Hence the sum assured during the first year is 1000−678·067= 321·933; and the amount for each succeeding year is found by adding 678·067 ÷ 10=67·807 to that for the year preceding. The sum assured during the eleventh year is thus £1000; and it remains at this amount during the rest of life.

The deduction at the outset seems here somewhat heavy; but it rapidly diminishes, and vanishes at the end of ten years. Were the term extended to twenty years the deduction at the outset would be only 365·053, and the assurance would consequently commence at 634·947.

I must defer till another opportunity the development of the schemes here shadowed forth. I will now merely mention, that they find their practical applications in cases in which it is arranged that a party who has been "rated up," instead of paying additional premium, shall be subjected to a temporary abatement of assurance.

I am, Sir,
Your most obedient servant,

*London*, 21 *Oct.* 1872. P. GRAY.

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## ON THE RELATION BETWEEN THE VALUE OF A POLICY AND THE RATE OF INTEREST.

*To the Editor of the Journal of the Institute of Actuaries.*

SIR,—In the paper on "Extra Premium," by Mr. J. R. Macfadyen, in the current volume of the *Journal*, that gentleman has given, in a footnote on p. 89, a demonstration intended to show that "in any given case it is practically certain that the value of a policy by a higher rate of interest must always be less than by a lower." Having, sometime ago, myself arrived at a similar result to Mr. Macfadyen's by a rather different process, I venture to send it you, with the hope that it may be of interest to some of your readers.

We have, by a well-known formula,

$$_nV_x=1-(1-V_x)(1-V_{x+1})\ldots\ldots(1-V_{x+n-1});$$

consequently, it will be sufficient to consider how the value of a policy one year old is affected by increasing or diminishing the rate of interest at which it is calculated.

Now,
$$V_x=1-\frac{1+a_{x+1}}{1+a_x}$$
$$=1-\frac{a_x}{vp_x(1+a_x)}$$

or, omitting the subscript $x$,

$$V=1-\frac{a}{vp(1+a)}.$$

Differentiating this with respect to $v$, we have

$$\frac{dV}{dv}=-\frac{1}{p}\cdot\frac{v(1+a)\frac{da}{dv}-a(1+a)-av\frac{da}{dv}}{v^2(1+a)^2}$$

$$=-\frac{1}{p}\cdot\frac{v\frac{da}{dv}-a(1+a)}{v^2(1+a)^2}.$$

Now, $a=vp+v^2{}_2p+v^3{}_3p+\ldots\ldots$

where ${}_np$=probability of $x$ living $n$ years.

$$\therefore \quad v\frac{da}{dv}=vp+2v^2{}_2p+3v^3{}_3p+\ldots\ldots$$

$$=a_x+vpa_{x+1}+v^2{}_2pa_{x+2}+\ldots\ldots$$

Also, $a(1+a)=a_x+vpa_x+v^2{}_2pa_x+\ldots\ldots$

Hence, if $a_x$ is greater than $a_{x+1}$, $a_{x+2}$,......*i.e.*, if the value of an annuity on $x$'s life is greater than the value of an annuity on any life older than $x$,

then $$a(1+a) > v\frac{da}{dv};$$

that is, $\frac{dV}{dv}$ is a positive quantity: in other words, as $v$ increases so also does V. But as $v\left(=\frac{1}{1+i}\right)$ increases, $i$ diminishes; therefore the lower the rate of interest the greater is the value of a policy of one year's standing on $x$'s life.

Similarly, if $a_{x+1}$ is greater than $a_{x+2}$, $a_{x+3}$......, $V_{x+1}$ increases in value as the rate of interest diminishes; and generally, if the values of annuities on the lives $x$, $x+1$, $x+2$,...... form a continually decreasing series, the value of a policy of one year's standing taken out at any age from $x$ upwards is greater the less the rate of interest. Whence it follows from (1) that ${}_nV_x$ increases as the rate of interest decreases.

I am, Sir,
Your obedient servant,
W. SUTTON.

18 *Lincoln's Inn Fields,*
26 *August* 1872.

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## ERRATUM.

The last line on p. 4 of this volume is misplaced and should be carried over so as to be the last line on p. 8.

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