s^3:

$$R^{I=1}(s) = \frac{3}{2}\left\{1 + a_s + F_3 a_s^2 + F_4 a_s^3 + \mathcal{O}\left(a_s^4\right)\right\}, \tag{5}$$

where, for 3 flavours: $F_3 = 1.623$ [23], $F_4 = 6.370$ [24]; the expression of the running coupling to three-loop accuracy is:

$$\begin{aligned} a_s(\nu) = a_s^{(0)} \Bigg\{ & 1 - a_s^{(0)} \frac{\beta_2}{\beta_1} \log\log\frac{\nu^2}{\Lambda^2} \\ & + \left(a_s^{(0)}\right)^2 \left[\frac{\beta_2^2}{\beta_1^2}\log^2\log\frac{\nu^2}{\Lambda^2} - \frac{\beta_2^2}{\beta_1^2}\log\log\frac{\nu^2}{\Lambda^2} \right. \\ & \left. - \frac{\beta_2^2}{\beta_1^2} + \frac{\beta_3}{\beta_1}\right] + \mathcal{O}\left(a_s^3\right)\Bigg\}, \end{aligned} \tag{6}$$

with:

$$a_s^{(0)} \equiv \frac{1}{-\beta_1 \log(\nu/\Lambda)} \tag{7}$$

and β_i are the $\mathcal{O}(a_s^i)$ coefficients of the β-function in the $\overline{\mathrm{MS}}$ scheme for n_f flavours:

$$\begin{aligned} \beta_1 &= -\frac{11}{2} + \frac{1}{3}n_f \\ \beta_2 &= -\frac{51}{4} + \frac{19}{12}n_f \\ \beta_3 &= \frac{1}{64}\left[-2857 + \frac{5033}{9}n_f - \frac{325}{27}n_f^2\right]. \end{aligned} \tag{8}$$

For three flavours, we have:

$$\beta_1 = -9/2, \qquad \beta_2 = -8, \qquad \beta_3 = -20.1198\,. \tag{9}$$

In the chiral limit, the $D = 2$-contribution vanishes. It has also been proved recently [16] that renormalon-type contributions induced by the resummation of the QCD series at large order cannot induce such a term.

In the chiral limit, the $D = 4$ non-perturbative corrections read [10,3]:

$$\begin{aligned} \Delta_1^{(4)} &= \frac{\pi}{3}\tau^2\langle\alpha_s G^2\rangle\left(1 - \frac{11}{18}\frac{\alpha_s}{\pi}\right) \\ \Delta_2^{(4)} &= -\Delta_1^{(4)}. \end{aligned} \tag{10}$$

The $D = 6$ non-perturbative corrections read [10]:

$$\Delta_1^{(6)} = -\frac{448\pi^3}{81}\tau^3\rho\langle\bar{u}u\rangle^2$$
$$\Delta_2^{(6)} = -2\Delta_1^{(6)} . \tag{11}$$

We shall use, in the first iteration of the analysis, the conservative values of the condensates [9,3]:

$$\langle\alpha_s G^2\rangle = (0.06 \pm 0.03)\,\mathrm{GeV}^4,$$
$$\rho\langle\bar{u}u\rangle^2 = (3.8 \pm 2.0)10^{-4}\,\mathrm{GeV}^6, \tag{12}$$

and *high* values of Λ from LEP and tau-decay data [1–4] for 3 flavours:

$$\Lambda_3 = 375^{+105}_{-85}\,\mathrm{MeV}, \tag{13}$$

corresponding to $\alpha_s(M_Z) = 0.118 \pm 0.06$.

The phenomenological side of the sum rule has been parametrized using analogous data as [11] and updated using the data used in [13]. The confrontation of the QCD and the phenomenological sides of the sum rules is done in Fig. 1a and in Fig. 2a for a giving value of $\Lambda_3 = 375$ MeV and varying the condensates in the range given previously. One can conclude that one has a good agreement between the two sides of $\mathcal{L}_1$ for $M \geq 0.8$ GeV and of $\mathcal{L}_2$ for $M \geq 1.0 \sim 1.2$ GeV. The effects of the condensates are important below 1 GeV for $\mathcal{L}_1$ and below 1.3 GeV for $\mathcal{L}_2$. In Fig. 1b and Fig. 2b, we fix the condensates at their central values and we vary Λ_3 in the range given above. One can notice that a value of Λ_3 as high as 525 MeV is still allowed by the data, while the shape of the QCD curve for $\mathcal{L}_2$ changes drastically for a high value of Λ_3. This phenomena is not informative as, below 1 GeV, higher dimension condensates can already show up and may break the Operator Product Expansion (OPE).

