Matters for debate

School algebra and the challenge of the TI 92

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Introduction

Software such as *Derive* and *MathCad*, which will carry out the operations of algebra and calculus, have been available for some time, but are not widely used in schools. However, the Texas Instruments TI 92 calculator which has become available during 1996 is, I would predict, likely to be in common use among students in a fairly short time. This presents an interesting challenge to teachers: do we ignore it or do we try to make sensible use of it in the classroom?

I should like to consider some of the implications, possibilities and pitfalls, but first it is worth seeing something of the power offered by the machine. The two screens in Figure 1 show some of the algebraic and calculus operations that are available by pressing a few keys. I suspect that over half the questions in the exercises in a typical A level textbook can be done effortlessly with it!

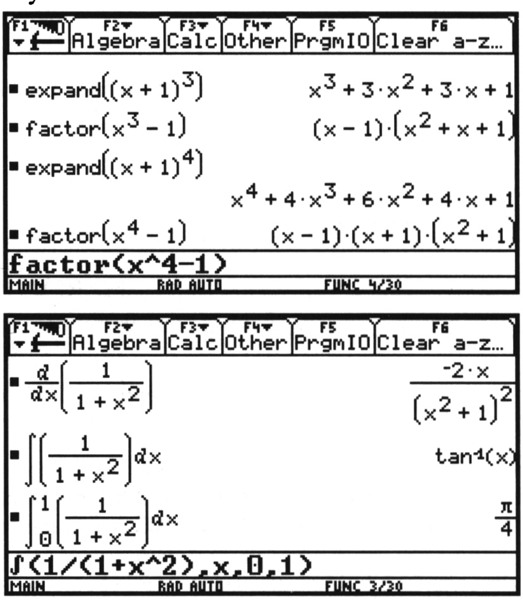

FIGURE 1

Initial reactions may be that such a machine will not be adopted widely in the near future because the current educational price of about £150 is too high. The advertisement in Figure 2, taken from the *Guardian* of 29 March 1974, illustrates very clearly what has happened with four-function calculators over a period of 20 years or so. The price has fallen from around £30 to around £1 for such machines, and that has happened with massive

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inflation over that period. Similar dramatic falls in price have occurred with scientific calculators, so, although inflation is unlikely to be so great, there is every reason to expect the same thing to happen with the TI 92 and its successors. It is not therefore wildly optimistic to suppose that every A level student will have access to an algebraic calculator in, say, five years time.

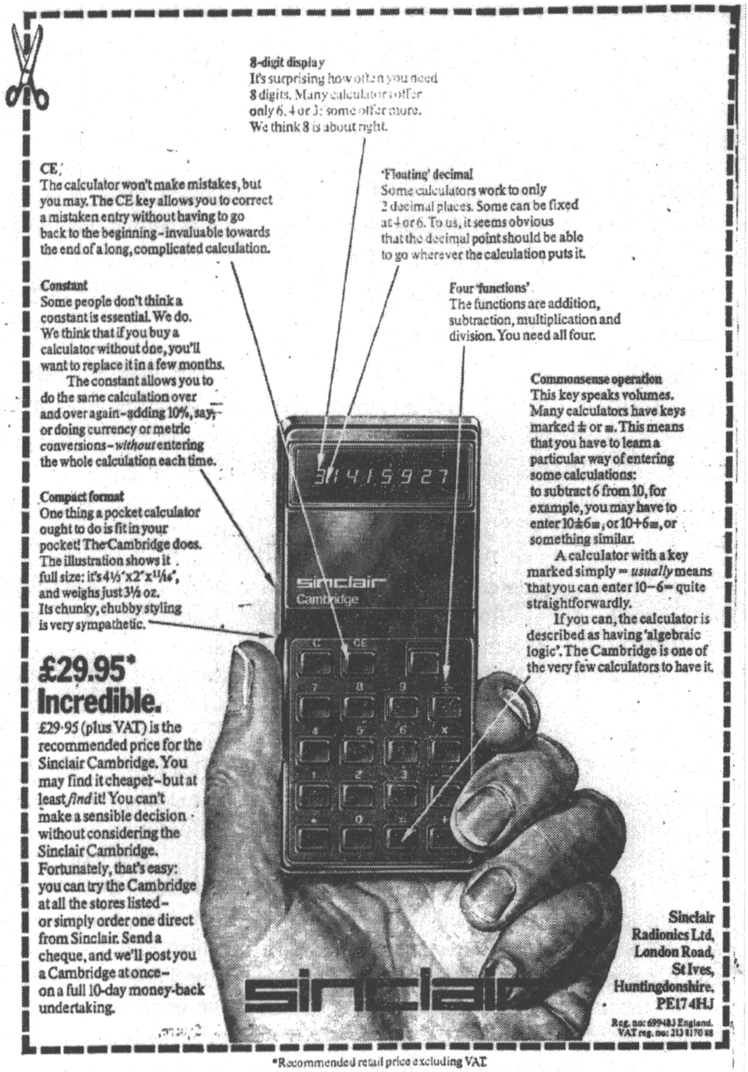

FIGURE 2

The debate about calculators

It is interesting to speculate how long it will take before calculators like the TI 92 are widely accepted as an essential tool for the student. Innovations should not be accepted uncritically, but it is inevitable that

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changes will take place. The challenge for all of us is to see that change takes place in a considered and appropriate way.

