SPHERIZATION OF THE REMNANTS OF ASYMMETRICAL SN EXPLOSIONS IN A UNIFORM MEDIUM

G.S. Bisnovatyi-Kogan
Space Research Institute
Moscow

S.I. Blinnikov
Institute of Experimental and Theoretical Physics
Moscow

The "snow-plough" approximation is applied to the problem of the propagation of the shock wave from a nonsymmetrical supernova explosion in a uniform medium. The spherization of the shape of the remnant is obtained. The results are in good agreement with the observed properties of Cas A.

Optical, X-ray and radio observations show that the shape of the projection of SNR's on the plane of the sky is close to a circle [1]. The deviations from a circle may be connected with asymmetry of the explosion, nonsymmetrical support from a central pulsar (for young SNR's) and nonuniformity of the interstellar matter. In the first and second cases we may expect a shape close to an ellipse, as observed in the Crab Nebula.

Asymmetry of the explosion is to be expected when the magnetorotational mechanism is producing the SN (Fig. 1). The transformation of the rotational energy into the energy of the explosion is so efficient as to give an energy output $\sim 10^{51}$ ergs [2,3].

We consider here the evolution of the remnant of an axisymmetrical (nonspherical) explosion in a uniform medium. The exact solution of the problem needs two-dimensional nonstationary hydrodynamical calculations, which is very complex. The problem may be simplified by taking into account the fact that during the propagation of the blast wave through the interstellar medium the main part of the mass is collected in a thin layer. This layer is treated within an approximation which may be called 1.5-dimensional hydrodynamics.

Let the density of the outer medium be ρ_o, and its pressure p_o be insignificant, so that the strong wave approximation holds:

$$p_o << p_i \; ; \qquad (1)$$

here p_i is the pressure which is taken uniform across the cavity inside the shock, $p_i = p_i(t)$. We consider at the initial time $t = t_o$ an axisymmetric form of the SNR:

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$$t = t_o \; ; \quad r = r_o(\theta) \tag{2}$$

and an axisymmetric velocity field

$$v_f = v_{fo}\ (\theta,\ r_o(\theta)) \ , \tag{3}$$

r and θ being spherical coordinates.

There are different approximations for solving this problem. The simplest one is Kompaneetz approximation [4], where not only $p_i = p_i(t)$, but also the pressure along the shock $p_{sh} = p_{sh}(t)$. In this case the velocity of the shock is also position-independent

$$v_{sh} = \left(\frac{\gamma+1}{2}\ \frac{p_{sh}}{\rho_o}\right)^{\frac{1}{2}} \ , \tag{4}$$

and is directed perpendicular to the shock front. Thus, all the matter on the shock moves with the same velocity and spherization occurs. The energy conservation law determines the time dependence of all quantities. The spherization of a flat disk in this model is shown in Fig. 2.

For a better approximation it is necessary to take into account the dependence $v(\theta,\ t,\ r(\theta,t))$ and the existence of a tangential velocity averaged across the shock.

The "snow-plough" model developed in plasma physics [5] is adequate for solving this problem. Let λ be a Lagrangian coordinate along the shock front (see Fig. 3); μ the surface density in Lagrangian coordinates; u_z, u_ω the average velocity components of the matter in the layer (in cylindrical coordinates ω, z); and D_n the normal component of the velocity of the shock relative to the interstellar medium. The set of equations includes the laws of mass and momentum conservation,

$$\frac{D\mu}{Dt} = \frac{\gamma+1}{2}\ \rho_o\omega\ \left(\frac{Dz}{Dt}\ \frac{\partial\omega}{\partial\lambda} - \frac{D\omega}{Dt}\ \frac{\partial z}{\partial\lambda}\right) \ , \tag{5}$$

$$\vec{u} = \left(\frac{D\omega}{Dt}\ ,\ \frac{Dz}{Dt}\right) \ , \tag{6}$$

$$\frac{D(\mu\ \vec{u})}{Dt} = p_i\omega\ \left(-\ \frac{\partial z}{\partial\lambda}\ ,\ \frac{\partial\omega}{\partial\lambda}\right) \ , \tag{7}$$

the integral energy conservation law

$$E_o = \frac{\pi p_i}{\gamma-1}\ \int_0^{\lambda max} \omega^2(\lambda)\ \left|\frac{\partial z}{\partial\lambda}\right|\ d\lambda\ +\ \pi\ \int_0^{\lambda max} \mu(u_\omega^2+u_z^2)d\lambda \tag{8}$$

and a condition on the shock front

$$D_n = \vec{D}\cdot\vec{n} = (\gamma+1)u_n/2 = (\gamma+1)\ \vec{u}\cdot\vec{n}/2 \ . \tag{9}$$