By comparing these results with the ones of [11], one can notice that our QCD prediction for $\mathcal{L}_1$ corresponding to the previous set of parameters is as good as the one of [11] obtained from a different set of the QCD parameters, while for that of $\mathcal{L}_2$, the agreement between the two sides of the sum rule is obtained here at a slightly larger value of M for high values of Λ_3.

One can clearly conclude from our analysis is that the exponential Laplace sum rules applied to $e^+e^- \to I = 1$ hadrons data *do not exclude* values of Λ_3 obtained from LEP and τ-decay data. Contrary to some claims in the literature, the sum rules cannot also give a *precise* information on the real value of Λ_3 if the condensates are allowed to move inside the conservative range of values given in Eq. (12). It is also important and reassuring, that our analysis supports the value of Λ_3 obtained from τ-decay and used in e^+e^- via CVC [22] for the stability-test of the prediction for different values of the τ-mass [13] as we shall see also below.

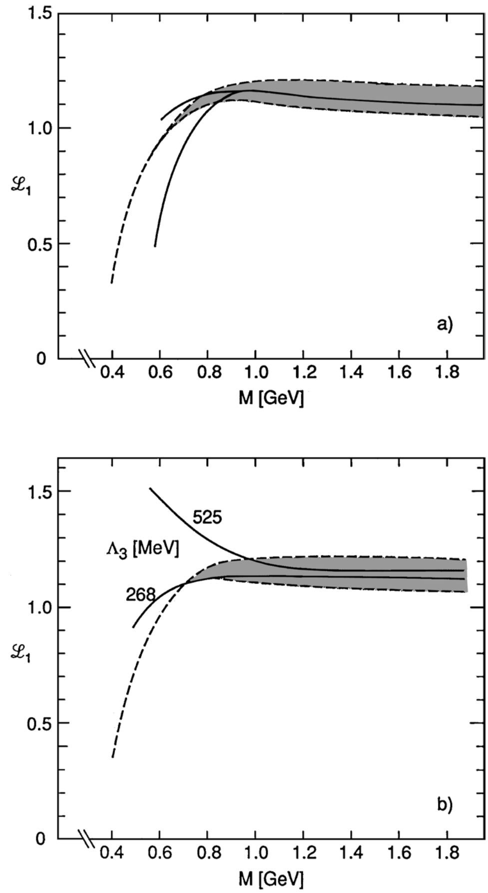

Fig. 1. (a) The Laplace sum rule $\mathcal{L}_1$ versus the sum rule parameter M. The dashed curves correspond to the experimental data. The full curves correspond to the QCD prediction for $\Lambda_3 = 375$ MeV, $\langle \alpha_s G^2 \rangle = 0.06 \pm 0.03$ GeV^4 and $\rho \alpha_s \langle \bar{u}u \rangle^2 = (3.8 \pm 2.0)10^{-4}$ GeV^6. (b) The same as Fig. 1a but for different values of Λ_3 and for $\langle \alpha_s G^2 \rangle = 0.06$ GeV^4 and $\rho \alpha_s \langle \bar{u}u \rangle^2 = 3.8\, 10^{-4}$ GeV^6.

3. The condensates from τ-like decays

In so doing, we shall work with the vector component of the τ decay-like quantity deduced from CVC [22]:

$$R_{\tau,1} \equiv \frac{3\cos^2\theta_c}{2\pi\alpha^2} S_{\mathrm{EW}} \times \int_0^{M_\tau^2} ds \left(1 - \frac{s}{M_\tau^2}\right)^2 \left(1 + \frac{2s}{M_\tau^2}\right) \frac{S}{M_\tau^2}\, \sigma_{e^+e^- \to I=1} \,, \tag{14}$$

where $S_{\mathrm{EW}} = 1.0194$ is the electroweak correction from the summation of the leading-log contributions [25].

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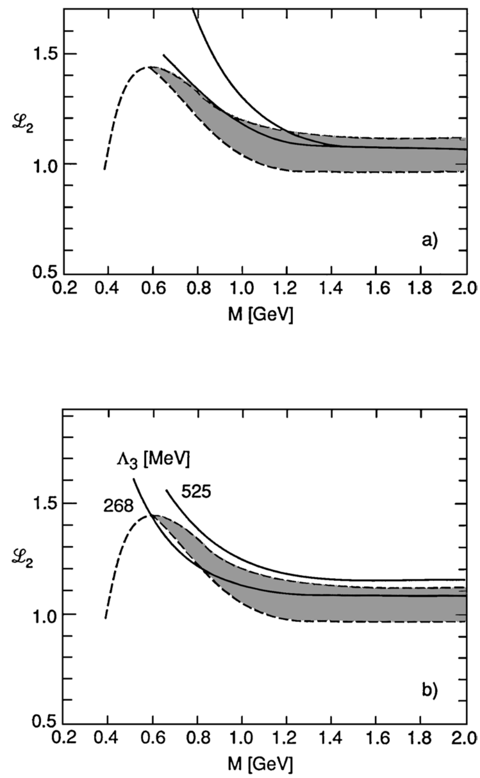

