VULL. AUSTRAL. MATH. SOC.
VOL. 9 (1973), 153-154.

76R99, 80A20
(86A25)

Thermal convection in a non-newtonian fluid

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Since the year 1900, the problem of thermal convection in horizontal layers of ordinary newtonian fluid heated from below has been studied intensively both theoretically and experimentally, the theoretical effort being mainly concerned with obtaining the conditions for the onset of convection, with predicting the flow pattern that first results when the temperature gradient across the layer exceeds the appropriate critical value and with determining the structure of the flow field for temperature gradients substantially higher than the critical. The "principle of exchange of stabilities" was found to be valid for this problem, so the instability is manifested as a steady, cellular convective motion.

More recently it was found that when subjected to a uniform rotation and/or an externally imposed magnetic field, the fluid layer becomes overstable in certain ranges of the governing parameters, the thermal instability giving rise to an oscillatory convective motion. Overstability is possible in their presence because they lend an elastic-like behaviour to the fluid enabling it thereby to sustain appropriate modes of wave propagation. It is therefore expected that a layer of non-newtonian [viscoelastic] fluid can become overstable solely due to heating from below.

The aim of this thesis is to investigate the problem of thermal convection in non-newtonian fluids whose rheological properties are described by the stress-relaxing Maxwell constitutive relation. This constitutive relation is sufficient to reveal the basic effects of

Received 16 April 1973. Thesis submitted to Monash University, January 1973. Degree approved, April 1973. Supervisor: Professor R.F.E. Van der Borght.

viscoelasticity on thermal instability, particularly in view of the extremely low shear rates involved in a natural convection experiment. Specifically, the analysis will be carried out with respect to both infinitesimal and finite amplitude disturbances.

In the first part of the thesis a linear theory is employed to investigate the onset of convection in the presence of a uniform rotation or magnetic field. It is shown that the critical constants for the onset of steady convection are identical to those for a newtonian fluid. However, unlike a newtonian fluid [for which both rotation or magnetic field inhibit the onset of overstability], it is found that rotation and magnetic field have conflicting effects on the onset of overstability - rotation having in the main a destabilizing effect. Moreover, unlike a newtonian fluid, overstability can occur in the presence of a magnetic field although the thermal diffusivity of the fluid is less than its resistivity. As in the absence of rotation or magnetic field, elasticity is found to destabilize the fluid.

The numerical solution of the double characteristic value problems of oscillatory convection is also studied.

In the remainder of the thesis a non-linear theory is employed to show that when subject to buoyancy-driven steady convection at low rates of strain, a Maxwell fluid with small relaxation time acts rather like a newtonian fluid with comparable viscosity; the Nusselt number at a given Rayleigh number being slightly but consistently higher than that for a newtonian fluid, in line with experiments.

The subject matter of the thesis is of particular interest for geophysics, geomagnetism and engineering.