The equations (5)-(9) have been solved for the following initial conditions:

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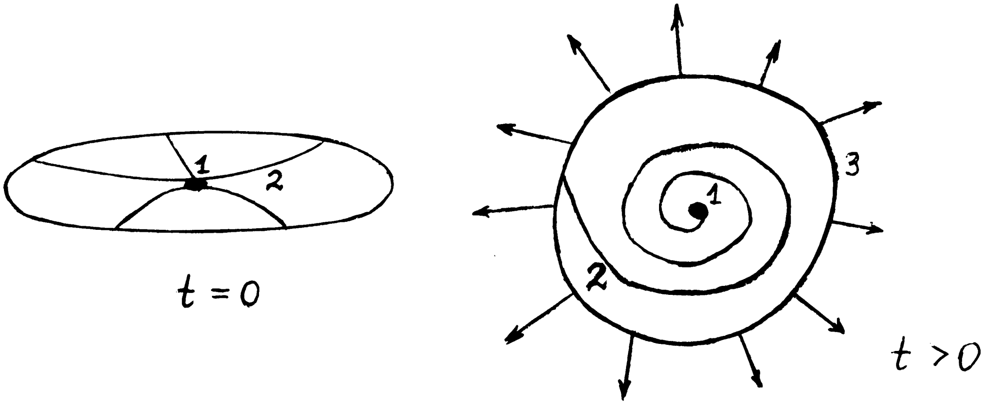

Fig. 1. Schematical model of magnetorotational supernova explosion. 1. neutron star; 2. flattened envelope with twisted magnetic field; 3. shock wave.

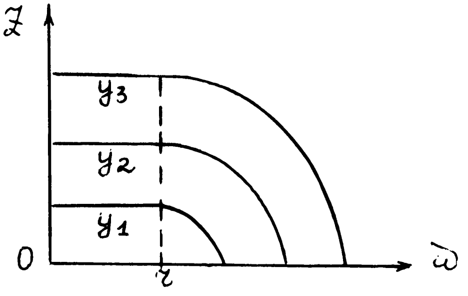

Fig. 2. The spherization of a flat disk in Kompaneetz model [4], y = time coordinate

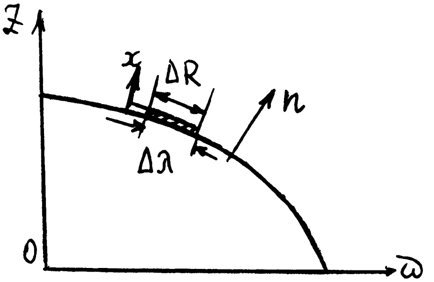

Fig. 3. For the derivation of the equations (5)- (9).

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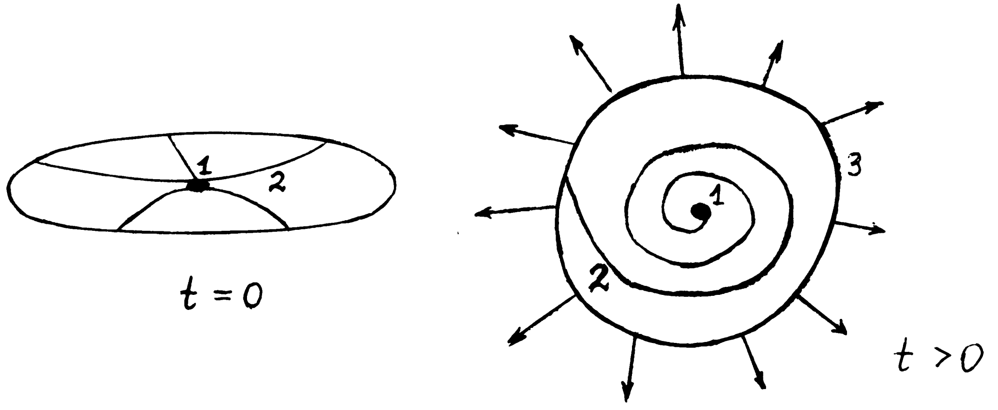

