MHD waves in magnetically twisted solar atmospheric flux tubes

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Abstract. The propagation of surface and body linear MHD modes in a twisted magnetic flux tube embedded in a magnetically twisted plasma environment is considered. The dispersion relation for surface and body modes is derived assuming constant external twisted field. Analytic approximate solutions to the dispersion equation are found for the long and short wave length cases.

It was found, that in case the twisted component of the magnetic field in the environment of the flux tube is constant the index of Bessel functions ν in the corresponding dispersion relation is not integer. In the particular case of a homogen magnetic twist the total pressure is found to be constant across the boundary of the flux tube.

Keywords. Twisted magnetic flux tube, MHD waves, solar atmosphere, magneto-seismology

The twisted equilibrium magnetic field, $\mathbf{B_0}(\mathbf{r})$, representing a vertical flux tube embedded in a magnetic environment is given as follows:

$$\mathbf{B_0}(\mathbf{r}) = \begin{cases} (0, Ar, B_{iz}), & r < a, \\ (0, B_{e\varphi}, B_{ez}), & r > a. \end{cases} \qquad (0.1)$$

where a - is the radius of the magnetic flux tube. For simplicity we consider that the azimuthal component $B_{e\varphi}$ is constant (though not equal to zero) outside the flux tube. At the tube boundary the conditions of continuity of the radial displacement and the total pressure $p_T = p + \mathbf{B_0}\mathbf{b}/\mu_0$ must be satisfied. Applying these boundary conditions to the linear MHD equations yields the dispersion relation

$$k\frac{I_1(m_i a)}{I_0(m_i a)} = \frac{m_i(\omega^2 - \omega_{iA}^2)K_\nu(ka)}{\frac{\rho_{e0}}{\rho_{i0}}(\omega^2 - \omega_{eA}^2)(\frac{1+\nu}{ak}K_\nu(ka) - K_{1+\nu}(ka)) + \frac{k}{a}\frac{(A^2 a^2 - B_{e\varphi}^2)}{\mu\rho_{i0}}K_\nu(ka)}, \qquad (0.2)$$

where I_0 and I_1 are the modified Bessel functions of the first kind of zero and first order; K_ν is the modified Bessel function of the second kind of ν order; ω_{iA} and ω_{eA} are the Alfvén frequencies inside and outside the flux tube; $m_i^2 = k^2\left(1 - \frac{4A^2\omega_{iA}^2}{\mu_0\rho_{i0}(\omega^2 - \omega_{iA}^2)^2}\right)$; $\nu^2 = 1 - 2k^2\frac{B_{e\varphi}^2}{\mu_0\rho_{e0}}\frac{\omega^2 + \omega_{Ae}^2}{(\omega^2 - \omega_{Ae}^2)^2}$. Equation (0.2) is the general dispersion relation for MHD waves in an incompressible uniformly magnetically twisted flux tube embedded in a *twisted magnetic environment*. In the limit of no magnetic environment, ($B_{e\varphi} = 0$) the dispersion equation reduces to equation (23) in Bennett, *et al.* (1999) for sausage modes. In the limit of no magnetic twist within the flux tube either one recovers the dispersion relation found by Edwin & Roberts (1983).

In the long-wavelength limit ($ka \to 0$) the equation (0.2) has its simplified form:

$$2C_{i\varphi}V_{iA}J_0(an_i) = \pm\left(\frac{\rho_{e0}}{\rho_{i0}}(V_{ph}^2 - V_{eA}^2)(1 - \nu) + C_{i\varphi}^2 - \frac{\rho_{e0}}{\rho_{i0}}C_{e\varphi}^2\right)J_1(an_i), \qquad (0.3)$$
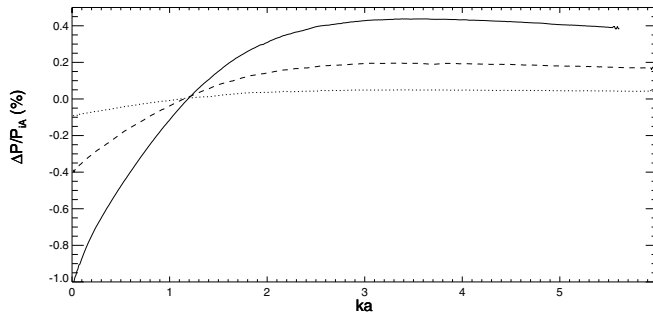
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Figure 1. The periods shift of oscillations are shown for internal twist $B_{i\phi}/B_{iz} = 0.3$ and external twist $B_{e\phi}/B_{iz} = 0.1$ (the dotted line), $B_{e\phi}/B_{iz} = 0.2$ (the dashed line), $B_{e\phi}/B_{iz} = 0.3$ (the solid line). The periods shift ΔP are normalized to the period of oscillation P_{iA} of the flux tube without any external twist.

where $n_i^2 = -m_i^2$, $V_{ph} = \omega/k$ is the phase speed; $C_{i\varphi} = B_{i\varphi}/(\mu_0\rho_{i0})^{1/2}$ is the Alfvén speed component associated with the magnetic twist inside; $C_{e\varphi} = B_{e\varphi}/(\mu_0\rho_{e0})^{1/2}$ is the Alfvén speed component associated with the magnetic twist in the surrounding environment.

When $ka \to \infty$ (the limit of short-wavelength) one can write for the sausage mode, an approximate expression:

$$V_{ph}^2 \approx C_k{}^2 + \frac{\rho_{e0}(C_k^2 - V_{eA}^2) + \rho_{i0}C_{i\varphi}^2 - \rho_{e0}C_{e\varphi}^2}{\rho_{i0} + \rho_{e0}}\frac{1}{ka}, \tag{0.4}$$

where $C_k^2 = \frac{V_{iA}^2\rho_{i0} + V_{eA}^2\rho_{e0}}{\rho_{i0} + \rho_{e0}}$ - is the kink mode speed.

Finally, the changes in wave periods caused by the magnetic twist is investigated. It is important for solar atmospheric magneto-seismologic observational studies to establish what is/are the dominant effect(s) for observable quantities (periods, amplitude, etc.). Magnetic twist, curvature, inhomogeneity, etc. are all possible accountable candidates, at least in theory, for the deviations between theoretically calculated and observationally derived quantities. Magnetic twist causes small, but ubiquitous shifts in periods. For small ka e.g. $ka < 1$, we found that increasing the external magnetic twist from 0 to 0.3 causes an increase in the normalized periods of oscillations approximately by 1-2% (see Figure 1). For solar magnetic loops with oscillatory periods $\sim 10 - 20$ mins. this would mean about $6 - 12$ s.; i.e. just about the time resolution of current space instrumentation onboard SOHO or TRACE. This may, however change, as Solar-B and SD0 will have improved cadence and time resolution.

Further detailed analysis is necessary in order to find the dispersion relation for more realistic cases $B_{e\varphi} \sim 1/r$ for compressible plasmas, where the magnetic twist diminishes with distance from the tube.

Acknowledgements

References

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