The Chandrasekhar Spitzer controversy and the (ir)relevance of distant interactions

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Abstract. We have done N-body simulations with N up to 10^6, with the aim to determine whether fluctuations in the force field of a globular cluster are caused by nearby or distant encounters. We find that distant encounters are insignificant, in agreement with Chandrasekhar's expectations, contrary to general opinion.

Keywords. Coulomb logarithm, diffusion, N-body simulations

1. Introduction

In this paper we reconsider an old problem of stellar dynamics: are the fluctuations in the orbital parameters of a given star the result of encounters with nearby stars or does the entire system play an essential role? More precisely: what is the relative importance of nearby and distant interactions in globular clusters? The choice of the maximum impact parameter b_{max} in the expression for the Coulomb logarithm ln Λ also depends on this.

There are two views: Chandrasekhar (1942) argues that velocity changes of cluster stars are caused by nearby stars (using the argument that long-range interactions are short lived). Therefore, b_{max} should be close to the mean distance D_0 between the stars, 'within a factor of 2 or 3', i.e. local interactions dominate in Chandrasekhar's view. See also Kandrup (1980).

Spitzer (1987) argues that velocity changes of cluster stars result from the cumulative sum of many very small velocity changes (using the argument that there is no screening in gravitational systems): the size of the system, e.g. the half-mass radius R_h, should therefore be used. But the theoretical foundations of both choices are weak.

The idea that many very weak encounters dominate the relaxation process has prevailed till now, and results from N-body simulations appear to confirm this (e.g. Theuns 1996, see also Binney & Tremaine 2008, p. 581). We feel however that there is room for doubt because the potentials of the particles in these N-body simulations are softened considerably, with an amount of the same order as D_0.

Note that the minimum impact parameter in calculations involving the Coulomb logarithm is $b_{min} \simeq Gm/<v^2> \simeq R_h/N$ and $D_0 \simeq R_h/N^{1/3}$.

2. Our approach

To get a better understanding of the diffusion of stellar orbits in globular clusters we have done the following computer experiment. A test particle moves in a force field generated by N non-interacting field stars, mimicking a Plummer model with the proper $f(E)$. The force field of the field particles does not affect the orbit of the test particle,

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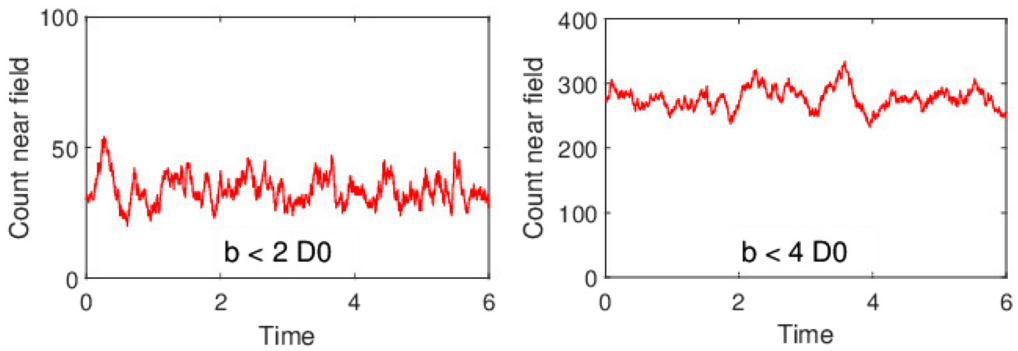
Figure 1. Number of field stars in the near field as a function of time. Left: boundary of near field at $2\,D_0$, right $4\,D_0$. The orbital period of the test star is 5.987.

its orbit remains undisturbed. This impulse approximation makes it possible to avoid the use of softened particle potentials and allows the separation of the forces acting on the test particle in a contribution from a near and a far field. A high precision ODE solver for the orbits of the field particles can be used with this approach. (We use Matlab 'ode45' with absolute tolerance 10^{-10}). These field stars generate the force field acting on the test star. Whereas in the classical approach one sums the effects of independent two-body encounters between field stars in a homogeneous background and a test star, in our approach the test particle sees the entire gravitational field of all field particles simultaneously, thus including the 'Poisson – noise' in the gravitational field at all spatial scales (cf. Woltjer 1967). And instead of a homogeneous background we use the spherical geometry appropriate for a globular cluster. Otherwise our approach and the classical derivation are similar.

In the results presented here the massless test star moves in a circular orbit at the half-mass radius of the Plummer model. At each instant during its motion, using typically 4000 to 16000 time intervals per revolution, we split the field stars into two groups: a 'near field' and a 'far field'. The near field comprises the field stars inside 1, 2, 3 or $4\,D_0$. Thus, the near field typically contains about 4, 35, 115 or 275 stars. Forces resulting from distances less than $2\,b_{min}$ are set equal to zero, i.e. we suppress very close encounters.