Four-function calculators have been readily available since the mid-1970s and have been used increasingly in classrooms since that time. Their use in schools at all levels is still controversial and the subject of much debate like that generated by Tony Gardiner [1] in the *Gazette*, because of the implications for children's learning of arithmetical skills and their wider understanding of mathematics.

Scientific calculators have likewise been used widely from a slightly later date and have effectively eliminated the use of books of tables. The table-replacing aspect of scientific calculators seems to have attracted little attention and appears to be widely accepted as a useful development. It is perhaps surprising that nobody seems to comment on the advantages of having learnt to use logarithms for multiplication and division in the lower school for the understanding of work on the logarithm function in the sixth form!

Graphical calculators have only been used on a wide scale for a few years, although a variety of simple graph plotters have been available for computers for a long time. There are clearly issues relating to graphical skills and understanding similar to those relating to arithmetical skills raised by four-function calculators, but, surprisingly perhaps, concerns have not yet been voiced widely.

The advent of the TI 92 raises much bigger issues, because it calls into question many aspects of the content of the traditional A level curriculum, and the way it is presented. It does this at a time when there is much concern about the preparation of students for higher education, as is evident, for instance, in the recent report by the group set up by the London Mathematical Society [2]. It is noteworthy, and a matter for some surprise, and concern that there is no discussion in that report of the issues raised by such technological developments.

Neil Bibby [3] has described neatly how changes in technology are 'gradually driving a wedge between mathematical content and the associated traditional mathematical skills'. He states the pedagogical problem as: 'if we can automate an algorithm, what does that imply for the teaching and learning of the "manual" algorithm?' and discusses how this serves to highlight the dilemma involved in seeking to develop both technique and insight. To the student it may seem pointless to rehearse endlessly a set of pencil and paper skills which can be readily performed with a calculator, and yet, to the teacher, it is evident that the development of insight does require some technical facility.

As a result of experience with four-function calculators, it is now widely accepted that mental arithmetic needs to be given strong emphasis, partly to develop an appropriate facility when operating with simple numbers, but also, and perhaps more importantly, to help develop insight and understanding. By analogy we can argue that something similar is required in relation to algebra.

A historical aside: square roots

Whenever there have been innovations in education, technological or otherwise, there has often been a long period before they have been accepted and become widely adopted. When I was at secondary school in the 1950s I was taught how to work out square roots 'by hand' using the method shown in Figure 3, taken from a standard school textbook by Durell [4]. We were also shown how to use square root tables like those of Godfrey and Siddons [5] shown in Figure 4, and that became our standard procedure when a square root was needed.

Example 1. Find the square root of 69169.

Mark off 69169 in periods of *two* digits *starting from the decimal point*, in this case from the right.

```
       2| 6| 3
       6|91|69
       4
   46) 2 91
       2 76
  523)  15 69
        15 69
```

The greatest integer whose square is not more than 6 is 2; write 2 in the first period of the answer and 2^2, that is 4, under the 6, and subtract. Bring down the digits 91 from the next period. Write down *twice* 2, that is 4, and find the approximate value of $291 \div 40$, that is 7. $47 \times 7 = 329$, but this is too large; $46 \times 6 = 276$. Write 6 in the trial divisor and the second period of the answer, 276 under 291, and subtract. Bring down the digits 69 from the next period. Write down *twice* 26, that is 52, and find the approximate value of $1569 \div 520$, that is 3. $523 \times 3 = 1569$. Write 3 in the trial divisor and in the third period of the answer.

$\sqrt{69169} = 263$, exactly.

