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BOOK REVIEWS

COTTET, G.-H. AND KOUMOUTSAKOS, P. D. *Vortex methods: theory and practice* (Cambridge, 2000), xiii+313 pp., 0 521 62186 0 (hardback), £37.50 (US$59.95).

The book presents and analyses vortex methods for the direct numerical simulation of incompressible viscous flows. Vortex methods consist of the discretization of the vorticity field and the Lagrangian formulation of the governing equations. Until fairly recently disadvantages of vortex methods such as high computational cost and the inability to provide an accurate treatment of viscous effects had limited their application to modelling the evolution of the vorticity field of unsteady high Reynolds number flows with a fairly small number of computational elements. These difficulties have been overcome through the advent of fast summation algorithms that have reduced the computational cost, and also through recent developments in numerical analysis that have facilitated the accurate treatment of viscous effects. Vortex methods now offer a viable alternative to finite difference and spectral methods for high-resolution numerical solutions of the Navier–Stokes equations. Over the last few years developments in the analysis of vortex methods have yielded a sound mathematical background for understanding the accuracy and stability of the methods.

The modern developments of vortex methods originate in the works of A. J. Chorin in the 1970s and in the three-dimensional calculations of A. Leonard in the 1980s. These practical works stimulated interest in the analysis of vortex methods, and the first complete convergence analyses were provided by O. Hald, J. T. Beale and A. Majda in the late 1970s and early 1980s. Since around 1980 there has been a significant development in fast multiple methods for the efficient evaluation of the velocity field (by L. Greengard) and a deeper understanding of convergence properties, with convergence proofs of random-walk methods by D. G. Long and J. Goodman and convergence proof of point vortex methods by T. Y. Hou and co-workers. This well-written text by Cottet and Koumoutsakos covers these developments of vortex methods from the dual viewpoint of numerical analysis and fluid dynamics. It clearly demonstrates that, unlike other discretization methods such as finite differences or spectral methods, vortex methods are intimately linked to the physics that they aim to mimic.

Vortex methods are based on the Lagrangian formulation of the equations of motion of a fluid: in particular, they make use of Kelvin's theorem which asserts the conservation of circulation along material elements moving with the fluid. The basic idea is to sample the computational domain into cells in each of which the initial circulation is concentrated on a single point or particle. The transport equation is dealt with exactly, and the approximation amounts to a replacement of the initial vorticity by a set of particles and a smoothing of the velocity field that carries these particles. Since the vorticity field is sampled on a grid that evolves with time, vortex methods are sensitive to the smoothness of the velocity field.

Chapter 1 introduces the governing equations and Chapter 2 gives a convergence theory for two-dimensional inviscid flows. The discussion on convergence features of vortex methods brings out the conservation properties of the methods. Many inviscid flow invariants are conserved under vortex approximation methods. These conservation properties provide a guarantee

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that, even when used in underresolved situations, vortex methods will give a correct qualitative answer.

The energy conservation in two-dimensional schemes, which follows from the Hamiltonian nature of the particle motion in the velocity field, is a feature that distinguishes vortex methods from the familiar Eulerian methods. For the three-dimensional schemes that are dealt with in Chapter 3 the conservation of circulation has generally led to a preference for vortex filament methods rather than vortex particle methods. This chapter discusses several ways that are now available for enforcing conservation in the three-dimensional vortex particle methods. Chapter 4 deals with the implementation of boundary conditions for inviscid flow vortex simulations. The kinematic boundary condition (no-through flow) is enforced by means of an extended vorticity field. The concept of a surface vortex sheet is introduced and Poincaré's identity is invoked, relating the values of the velocity field with its values at the boundary. The resulting integral equations are analysed and their approximation by panel methods is discussed.

Early simulations using inviscid vortex methods predicted linear growth in the mixing layer and they were able to predict Strouhal frequency in a variety of bluff-body flow simulations. However, the inviscid approximation of high Reynolds number flows has its limitations. In bluff-body flows viscous effects are responsible for the generation of vorticity at the boundaries, and an approximation of viscous effects is necessary, at least in the neighbourhood of the body. In three dimensions the transfer of energy to small scales produced by vortex stretching produces complex patterns of vortex lines. This complexity increases as time evolves, and viscous effects provide the only limit in the increase of complexity. The simulation of viscous effects in the context of vortex methods is discussed in Chapter 5. It is shown that vortex methods are able to simulate viscous effects accurately, while maintaining the Lagrangian character of the methods. In Chapter 6 vortex methods are discussed for unsteady flows in domains containing solid boundaries. This chapter deals with the no-slip boundary condition and its equivalence with the vorticity boundary condition. Integral methods are presented for the implementation of boundary conditions and the techniques are illustrated by direct simulation of bluff-body flows.

Chapter 7 deals with the problem of particle distortion in Lagrangian methods. In numerical simulation the clustering and spreading of the particles have various consequences, depending on the specific numerical schemes that are being adopted. For two-dimensional inviscid flows the result is a loss of accuracy in the computed velocity, which may lead to the appearance of undesirable small scales. For three-dimensional flows in regions of high strain the depletion of particles becomes fairly severe as the flow is generally associated with vorticity intensification. To overcome these difficulties there are two possible strategies, which may be used independently or in combination. The first approach consists of restarting the particles every few time steps at revised locations where the distortions are well controlled. The second one consists of processing the circulation carried by the particles in order to correct the effect of the distortion of the flow and to allow particles to continue to give an accurate description of the vorticity. Both strategies are considered in this chapter. Finally, Chapter 8 deals with hybrid schemes. A scheme of this type is formed by combining a vortex method and an Eulerian method with a view to by-passing the difficulties inherent in particle methods near boundaries.

This is a well-presented, readable text that illustrates some of the recent advances in these powerful methods for simulating incompressible flows.

D. M. SLOAN

CONSTANDA, C. *Direct and indirect boundary integral equation methods* (Monographs and Surveys in Pure and Applied Mathematics no. 107, Chapman & Hall/CRC, 2000), vi+201 pp., 0 8493 0639 6, £43.99.

The book is a detailed study of boundary integral equation methods in application to three different two-dimensional mathematical models: the Laplace equation, plane strain linearized