CORRESPONDENCE

Multiple Decrement Tables

The Editor, *8 April* 1948

The Journal of the Institute of Actuaries Students' Society

Sir,

We are dismayed by the thought that, whereas hitherto we imagined that we had only one critic to convince, we now have two, yourself and your reviewer, unless, of course, they should happen to be one and the same.

There are two points involved, and both you and your reviewer are making a great fuss about the smaller of them.

The footnote on p. 13 of our booklet has very little to do with Karup's theorem. It simply says that if

$$ {}_tp_x^\alpha \times {}_tp_x^\beta \times {}_tp_x^\gamma \times \ldots = {}_t(ap)_x $$

is given, then it might be incautiously inferred *from this relationship alone* that $\mu = (a\mu)^\alpha$, etc. All that the formula by itself will give is

$$ \sum_\alpha \mu_x^\alpha = \sum_\alpha (a\mu)_x^\alpha, $$

and we maintain that without further conditions there are an infinite number of solutions to this equation. The further conditions are either

(1) the μ_x^α, etc., are independent, i.e. they are not selective,

or (2) the μ_x^α have been measured from multiple decrement data, in which case they are in fact $(a\mu)^\alpha$, etc.

Taking the first case, Karup showed that, on the assumptions that the μ's were independent and continuous functions, it was legitimate to add them together in order to obtain the combined effect. Every Part II student acknowledges this proof when he adds δ to μ in order to obtain the combined effect of interest and mortality. It should be noted that there can be temporal discontinuities in the μ^α, etc., provided that the periods of discontinuity are the same for each μ^{α}. For example, it would be legitimate to add point-forces together provided that they all acted at the same instants in time.

It is equally true that if $(a\mu)^{\alpha}$, etc., have been measured from multiple decrement data, then provided that the forces operate together over the same time periods the combined effect is given by $\Sigma (a\mu)^{\alpha}$.

On the other hand, if μ^{α} and μ^{β} are not continuous and act at different points in the time interval, then it is true that the addition of the two forces will not give the combined effect.

We believe that these points are all covered adequately in our definitions and in the method of constructing the model, and thereby the limitations of Karup's approach are exposed.

In the contributions to 7th International Congress the paper by Van den Belt which summarizes Karup's position is confusing precisely because the problems of selection and interpretation are not faced. All the so-called 'proofs' aim at proving that continuous forces are additive in the sense discussed above. Van den Belt himself, however, hints at the correct approach when in his opening paragraph (p. 389) he says:

'When the diminution of a group consisting of A_x persons all of the same age x, is caused by death, invalidity, marriage, etc., the direct observation yields the probabilities (in our notation)

$$(aq)_x^{\alpha}, \quad (aq)_x^{\beta}, \quad (aq)_x^{\gamma}, \quad \text{etc.,}$$

so that the number of persons of the group, A_{x+1}, at the beginning of the following year is obtained by the product $A_x.(ap)_x$, where

$$(ap)_x = 1 - (aq)_x^{\alpha} - (aq)_x^{\beta} - (aq)_x^{\gamma} \ldots.$$

The theory of independent probabilities (Karup) substitutes for this formula the product

$$(1 - q_x^{\alpha})(1 - q_x^{\beta})(1 - q_x^{\gamma}) \ldots,$$

and treats the formulas for the calculation of the probabilities $q_x^{\alpha}, q_x^{\beta}, q_x^{\gamma}$, etc., which are to be used *as if* [our italics] they were mutually independent.'

The two little words 'as if' are important and in fact are the foundation of all our work on this subject. They imply that if forces are measured directly from multiple decrement data, then in relation to those data and those data only, they can be manipulated

https://doi.org/10.1017/S0020269X00003455 Published online by Cambridge University Press

mathematically as if they were independent whether in fact they are independent or not. If, however, they are related to other data, or if one set of forces is isolated in order to construct a single decrement table, then a special interpretation is required. Your reviewer seemed to think that this interpretation was in some sense beside the point; we, on the contrary, regard it as essential and the most important practical point to consider when the theory is applied.

Yours faithfully,

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https://doi.org/10.1017/S0020269X00003455 Published online by Cambridge University Press