FIGURE 3

SQUARE ROOTS

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
10	1000	1005	1010	1015	1020	1025	1030	1034	1039	1044	0 1 1	2 2 3	3 4 4
	3162	3178	3194	3209	3225	3240	3256	3271	3286	3302	2 3 5	6 8 9	11 12 14
11	1049	1054	1058	1063	1068	1072	1077	1082	1086	1091	0 1 1	2 2 3	3 4 4
	3317	3332	3347	3362	3376	3391	3406	3421	3435	3450	1 3 4	6 7 9	10 12 13

FIGURE 4

As a student it was difficult to see the purpose of being shown the long method of calculation, except perhaps as a historical curiosity, but one suspects that there were people who said that square roots would not be understood if the 'proper' procedure was not taught first. Indeed, I note from the Second Report on the Teaching of Arithmetic [6], published by the Mathematical Association in 1964, that 'it may well be argued that there is no need for this somewhat cumbersome and lengthy process to be undertaken at all'. Clearly there was still active debate about the issue at that time!

Nowadays we use a calculator instead of tables and that has been accepted with remarkably little question. Moreover the calculator has made it possible to use simple iterative procedures for finding square roots which can give much greater insight to the average student than the 'long method' ever did.

The significant point here is that tables were regularly used in schools for well over 50 years before the long method for doing square roots was dropped from the curriculum. This serves to illustrate how something, which today we appear to accept without question, in fact took a very long time to gain general acceptance.

It is not difficult to see that issues of this kind are going to arise with the advent of the TI 92. For instance, will it be necessary to spend so much time in an A level course learning a variety of integration techniques? How do we distinguish between what is essential in order to develop understanding and the ability to use ideas effectively, and what can reasonably be left to the machine? So we return to the idea of mental algebra.

Mental algebra

Students need an understanding, knowledge and certain skills that they have 'at their fingertips' in the sense that they can immediately call to mind particular key ideas, explain them simply and do simple calculations with them, without reference to text or machine, and without extensive written working. This is more than just technical facility, because it embraces the idea of understanding in its widest sense. The analogy with mental arithmetic is apt, because mental calculation requires understanding of underlying principles as well as a ready familiarity with a range of simple facts and skills.

Mathematics at A level does, of course, involve a far wider range of ideas and skills than those involved in this 'fingertip' knowledge. For instance, problem solving, proving, and modelling, all involve an ability to follow and create extensive chains of reasoning and to look at alternative strategies, but I am concerned here with a background of understanding, knowledge and skills, which is needed if students are to use a powerful calculator intelligently. Various aspects involved in this 'mental algebra' can be identified:

- familiarity with, and understanding at a simple level of, various mathematical functions and their properties;
- familiarity with, and recall of, key identities and formulae;
- fluency in mental arithmetic;
- fluency in algebraic manipulation with simple examples;
- fluency in sketching and interpreting graphs;
- awareness of links and connections;
- some sense of what could be right and of what is obviously wrong.

These requirements fall into three categories which we can usefully describe as *familiarity*, *fluency* and *feel* and it is these three 'f-words' that characterise what we should be aiming at with all our students, and which have much in common with the ideas of 'symbol sense' developed in a paper by Abraham Arcavi [7].

Some examples

Availability of TI 92s seems to me to make remarkably little difference to any list constituting a core for A level mathematics, because such a list should be largely concerned with precisely those items that students should be expected to have at their 'fingertips', in the ways suggested above, together with ideas about the sort of contexts in which that knowledge can be applied. Such a background is essential if students are to use a TI 92 intelligently. For example, I have indicated below with some examples what might be regarded as the essential ideas involved in section 3.1 of the current A level core [8].

Addition, subtraction, multiplication and factorisation of polynomials.

Difference of two squares: $a^2 - b^2 = (a + b)(a - b)$.

Recognition and application to a range of simple examples:

$$12.5^2 - 2.5^2 \qquad (n + 1)^2 - (n - 1)^2 \qquad 4x^2 - 9y^2 \qquad (x + 2)^2 - 9.$$

Similarly with $(a \pm b)^2 = a^2 \pm 2ab + b^2$.

Expanding, factorising and completing the square for quadratics with unit coefficient of x^2:

$$x^2 + 4x - 5 = (x + 5)(x - 1) = (x + 2)^2 - 9.$$

Graphical interpretation and equation-solving implications of the above three forms.

Simplifications such as:

$$(x + 2)(x + 3) - (x + 1)(x + 4) = (x^2 + 5x + 6) - (x^2 + 5x + 4) = 2.$$

Operations with algebraic fractions:

$$\frac{x^2 - 4}{x - 2} = x + 2 \qquad \frac{1}{x - 1} - \frac{1}{x + 1} = \frac{2}{x^2 - 1} \qquad \frac{x - 1}{x - 2} = 1 + \frac{1}{x - 2}.$$

TI 92s have a central role in the debate about how mathematics is to be taught and learnt and in how it is to be assessed. It is clearly not satisfactory to spend time using the calculator to work through routine exercises involving, say, finding factors or integration. More imaginative approaches are required where the calculator is used as a *learning tool* with which to explore ideas and to develop and extend understanding, rather than just as a quick way of getting answers.

