H. THE EVOLUTION OF THE MOON'S ORBIT

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THE ROLE OF OCCULTATIONS IN THE IMPROVEMENT OF THE LUNAR EPHEMERIS

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Abstract. Analyses of occultation timings show that periodic correction terms with semi-amplitude as great as 0.″18 arise from corrections required to the empirical constants of the Brown/Eckert theory. Using the atomic time-scale, some of the occultation data have been used to determine a correction of $-30 \pm 16''/\mathrm{cy}^2$ to Spencer Jones' value for the secular acceleration of the Moon. In the light of this correction, and previous determinations, attention is drawn to the possible weakness of Spencer Jones' value, which is not reflected in his quoted error of $\pm 1''/\mathrm{cy}^2$. Further analyses of 50000 occultations observed since 1943 promise to reveal more accurately-determined corrections.

1. Introduction

Since the time of Ptolemy, occultations of stars have been used to provide a simple and effective method of monitoring the angular motion of the Moon relative to the stellar background. Naked-eye observations of a few bright stars made some two thousand years ago are used today in studying secular changes in the lunar motion. Nowadays small portable telescopes and a method of recording time to a precision of about 0.1 s are necessary to make a useful observation. From the star position, the time of the occultation to 0.1 s and the geodetic co-ordinates of the observer, we can (ideally) obtain a fix on a point on the limb of the Moon to $\pm 0''.05$. With two or more timings, the position of the centre of figure is known after allowing for the difference between the limb profile and a mean sphere using Watts' data (1963).

2. Accuracy of Occultation Observations

The standard error of the residuals ($\Delta\sigma$) formed by taking the difference between the observed position of the limb, deduced from the occultation time and star position, and the calculated position using the lunar ephemeris, is about $\pm 0''.43$ (s.e.). Analyses (Morrison, 1970) of occultations of the Pleiades group on 1969 March 23 have shown that this value is comprised of the following errors:

(1) Timing	$\pm 0''.20$
(2) Star place (Robertson)	± 0.26
(3) Lunar ephemeris ($j=2$)	± 0.16
(4) Profile corrections (Watts)	± 0.20
(5) Observer's position	± 0.10

The error for the lunar ephemeris $j=2$ is an estimate based on the effect of the corrections to Brown's constants given in Table I (see later) and the results of comparisons with numerical integrations made by Garthwaite *et al.* (1970) which reveal the deficiencies of the ephemeris due to Brown's truncation of planetary terms.

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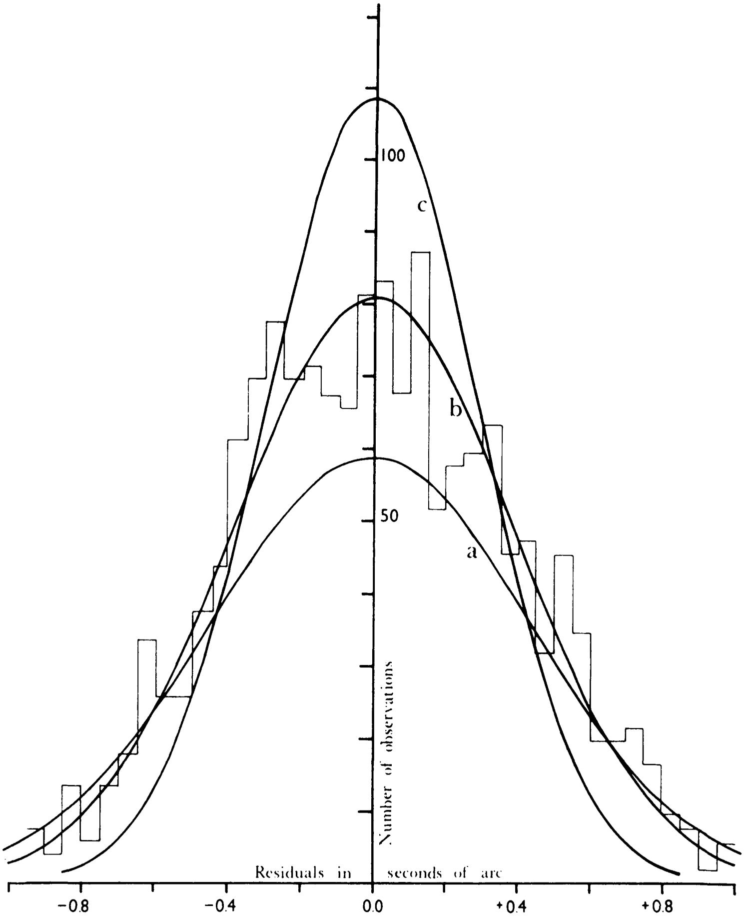

Fig. 1. Normal distribution curves for occultation residuals (see text).