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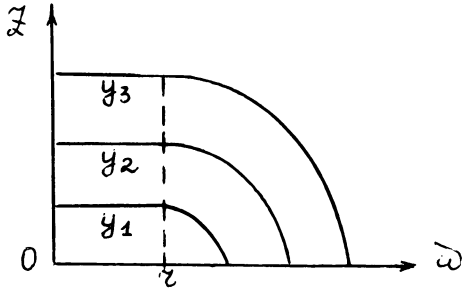

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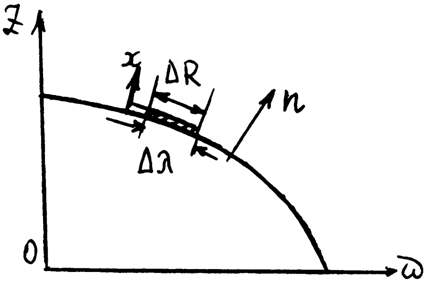

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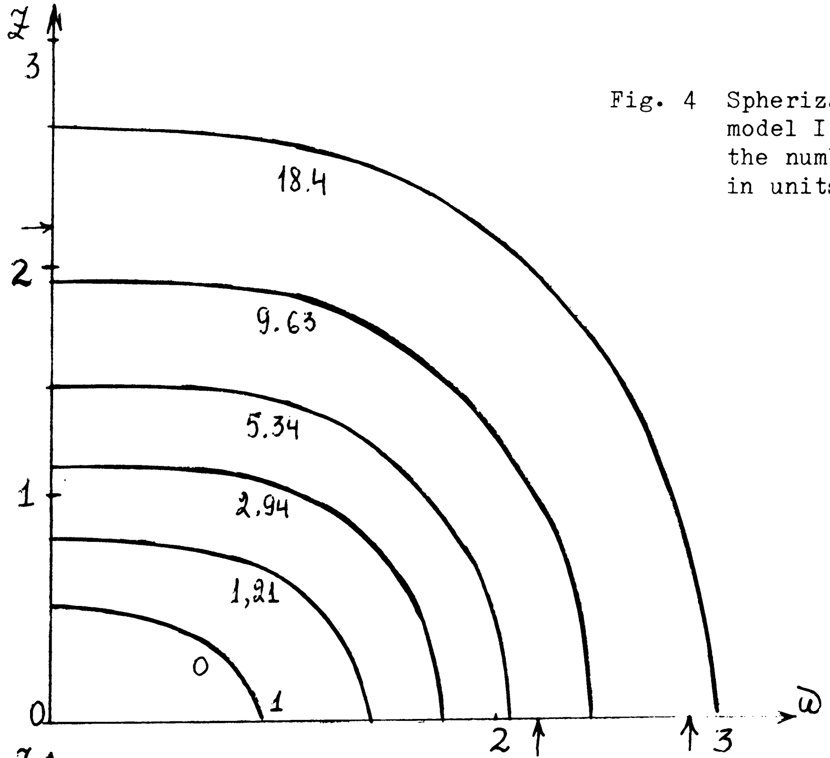

Fig. 4 Spherization of model I (see text); the numbers are time in units of τ_0.