We measure the dependence of the various diffusion coefficients on N over the range $N = 10^2$ to 10^6, separately for the near field and the far field. Our results show that interactions of the field stars nearest to the test star dominate for all N. For $N = 10^5$ the stars inside $2\,D_0$ (about 35) contribute about two thirds to the total $\Sigma(\Delta E)^2$. This agrees with Chandrasekhar's view that diffusion is mainly a local process and not a global one.
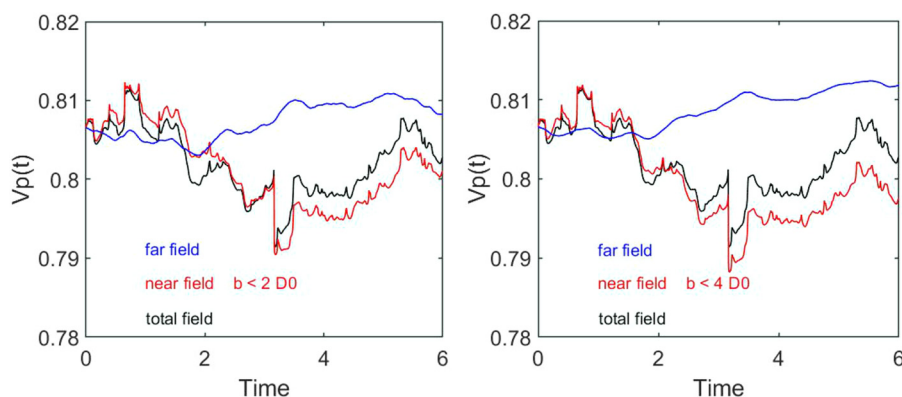
3. Results

We have studied numerous systems with N up to 10^6. Here we present some results for two cases with $N = 10^5$. Units are the standard N-body units: $G = 1, M = 1, E = -1/4$.

Figures 1 to 3 give respectively the number of stars in the near field, the parallel component of the force field along one circular orbit of the test star, and the resulting velocity change, i.e. the velocity change that the test star would have incurred as a result of the perturbations: $V_p(t) = V_{cir} + \int_0^t \delta F_p(t)dt$; here calculated as a sum over the 4000 time intervals. Two values for the boundary of the near field are used: $2\,D_0$ and $4\,D_0$.

The forces in Fig 2 are perturbations caused by the cumulative effect of density perturbations, i.e. Poisson noise; the smooth component of the force field is zero. The spikes are caused by individual stars crossing the near field on a timescale of $2\,D_0/<v^2>^{0.5} \simeq 0.05$
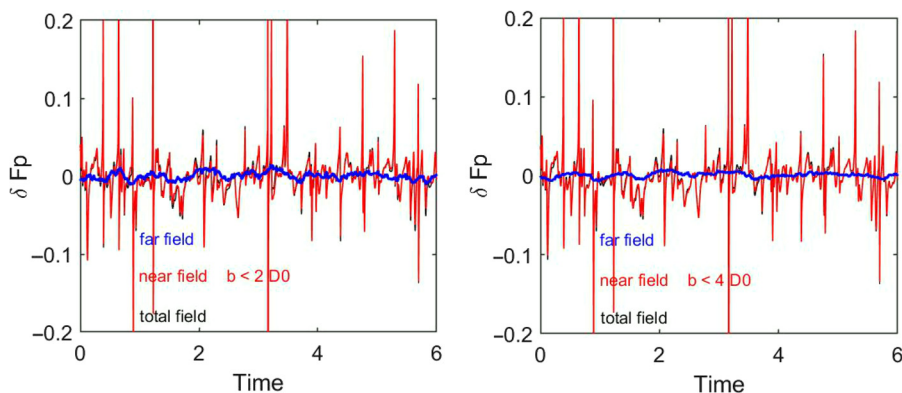
Figure 2. Parallel component of the force on the test particle vs. time.

Figure 3. Resulting parallel component of the velocity change of the test particle.

for $N = 10^5$, but they are resolved by our time resolution, which in this case is $6/4000 = 0.0015$. These spikes are already present in the case that the boundary of the near field lies at $1D_0$. Going from 1 to $4\,D_0$ the near field doesnt change too much, but the far field weakens considerably. The total field is almost indistinguishable from the near field.

In the simulation shown here, at time somewhat larger than 3, the parallel velocity V_p drops strongly. This drop is caused by a nearby passing star giving a strong peak in δF_p in Fig 2. We have identified this encounter by repeating the simulation with various cutoffs for the minimum impact parameter, that is, we set the forces equal to zero when the distance of the field star to the test star is less than some factor times b_{min}. It turns out that the drop in V_p is still there with a cutoff at $64\,b_{min}$. But with a cutoff of $128\,b_{min}$ it has disappeared. An impact parameter of $60\,b_{min}$ corresponds to a passage of the field star at about $0.03\,D_0$ from the test star. This is a rare event, but it produces a deflection of the orbit of only 2 degrees (see Spitzer 1987, table 2.1). In other words, all the encounters that we see in the near field in these figures are weak encounters.

Averaging over a large number of cases one can calculate $<E(t)>$ and $<(E(t) - E(0))^2>$, (cf. Spitzer 1987, eq. 2-51). Preliminary results show that the energy of our testparticle performs a random walk in time. (McMillan *et al.* 1988; Huang *et al.* 1993).

4. Conclusions

Our results show that the very weak encounters envisaged by Spitzer and others have no appreciable effect on the force field experienced by a star at the half mass radius of a Plummer model with 10^3 to 10^5 equal mass stars. The dominant contribution by far comes from the several tens of stars surrounding the star in question. This implies that energy relaxation, or diffusion, is basically a local process, as envisaged by Chandrasekhar and Kandrup. For small N there is no clear separation between the near field and the overall field. For N $= 10^3$ for example, D_0 is only about one tenth of the half mass radius.

Our results are also in broad agreement with Hénon's (1960) claim that stars can only escape from a globular cluster as a result of close encounters causing an abrupt increase in energy, which implies that the tail of the velocity distribution in a cluster has been formed in a similar way.

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