Figure 1 shows three normal distribution curves corresponding to the standard deviations calculated from the residuals ($\Delta\sigma$) of occultations of the Pleiades group on 1969 March 23. The three curves are for the following cases:

(a) the residuals, uncorrected for errors 1 to 5 above s.e. $=0''.43$

(b) the residuals, corrected for error 3 s.e. $=0.40$

(c) the residuals, corrected for errors 3 and 2 s.e. $=0.30$

The frequency distribution of the residuals for case (b) is also shown to indicate the reliability of a normal curve for this sample. The other two cases are equally reliable, but are not shown to avoid confusion in the figure.

The error due to the catalogue star places, which is the largest part of the standard error for one observation, is probably even greater for fainter stars. The desirability of improving the star places of Robertson's Zodiacal Catalogue (1940), which has a

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mean epoch of place around 1905, is therefore apparent. A programme of re-observation of these stars is now under way at the Royal Greenwich Observatory and the U.S. Naval Observatory.

3. Corrections to Brown's Constants from Occultations

Recently two independent analyses have been made of some of the occultation data which has been collated and coded at HM Nautical Almanac Office since 1943. Morrison and McBain Sadler (1969) analysed 10000 observations made during 1960–66, and Van Flandern (1971) analysed 7000 observations made during 1950–69. Their results for corrections to the adopted values of four of Brown's arbitrary constants (ILE, 1954), e, π, i, Ω (in elliptic motion, eccentricity, mean longitude of perigee, inclination, mean longitude of node) are given in Table I for epoch around 1960.0.

TABLE I

Summary of corrections to Brown's constants from occultations

Author	$\delta e \times 10^6$	$\delta\tilde{\omega}$	δi	$\delta\Omega$
Morrison and McBain Sadler	$+0.42$	$-1.''57$	$-0.''03$	$+2.''31$
	± 0.02	± 0.07	± 0.02	± 0.20
Van Flandern	$+0.24$	-0.7	-0.12	$+1.6$
	± 0.07	± 0.3	± 0.04	± 0.5

The differences between the corrections of Table I are probably due to two causes:

(1) Morrison and McBain Sadler's data only extends over seven years, thus, perhaps, not allowing complete separation of the unknowns in their solution; and

(2) the corrections given here are only four out of two different sets comprising 12 and 26 unknowns, respectively.

If we take mean values for the corrections to the constants in Table I this gives rise to the following correction terms in longitude and latitude to the ephemeris $j=2$ (IAU, 1967) with coefficients greater than $0.''01$:

$$\delta\lambda = +0.''14 \sin l + 0.''13 \cos l - 0.''03 \sin(l-2D) - 0.''03 \cos(l-2D)$$
$$\delta\beta = -0.''07 \sin F - 0.''18 \cos F,$$

where l, F and D have the usual meaning in Brown's notation.

The values in Table I do give an indication of the solution we might expect from a comprehensive analysis of about 50000 observations made since 1943 which is now under way. The unknowns to be determined will include the motions of the perigee and node, and also the secular acceleration in mean longitude.

4. Determination of the Secular Acceleration of the Moon from Recent Occultations

Much interest arises from the possibility of obtaining a reliable value for the secular acceleration in mean longitude, $\dot{n}_M$, using the occultation data after 1955.5 when a precise atomic-time scale is available to remove the effects of variations in the rate of rotation of the Earth from the universal time-scale. A range of 17 yr of data will be available in which to search for a possible correction to Spencer Jones' (1939)* value $-22''/\text{cy}^2$ which is incorporated in the lunar ephemeris. The residuals in longitude after 17 yr resulting from a correction of, say, $10''/\text{cy}^2$ would attain a value of 0.″04, and the best straight line through the residuals would have a maximum departure from the postulated second degree curve of 0.″02. Taking the standard error of an observation to be 0.″40 [case (b) of Figure 1], and an average of about 1600 observations a year since 1955, we have 17 annual mean points, each with a standard error of 0.″01. So the second degree correction will be detectable from the data, but only if it is not confused by other long-period corrections. For instance, the above correction is not large in comparison with the oscillatory variations (with a period of approximately 18 yr) in the residuals in longitude formed by the difference between a numerical integration ephemeris (JPL Lunar Ephemeris 16, Garthwaite *et al.*, 1970) and $j=2$ (Brown/Eckert theory). Confusion of this oscillation, which is not due to an erroneous secular acceleration, might lead to the doubtful significance of a correction derived from 17 yr of data. But if the oscillation is nearly sinusoidal with a period of 18 yr, then one might expect to separate the parabolic correction from this, provided that the observations are well distributed over the period.