Fig. 2. (a) The same as Fig. 1a but for $\mathcal{L}_2$. (b) The same as Fig. 1b but for $\mathcal{L}_2$.

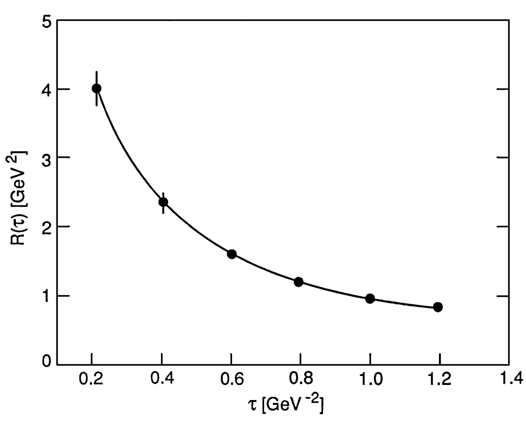

Fig. 3. Experimental value of the ratio of Laplace sum rules $\mathcal{R}(\tau)$ versus the sum rule variable $\tau = 1/M^2$.

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This quantity has been used in [13] in order to test the stability of the α_s-prediction obtained at the τ-mass of 1.78 GeV. It has also been used to test CVC for different exclusive channels [13,26]. Here, we shall again exploit this quantity in order to deduce *model-independent* informations on the values of the QCD condensates. The QCD expression of $R_{\tau,1}$ reads:

$$R_{\tau,1} = \frac{3}{2}\cos^2\theta_c S_{\rm EW} \times \left(1 + \delta_{\rm EW} + \delta^{(0)} + \sum_{D=2,4,\ldots} \delta_1^{(D)}\right) , \tag{15}$$

where $\delta_{\rm EW} = 0.0010$ is the electroweak correction coming from the constant term [27]; the perturbative corrections read [3]:

$$\delta^{(0)}\left(a_s \equiv \frac{\alpha_s(M_\tau)}{\pi}\right) + 5.2023a_s^2 + 26.366a_s^3 + \cdots , \tag{16}$$

The a_s^4 coefficient has also been estimated to be about 103 [14,15], though we shall use $(78 \pm 25)\, a_s^4$ where the error reflects the uncalculated higher order terms of the D-function, while the first term is induced by the lower order coefficients after the use of the Cauchy integration.

In the chiral limit $m_i = 0$, the quadratic mass-corrections contributing to $\delta_1^{(2)}$ vanish. Moreover, it has been proved [16] that the summation of the perturbative series cannot induce such a term, while the one induced eventually by the freezing mechanism is safely negligible [28,18]. Therefore, we shall neglect this term in the first step of our analysis. We shall test, later on, the internal consistency of the approach if a such term is included into the OPE.

In the chiral limit $m_i = 0$, the $D = 4$ contributions read [3]:

$$\delta_1^{(4)} = \frac{11}{4}\pi a_s^2 \frac{\langle \alpha_s G^2\rangle}{M_\tau^4} , \tag{17}$$

which, due to the Cauchy integral and to the particular s-structure of the inclusive rate, the gluon condensate starts at $\mathcal{O}(a_s^2)$. This is a great advantage compared with the ordinary sum rule discussed previously. The $D = 6$ contributions read [3]:

$$\delta_1^{(6)} \simeq 7\frac{256\pi^3}{27}\frac{\rho\alpha_s\langle\bar\psi_i\psi_i\rangle^2}{M_\tau^6} , \tag{18}$$

The contribution of the $D = 8$ operators in the chiral limit reads [3]:

$$\delta_1^{(8)} = -\frac{39\pi^2}{162}\frac{\langle\alpha_s G^2\rangle^2}{M_\tau^8} . \tag{19}$$

The phenomenological parametrization of $R_{\tau,1}$ has been done using the same data input as in [18,13]. We give in Table 1 its value for different values of the tau-mass. Neglecting the $D = 4$-contribution which is of the order α_s^2, we perform a two-parameter fit of the data for each value of Λ_3 corresponding to the world average value of $\alpha_s(M_Z) = 0.118 \pm 0.006$