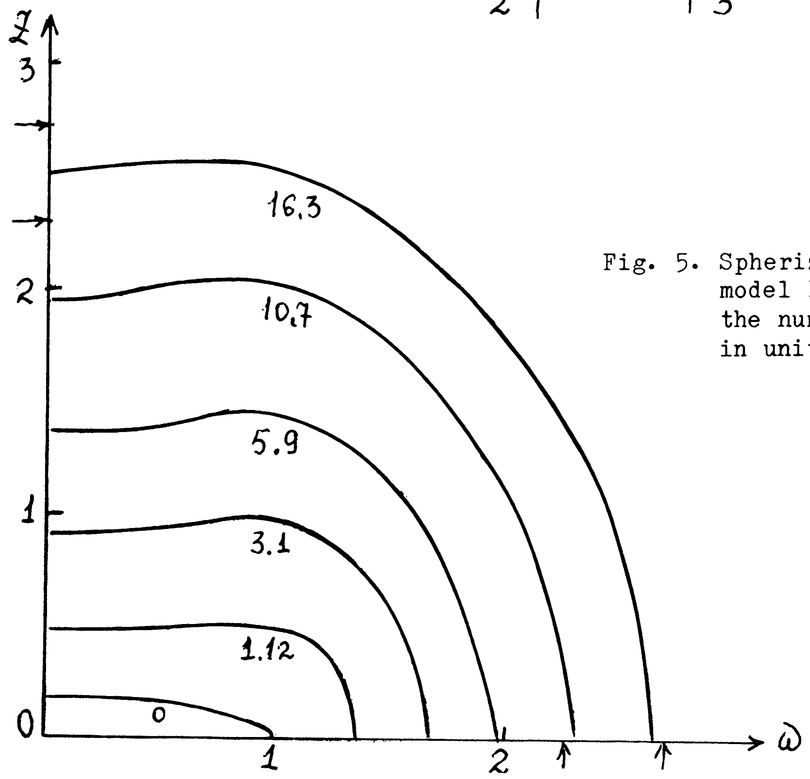

Fig. 5. Spherization of model II (see text); the numbers are time in units of τ_0.

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$$t=0: \quad u(\lambda) = u_o(1-b\cos 2\lambda), \; \lambda(t=0) = \theta = \text{spherical colatitude},$$

$$\omega = R_e \sin\lambda, \; z = R_p \cos\lambda, \; \sigma_o = \text{const.} \tag{10}$$

The additional dimensional parameters E_o - the total energy of the explosion -, ρ_o, M_o and the adiabatic index γ determine the problem. In nondimensional calculations only the following values must be given at $t = t_o$:

$$u_e/u_p, \; E_{kin}/E_o, \; M_o/M_H, \; R_e/R_p, \tag{11}$$

where $M_H = \frac{4}{3}\pi\rho_o R_e^2 R_p$ is the swept-up mass added to the initial mass of the shock M_o. We have calculated the variants with:

	γ	M_o/M_H	E_{kin}/E_o	R_e/R_p	u_e/u_p
model I	1.2	1.76	0.64	2	2
model II	1.2	4.47	0.47	5	2

The form of the shock wave for these models is shown in Figs. 4,5. The time of spherization is about ten characteristic times τ_o; for $E_o = 10^{50}$ ergs, $\rho_o = 10^{-24}$ g/cm^3, $M_o = 1M_\odot$ $\tau_o \approx 600$ years in both cases. Large differences of a factor 2 to 3 between σ_e and σ_p are still preserved. This may explain the observations of Cas A [6], where sphericity exists together with differences in surface brightness. The time τ_o may be much smaller when spherization occurs during the propagation of the shock wave through the extended envelope of the pre-SN star with $\sigma_o >> 10^{-24}$ g/cm^3, as it could have been in Cas A. An extended version of this work is given in [7].

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DISCUSSION

McKEE: Is the swept up gas also in a thin shell? If so, and if its mass is dominant, could you give a physical explanation of why the surface density varies by a factor of several over the surface of the remnant when it has become spherical?

BISNOVATYI-KOGAN: It is explained by the nonspherical velocity field given by the initial conditions. At the time of spherization there is a 2-3 times difference in the surface density. During further, almost spherical, expansion this difference decreases and goes to zero with time. Therefore, we may expect to observe a nonuniform surface density together with an almost spherical shape only in rather young SNR.

COX: Doesn't the spherization depend almost entirely on the assumption of homogeneity?

BISNOVATYI-KOGAN: Yes, it depends on it very strongly. The almost spherical shape of many of the observed SNR indicates that the interstellar medium in their vicinity is rather uniform.