5. Discussion of Several Determinations of $\dot{n}_M$

Using some of the occultation data in the period 1955.5–1969, Van Flandern (1970) has found $\dot{n}_M = -52 \pm 16''/\text{cy}^2$, which is better in agreement with Newton's (1970) recent investigation of the 'ancient' solar eclipses where he finds $\dot{n}_M = -42 \pm 4''/\text{cy}^2$, circa AD 0. Stephenson (1971) finds $\dot{n}_M = -32 \pm 5''/\text{cy}^2$, circa 100 BC from a discussion of ancient total solar eclipses. Stephenson has rejected as unreliable some of the data included in Newton's solution, and has added new observations. In all, about 50% of the data is common to both discussions. Fotheringham (1920) does not actually give a value for $\dot{n}_M$ from solar eclipses alone, but a value of $-26''/\text{cy}^2$ is consistent with his procedure. However, the triangular area of most probability in his famous diagram (p. 123) gives values of $-21''/\text{cy}^2$ and $-33''/\text{cy}^2$ for $\dot{n}_M$ at its extremities. De Sitter (1927) reconsidered Fotheringham's work and found $\dot{n}_M = -37.''7/\text{cy}^2$ which, until recently, has usually been taken as the 'ancient' value. Thus, three different discussions of the ancient solar eclipses lead to negative values of $\dot{n}_M$ greater than $30''/\text{cy}^2$. If one postulates that the secular acceleration of the Moon has remained nearly constant over the past 3000 yr, then, as has often been pointed out, the 'ancient' value is at variance with Spencer Jones' (1939) 'modern' value of $-22 \pm 1''/\text{cy}^2$ based on observations of the position of the Moon, Sun, Mercury and Venus over the past 300 yr. The various values of $\dot{n}_M$ are collected together in Table II.

* Throughout this note the secular acceleration is the value of twice the coefficient of T^2 in the expression for the mean longitude of the Moon. In the lunar ephemeris the expression (due to Spencer Jones) is $-11.''22\, T^2$.

TABLE II

Values of $\dot{n}_M$ and their probable errors

Data	Author	$\dot{n}_M$ ($''$/cy^2)
From ancient total solar eclipses circa AD 0	Fotheringham	-26 (± 5)
	de Sitter	-38 ± 4
	Newton	-42 ± 4
	Stephenson	-32 ± 5
From modern meridian observations 1700–1930	Spencer Jones	-22 ± 1
From recent occultations 1955–1969	Van Flandern	-52 ± 16

6. The Secular Acceleration derived by Spencer Jones

Spencer Jones' solution for $\dot{n}_M$ is heavily dependent on the analysis of the transits of Mercury and meridian circle observations of the declination of the Sun. Observations of the Sun's declination taken together with the theory of its motion lead to the deduction of corrections to the Sun's longitude. Timing of the transits of Mercury across the face of the Sun also lead to corrections to the Sun's longitude and these corrections are interpreted as arising from the retardation of the Earth's rate of rotation. The combination of these observations with those of the longitude of the Moon made over the same period give a value of $\dot{n}_M$.

He took weighted mean values of the observed minus tabular declinations of the Sun over periods of 4 yr around 1930, extending to 10 yr at 1760. Thus, his quoted probable error for the value of $\dot{n}_M$ does not reflect the scatter of the original observations and would undoubtedly be greater than the value from the fit to the means. Moreover, it is well-known that large systematic errors are present in meridian circle observations of the Sun's position and the inevitable dependence of the T^2 term on the earlier observations, despite their lower weighting factors, makes one approach the solution with extreme caution. It might be valuable to repeat Spencer Jones' work by omitting the 18th-century observations of the Sun and extending the 20th-century observations to the present day, using the individual observations, rather than smoothed values, in deriving a solution. However, the solution from the Sun's declination observations agrees closely with the independent solution from Mercury's November transits*.