Table 1. *Phenomenological estimate of $R_{\tau,1}$*

M_τ [GeV]	$R_{\tau,1}$
1.0	1.608 ± 0.064
1.2	1.900 ± 0.075
1.4	1.853 ± 0.072
1.6	1.793 ± 0.070
1.8	1.790 ± 0.081
2.0	1.818 ± 0.097

Table 2. *Estimates of d_6 and d_8 from $R_{\tau,1}$ for different values of Λ_3*

Λ_3 [MeV]	d_6 [GeV^6]	$-d_8$ [GeV^8]
480	-0.07 ± 0.43	1.15 ± 0.40
375	0.27 ± 0.34	0.69 ± 0.31
290	0.58 ± 0.29	0.83 ± 0.27

[1,2] and by letting the $D = 6$ and $D = 8$ condensates as free-parameters. We show the results of the fitting procedure in Table 2 for different values of Λ_3.

The errors take into account the effects of the τ-mass moved from 1.6 to 2.0 GeV, which is a negligible effect, and the one due to the data. One can notice that the estimate of the $D = 8$ condensates is quite accurate, while the one of the $D = 6$ is not very conclusive for $\Lambda_3 \geq 350$ MeV. Indeed, only below this value, one sees that the $D = 6$ contribution is clearly positive as expected from the vacuum saturation estimate. This fact also explains the anomalous low value of $-d_8$ around this transition region. Using the average value of Λ_3 in Eq. (13), we can deduce the result:

$$d_8 \equiv M_\tau^8 \delta_1^{(8)} = -(0.85 \pm 0.18)\,\text{GeV}^8$$
$$d_6 \equiv M_\tau^6 \delta_1^{(6)} = (0.34 \pm 0.20)\,\text{GeV}^6\,, \tag{20}$$

which we shall improve again later on once we succeed to fix the value of d_6.

4. The condensates from the ratio of the Laplace sum rules

Let us now improve the estimate of the $D = 6$ condensates. In so doing, one can remark that, though there are large discrepancies in the estimate of the absolute values of the condensates from different approaches, there is a consensus in the estimate of the ratio of

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the $D = 4$ over the $D = 6$ condensates:[2]

$$r_{46}\,[\text{GeV}^{-2}] \equiv \frac{\langle\alpha_s G^2\rangle}{\rho\alpha_s\langle\bar{u}u\rangle^2} = \begin{array}{l} 94.80 \pm 23\ [29] \\ 96.20 \pm 35\ [12] \\ 114.6 \pm 16\ [30] \\ 92.50 \pm 50\ [32]\,. \end{array} \tag{21}$$

from which we deduce the *average*:

$$r_{46} = (105.9 \pm 11.9)\ \text{GeV}^{-2}\,. \tag{22}$$

We use the previous informations on d_8 and r_{46} for fitting the value of the $D = 4$ condensates from the ratio of the Laplace sum rules:

$$\mathcal{R}(\tau) \equiv \tau^{-2}\frac{\mathcal{L}_2}{\mathcal{L}_1}\,, \tag{23}$$

used previously by [29] for a simultaneous estimate of the $D = 4$ and $D = 6$ condensates. We recall that the advantage of this quantity is its less sensitivity to the leading order perturbative corrections. The phenomenological value of $\mathcal{R}(\tau)$ is given in Fig. 2. Using a one-parameter fit, we deduce:

$$\langle\alpha_s G^2\rangle = (6.1 \pm 0.7)\ 10^{-2}\ \text{GeV}^4\,. \tag{24}$$

Then, we re-inject this value of the gluon condensate into the tau-like width in Eq. (14), from which we re-deduce the value of the $D = 8$ condensate. After a re-iteration of this procedure, we deduce our *final* results:

$$\begin{aligned} \left\langle\alpha_s G^2\right\rangle &= (7.1 \pm 0.7)\ 10^{-2}\ \text{GeV}^4\,, \\ d_8 &= -(1.5 \pm 0.6)\ \text{GeV}^8\,. \end{aligned} \tag{25}$$

Using the mean value of r_{46}, we also obtain:

$$\rho\alpha_s\langle\bar{u}u\rangle^2 = (5.8 \pm 0.9)\ 10^{-4}\ \text{GeV}^6\,. \tag{26}$$