7. Discussion of the Values of $\dot{n}_M$ derived from Transits of Mercury

It is easier to inspect the solution from Mercury's transits than that from the Sun's declination. Here again, Spencer Jones preferred to use the weighted mean times of contacts II and III (contacts I and IV are very unreliable). He took these values from de Sitter's (loc. cit.) discussion, which was in turn based on the reduction of the times

* Transits can only occur in May and November when the Earth passes through the line of nodes of Mercury's orbit.

of transit given by Innes (1925). Williams (1939), later supplemented by Clemence (1943), published a thorough discussion, independently of Innes. I have summarized the values of $\dot{n}_M$ deduced from their analyses in Table III. The solutions and standard errors in Table III result from the different treatment of the observations rather than the differences in the times of contact used in the analyses.

Besides the unknown in T^2, a constant and linear term in T are introduced into the equations of condition when analysing the observations of Mercury's transits. Williams and Clemence also included five other unknowns: corrections to the longitude and motion of the node of Mercury's orbit; corrections to the adopted semi-diameters of the Sun and Mercury; and a correction to the mass of Venus. It is questionable whether all of these additional unknowns should now be included. The adopted inverse mass of Venus, 408 000, is relatively close to the recently determined value of $408\,522 \pm 3$. The difference of 522 leads to differences in the times of contact of usually less than 0''1, which can certainly be ignored. From the duration of transits, Innes made preliminary solutions for the semi-diameters to be used in the analysis. He adopted values increasing with time to allow for the effect of the increasing optical power of telescopes, which reduced the effects of diffraction and irradiation on the observed diameters. Williams and Clemence's solution only permitted the derivation of mean values for the period. Whichever method is followed, the diameters are not well determined, but this is not of much consequence to the solution for the secular acceleration since the effects due to the diameters cancel out in the mean times of contacts II and III. When comparing the discordant solutions for the longitude and motion of Mercury's node from the transit and meridian observations, Clemence (loc. cit.) points to the weakness of the former which is largely dependent on the duration of the transits, and here there are inexplicable discrepancies in some of the observations, especially in the well-observed transit of 1940.

Provided that these corrections to the node and diameters are not closely correlated with the correction to the coefficient of T^2, it will be safe to allow them to be absorbed in the constant and linear term in T (with which they are correlated) in a solution which is primarily aimed at finding the coefficient of T^2.

Mainly to re-estimate the standard error of the solution made by Spencer Jones, I took Innes' reduced times of contact and weights and made a similar solution to Spencer Jones', but keeping the times of contact II and III separate, rather than taking weighted means. My solution for the November transits is shown in Table III.

TABLE III

Values of $\dot{n}_M$ derived from Transits of Mercury

	$\dot{n}_M$ (''/cy^2)	s.e.
Spencer Jones	−22.7	±1.6
Williams; Clemence	−17.9	±6.5
Morrison (Nov. only) unpublished	−21.1	±3.2

The value of $\dot{n}_M$ is not significantly different from Spencer Jones', but the standard error is doubled. The solution for the May transits gave an appreciably smaller value in accord with Spencer Jones' solution on page 554 of his paper. When I tried solutions by combining the November and May transits (with de Sitter's high weights for the latter drastically reduced) the residuals from the solutions differed systematically between the two sets of transits, and always gave an increased error for the solution of the secular acceleration. Spencer Jones arrived at his final value by taking a weighted mean of the separate solutions for November and May transits, which came close to the November value because of its appreciably smaller probable error. The combination of the November and May transits in the equations of condition probably accounts for the reduced value for $\dot{n}_M$ and its much greater standard error in the solution of Williams and Clemence.

8. Spencer Jones' Solution for $\dot{n}_M$ in Relation to the 'Ancient' Value

Unless one can reject the May transits – and there seems to be no obvious reason why one should – one should adopt Williams and Clemence's solution and standard error as being the most realistic solution for $\dot{n}_M$ from the transits of Mercury. Spencer Jones' value for $\dot{n}_M$ falls within Williams and Clemence's solution and standard error, but perhaps a figure of about $\pm 6''/\text{cy}^2$, rather than $\pm 2''/\text{cy}^2$ for the standard error should be borne in mind when comparing values of $\dot{n}_M$ from Mercury's transits with those from ancient eclipses, etc. Hence Spencer Jones' solution may be considered to be consistent with a value of, say $-30''/\text{cy}^2$, but not with a value as high as $-40''/\text{cy}^2$.

There still remains the independent solution from the Sun's declination, with its accompanying small probable error in Spencer Jones' analysis, but an extended re-analysis of the individual observations might well alter the position there also.

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