We consider these results as an improvement and a confirmation of the previous result in Eq. (12). It is also informative to compare these results with the ALEPH and CLEO II measurements of these condensates from the moments distributions of the τ-decay width. The most accurate measurement leads to [5]:

$$\langle\alpha_s G^2\rangle = (7.8 \pm 3.1)\ 10^{-2}\ \text{GeV}^4\,, \tag{27}$$

while the one of d_6 has the same absolute value as previously but comes with the wrong sign. Our value of d_8 is in good agreement with the one $d_8 \simeq -0.95\ \text{GeV}^8$ in [13,6] obtained from

[2] We have multiplied the original error given by [30] by a factor 10. The constraint obtained in [31] is not very conclusive as it leads to $r_{46} \leq 110\ \text{GeV}^{-2}$ and does not exclude negative values of the condensates which are forbidden from positivity arguments.

the same quantity, but it is about one order of magnitude higher than the vacuum saturation estimate proposed by [33] and about a factor 5 higher than the CLEO II measurement. However, it is lower by a factor 2∼3 than the FESR result from the vector channel [32][3]. The discrepancy with the vacuum saturation indicates that this approximation is very crude, while the one with the FESR is not very surprising, as the FESR approach done in the vector and axial-vector channels [12,32] tends *always to overestimate* the values of the QCD condensates. The discrepancy with the CLEO II measurement can be understood from the wrong sign of the $D = 6$ condensate obtained there and to its correlation with the $D = 8$ one.

5. Instanton contribution

Let us now extract the size of the instanton-like contribution by assuming that it acts like a $D \geq 9$ operator. A good place for doing it is $\mathcal{R}_{\tau,1}$ as, in the Laplace sum rules, this contribution is suppressed by a 8! factor implying a weaker constraint. Using the previous values of the $D = 6$ and $D = 8$ condensates, we deduce:

$$\delta_1^{(9)} \equiv \delta_V^{\text{inst}} = -(7.0 \pm 26.5)\, 10^{-4}(1.78/M_\tau)^9 \,, \tag{28}$$

which, though inaccurate indicates that the instanton contribution is negligible for the vector current and has been overestimated in [34] ($\delta_V^{\text{inst}} \approx 0.03 \sim 0.05$). Our result supports the negligible effects found from an alternative phenomenological [35] ($\delta_V^{\text{inst}} \approx 3 \times 10^{-3}$) and theoretical [36] ($\delta_V^{\text{inst}} \approx 2 \times 10^{-5}$) analysis. Further cancellations in the sum of the vector and axial-vector components of the tau widths are however expected [34,35] ($\delta^{\text{inst}} \approx \frac{1}{20}\delta_V^{\text{inst}}$).

6. Test of the size of the $1/M_\tau^2$-term

Let us now study the size of the $1/M_\tau^2$-term. From the QCD point of view, its possible existence from the resummation of the PTS due to renormalon contributions [28] has not been confirmed [16], while some other arguments [28,37] advocating its existence are not convincing and seems to be a pure speculation. Postulating its existence (whatever its origin!), [18] has estimated the strength of this term by using FESR and the ratio of moments $\mathcal{R}(\tau)$. As already mentioned earlier, the advantage in working with the ratio of moments is that the leading order perturbative corrections disappear such that in a *compromise* region where the high-dimension condensates are still negligible, there is a *possibility* to pick up the $1/M_\tau^2$-contribution. Indeed, using usual stability criteria and *allowing a large range of values around the optimal result*, [18] has obtained the *conservative* value:

$$d_2 \equiv C_2 \equiv \delta_1^2 M_\tau^2 \simeq (0.03 \sim 0.08)\ \text{GeV}^2 \,, \tag{29}$$

[3] In the normalization of [32], our value of d_8 translates into $C_8\langle O_8\rangle = (0.18 \pm 0.04)\ \text{GeV}^8$.

Table 3. *Estimates of* $\langle \alpha_s G^2 \rangle$ *from* $R(\tau)$ *for different values of* d_2

d_2 [GeV]^2 [18]	$-d_2$ [GeV]^2 [19]	$\langle \alpha_s G^2 \rangle$ 10^2 [GeV^4]
0.03		7.8 ± 0.5
0.05		8.1 ± 0.5
0.07		8.6 ± 0.5
0.09		9.1 ± 0.5
	0.2	3.2 ± 0.29
	0.3	1.2 ± 0.29
	0.4	-0.7 ± 0.6

while the estimate of [18] from FESR applied to the vector current has not been very conclusive, as it leads to the inaccurate value:

$$d_2 \simeq (0.02 \pm 0.12)\ \mathrm{GeV}^2\ . \tag{30}$$

However, the recent FESR analysis from the axial-vector current [19] obtained at about the same value of the continuum threshold t_c satisfying the so-called evolution test [12], disagrees in sign and magnitude with our previous estimate from the ratio of moments and is *surprisingly* very precise compared with the result in Eq. (30) obtained from the same method for the vector current. If one assumes like [19] a quadratic dependence in Λ_3, the result of [19] becomes for the value of Λ_3 in Eq. (13):

$$d_2 \simeq -(0.3 \pm 0.1)\ \mathrm{GeV}^2\ , \tag{31}$$

We test the reliability of this result, by remarking that d_2 (if it exists!) is strongly correlated to d_4 in the analysis of the ratio of Laplace sum rules $R_{(\tau)}$, while this is not the case between d_2 or d_4 with d_6 and d_8. Using our previous values of d_6 and d_8, one can study the variation of d_4 given the value of d_2. The results given in Table 3 indicate that the present value of the gluon condensate excludes the value of d_2 in Eq. (31) and can only permit a negligible fluctuation around zero of this contribution, which should not exceed the value $0.03 \sim 0.05$. This result rules out the possibility to have a sizeable $1/M_\tau^2$-term [28,37] and justifies its neglect in the analysis of the τ-width. More precise measurement of the gluon condensate or more statistics in the τ-decay data will improve this constraint.

7. Sum of the non-perturbative corrections to R_τ

Using our previous estimates, it is also informative to deduce the sum of the non-perturbative contributions to the decay widths of the observed heavy lepton of mass 1.78 GeV. In so doing, we add the contributions of operators of dimensions $D = 4$ to $D = 9$ and we neglect the expected small $\delta^{(2)}$-contribution.

Table 4. *QCD predictions for R_τ using the contour coupling-expansion*

$\alpha_s(M_\tau)$	a_s^3	a_s^4	a_s^6	a_s^8
0.26	3.364 ± 0.022	3.370	3.380 ± 0.019	3.381
0.28	3.402 ± 0.024	3.411	3.426 ± 0.019	3.426
0.30	3.442 ± 0.026	3.453	3.474 ± 0.021	3.472
0.32	3.484 ± 0.030	3.498	3.526 ± 0.023	3.520
0.34	3.526 ± 0.033	3.546	3.582 ± 0.031	3.568
0.36	3.571 ± 0.040	3.594	3.640 ± 0.045	3.613
0.38	3.616 ± 0.040	3.645	3.706 ± 0.069	3.655
0.40	3.664 ± 0.040	3.700	3.775 ± 0.108	3.685

For the vector component of the tau hadronic width, we obtain:[4]

$$\delta_V^{\rm NP} \equiv \sum_{D=4}^{9} \delta_1^{(D)} = (2.38 \pm 0.89)10^{-2} , \qquad (32)$$

while using the expression of the corrections for the axial-vector component given in [3], we deduce:

$$\delta_A^{\rm NP} = -(7.95 \pm 1.12)10^{-2} , \qquad (33)$$

and then:

$$\delta^{\rm NP} \equiv \frac{1}{2}\left(\delta_V^{\rm NP} + \delta_A^{\rm NP}\right) = -(2.79 \pm 0.62)10^{-2} , \qquad (34)$$

Our result confirms the smallness of the non-perturbative corrections measured by the ALEPH and CLEO II groups [5]:

$$\delta_{\rm exp}^{\rm NP} = (0.3 \pm 0.5)10^{-2} , \qquad (35)$$

though the exact size of the experimental number is not yet very conclusive.

8. Implication on the value of α_s from R_τ

Before combining the previous non-perturbative results with the perturbative correction to R_τ, let us test the accuracy of the resummed $(\alpha_s\beta_1)^n$ perturbative result of [16]. In so doing, we fix $\alpha_s(M_\tau)$ to be equal to 0.32 and we compare the resummed value of $\delta^{(0)}$ including the δ_s^3-corrections with the one where the coefficients have been calculated in the $\overline{\rm MS}$ scheme [23]. We consider the two cases where R_τ is expanded in terms of the usual coupling α_s or in terms of the *contour coupling* [4]. In both cases, one can notice that the approximation

[4]We have used, for $M_\tau = 1.78$ GeV, the conservative values: $\delta_V^{(9)} \approx \delta_A^{(9)} \simeq -(0.7 \pm 2.7)10^{-3}$ and $\delta^{(9)} \approx 1/20\delta_V^{(9)}$ [34].

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used in the resummation technique tends to overestimate the perturbative correction by about 10%. Therefore, we shall reduce systematically by 10%, the prediction from this method from the α_s^5 to α_s^9 contributions. We shall use the coefficient 27.46 of α_s^4 estimated in [14,15]. Noting that, to the order where the perturbative series (PTS) is estimated, one has alternate signs in the PTS, which is an indication for reaching the asymptotic regime. Therefore, we can consider, as the best estimate of the resummed PTS, its value at the minimum. That is reached, either for truncating the PTS by including the α_s^6 or the α_s^8 contributions. The corresponding value of R_τ including our non-perturbative contributions in Eq. (34) is given in Table 4. We show for comparison the value of R_τ including the α_s^3-term, where we have used the perturbative estimate in [6] (the small difference with the previous papers [4,13,6,7,20] comes from the different non-perturbative term used here), while the error quoted there comes from the naïve estimate $\pm 50 a_s^4$. However, one can see that the estimate of this perturbative error has taken properly the inclusion of the higher order terms, while the truncation of the series at α_s^3 already gives a quite good evaluation of the PTS. One can also notice that there is negligible difference between the PTS to order α_s^6 and α_s^8 for small values of α_s, while the difference increases for larger values. We consider, as a final perturbative estimate of R_τ, the one given by the PTS including the α_s^6-term at which we encounter the first minimum. The error given in this column is the sum of the non-perturbative one from Eq. (34) with the perturbative conservative uncertainty, which we have estimated like the effect due to the last term i.e $\pm 34.53(-\beta_1 a_s/2)^6$ at which the minimum is reached, which is a legitimate procedure for asymptotic series [38]. We have also added to the latter the one due to the small fluctuation of the minimum of the PTS from the inclusion of the α_s^6 or α_s^8-terms. One can notice that for $\alpha_s \leq 0.32$, the error in R_τ is dominated by the non-perturbative one, while for larger value of α_s, it is mainly due to the one from the PTS. Using the value of R_τ in Table 4, we deduce:

$$\alpha_s(M_\tau) = 0.33 \pm 0.030, \tag{36}$$

where we have used the experimental average [2]:

$$R_\tau = 3.56 \pm 0.03. \tag{37}$$

Our result from the optimized resummed PTS is in good agreement with the most recent estimate obtained to order α_s^3 [6,5,7]:

$$\alpha_s(M_\tau) = 0.33 \pm 0.030. \tag{38}$$

9. Conclusion

Our analysis of the isovector component of the $e^+e^- \rightarrow$ hadrons data has shown that there is a consistent picture on the extraction of α_s from high-energy LEP and low-energy τ and e^+e^- data.

It has also been shown that the values of the condensates obtained from QCD spectral sum rules based on *stability criteria* are reproduced and improved by fitting the τ-like decay widths and the ratio of the Laplace sum rules. Our estimates are in good agreement with the determination of the condensates from the τ-hadronic width moment-distributions [5], which needs to be improved from accurate measurements of the e^+e^- data or/and for more data sample of the τ-decay widths which can be reached at the τ-charm factory machine.

Finally, our consistency test of the effect of the $1/M_\tau^2$-term, whatever its origin, does not support the recent estimate of this quantity from FESR axial-vector channel [19] and only permits a small fluctuation around zero due to its strong correlation with the $D = 4$ condensate effects in the ratio of Laplace sum rules analysis, indicating that it cannot affect in a sensible way the accuracy of the determination of α_s from tau decays.

As a by-product, we have reconsidered the estimate of $\alpha_s(M_\tau)$ from the τ-widths taking into account the recent resummed result of the perturbative series. Our result in Eq. (36) is a further support of the existing estimates.

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References

[1] S. Bethke, talk given at the QCD94 Workshop, 7–13th July 1994, Montpellier, France and references therein; I. Hinchliffe, talk given at the 1994 Meeting of the American Physical Society, Albuquerque (1994); B. Weber, talk given at the IHEP-Conference, Glasgow (1994).
[2] PDG94, L. Montanet et al., Phys. Rev. D 50 (1994), Part 1, 1175.
[3] E. Braaten, S. Narison and A. Pich, Nucl. Phys. B 373 (1992) 581.
[4] F. Le Diberder and A. Pich, Phys. Lett. B 286 (1992) 147; B 289 (1992) 165.
[5] ALEPH Collaboration: D. Buskulic et al., Phys. Lett. B 307 (1993) 209; CLEOII Collaboration: R. Stroynowski, talk given at TAU94 Sept. 1994, Montreux, Switzerland; for a review of the different LEP data on tau decays, see e.g: L. Duflot, talks given at the QCD94 Workshop, 7–13th July 1994, Montpellier, France and TAU94 Sept. 1994, Montreux, Switzerland.
[6] A. Pich, talk given at the QCD94 Workshop, 7–13th July 1994, Montpellier, France.
[7] S. Narison, talk given at TAU94 Sept. 1994, Montreux, Switzerland.
[8] Hl Collaboration, Phys. Lett. B 346 (1995) 415.
[9] S. Narison, QCD spectral sum rules Lecture notes in physics, Vol 26 (1989).
[10] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B 147 (1979) 385, 448.
[11] S.I. Eidelman, L.M. Kurdadze and A.I. Vainshtein, Phys. Lett. B 82 (1979) 278.
[12] R.A. Bertlmann, G. Launer and E. de Rafael, Nucl. Phys. B 250 (1985) 61; R.A. Bertlmann, C.A. Dominguez, M. Loewe, M. Perrottet and E. de Rafael, Z. Phys. C 39 (1988) 231.
[13] S. Narison and A. Pich, Phys. Lett. B 304 (1993) 359.
[14] A.L. Kataev, talk given at the QCD94 Workshop, 7–13th July 1994, Montpellier, France.

[15] F. Le Diberder, talk given at the QCD94 Workshop, 7–13th July 1994, Montpellier, France.
[16] P. Ball, M. Beneke and V.M. Braun, CERN-TH/95-26 (1995) and references therein; C.N. Lovett-Turner and C.J. Maxwell, University of Durham preprint DTP/95/36 (1995).
[17] S. Narison, Montpellier preprint PM95/06 (1995).
[18] S. Narison, Phys. Lett. B 300 (1993) 293.
[19] C.A. Dominguez, Phys. Lett. B 345 (1995) 291.
[20] S. Narison, talk given at the QCD-LEP meeting on α_s (published by S. Bethke and W. Bernreuther as Aachen preprint PITHA 94/33).
[21] T.N. Truong, Ecole polytechnique preprint, EP-CPth.A266.1093 (1993); Phys. Rev. D 47 (1993) 3999; talk given at the QCD-LEP meeting on α_s (published by S. Bethke and W. Bernreuther as Aachen preprint PITHA 94/33).
[22] F.J. Gilman and S.H. Rhie, Phys. Rev. D 31 (1985) 1066; F.J. Gilman and D.H. Miller, Phys. Rev. D 17 (1978) 1846; F.J. Gilman, Phys. Rev. D 35 (1987) 3541.
[23] K.G. Chetyrkin, A.L. Kataev and F.V. Tkachov, Phys. Lett. B 85 (1979) 277; M. Dine and J. Sapirstein, Phys. Rev. Lett. 43 (1979) 668; W. Celmaster and R.J. Gonsalves, Phys. Rev. Lett. 44 (1980) 560.
[24] S.G. Gorishny, A.L. Kataev and S.A. Larin, Phys. Lett. B 259 (1991) 144; L.R. Surguladze and M.A. Samuel, Phys. Rev. Lett. 66 (1991) 560, 2416 (E).
[25] W. Marciano and A. Sirlin, Phys. Rev. Lett. 61 (1988) 1815; 56 (1986) 22.
[26] S.I. Eidelman and V.N. Ivanchenko, Phys. Lett. B 257 (1991) 437; talk given at the TAU94 Workshop, Sept. 1994, Montreux, Suisse.
[27] E. Braaten and C.S. Li, Phys. Rev. D 42 (1990) 3888.
[28] G. Altarelli, in: QCD-20 years later, eds. P.M. Zerwas and H.A. Kastrup, WSC (1994) 308; talk given at the TAU94 Workshop, Sept. 1994, Montreux, Suisse; G. Altarelli, P. Nason and G. Ridolfi, CERN-TH.7537/94 (1994).
[29] G. Launer, S. Narison and R. Tarrach, Z. Phys. C 26 (1984) 433.
[30] J. Bordes, V. Gimenez and J.A. Peñarrocha, Phys. Lett. B 201 (1988) 365.
[31] M.B. Causse and G. Menessier, Z. Phys. C 47 (1990) 611.
[32] C.A. Dominguez and J. Sola, Z. Phys. C 40 (1988) 63.
[33] E. Bagan, J.I. Latorre, P. Pascual and R. Tarrach, Nucl. Phys. B 254 (1985) 555.
[34] I.I. Balitsky, M. Beneke and V.M. Braun, Phys. Lett. B 318 (1993) 371.
[35] V. Kartvelishvili and M. Margvelashvili, Phys. Lett. B 345 (1995) 161.
[36] P. Nason and M. Porrati, Nucl. Phys. B 421 (1994) 518.
[37] M.A. Shifman, Minnesota preprint UMN-TH-1323-94 (1994); M. Neubert, CERN-TH.7524/94 (1994).
[38] G.N. Hardy, Divergent Series, Oxford University press (1949).