As a small contribution to this I offer two examples involving use of the TI 92 to extend students' understanding of some of the key ideas above. The

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machine's ability to factorise numbers, together with its acceptance of function notation and its graphical facilities, are utilised in the first example to help students see the links between algebraic expressions and operations, and corresponding numerical and graphical interpretations. The worksheet in Figure 5 and the accompanying screens in Figure 6 are self-explanatory. In the second example some aspects of the manipulation of algebraic fractions are considered.

FINDING FACTORS

- *Enter* define $f(x) = x^2 + 3x + 2$.
- *Enter* $f(20)$ to give 462.

  The function has been evaluated for $x = 20$ which can be checked by noting that $20^2 + 3 \times 20 + 2 = 462$.

- *Enter* factor(462) to give 2.11.7.3.

  462 has been written as a product of its prime factors. Notice that there are many possible ways in which 462 can be expressed as a product of two factors: 21×22, 6×77 and so on.

- Taking values of x from 21 to 30, evaluate $f(x)$ and factorise the result.
- Look carefully at the factors and see what you can find by expressing each as a product of two numbers. If necessary try some other values for x.
- *Enter* factor($f(x)$).
- How does this *explain* what you have observed about the products of factors?
- For what values of x will $f(x)$ be zero? How do the factors of $f(x)$ enable you to determine this?
- For what value of x does $f(x)$ have its least value? How is this value of x related to the values for which $f'(x)$ is zero?
- *Enter* graph $f(x)$.

  This command provides a quick way of drawing a graph. You may need to vary the ranges for the graph using either the window screen or the zoom menu.

- How does the graph help in answering the questions?
- Repeat this exercise for the functions defined by the following expressions:

  $f(x) = x^2 + 2x + 3$

  $f(x) = x^2 + x + 1$ (don't spend too long looking for products of two factors!)

  $f(x) = x^3 + 6x^2 + 11x + 6$ (a product of three factors)

FIGURE 5

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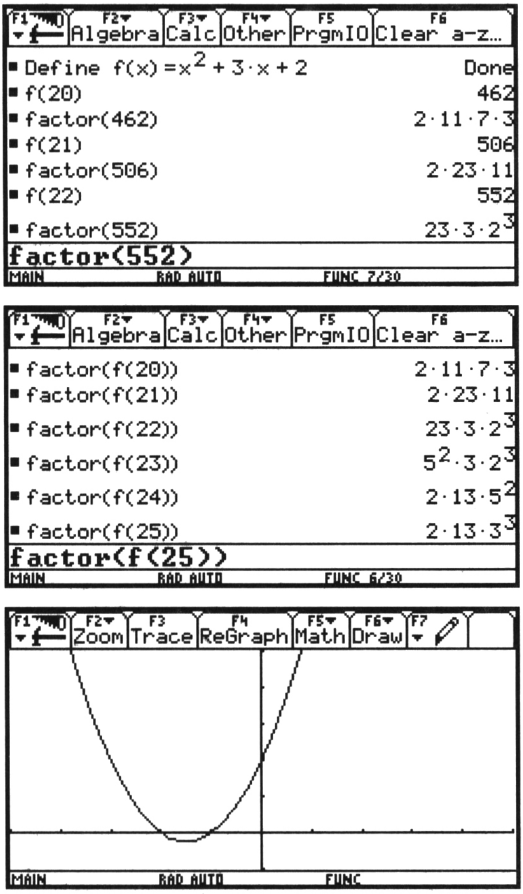

FIGURE 6

The aim of the task is to extend students' understanding of factorisation by asking them to *explain* what they find and to *make links* between the different algebraic expressions, numerical examples and graphical representations. The use of such tasks needs to be coupled with abundant opportunities to practise the mental skills of expanding and factorising simple expressions and to apply them in a wide variety of contexts.

As a second example I take one of the examples from the core list given previously. The extract from a TI 92 screen in Figure 7 shows the effect of applying the command propFrac to the fraction $\dfrac{x-1}{x-2}$.

FIGURE 7

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Rather than start by showing students how to perform a particular operation with algebraic fractions by hand we can take a result like this from the calculator as something to be *explained.* The explanation as given below simply involves finding an intermediate step or steps which link the two forms, something which the calculator will not supply for you.

$$\frac{x-1}{x-2} = \frac{1+x-2}{x-2} = \frac{1}{x-2} + 1.$$

It could be argued that learning to make such linking steps might be at least as effective in developing skills and understanding as more traditional tasks.

The way forward

It is difficult to predict what will happen, even in the near future, but past experience with technological developments suggests that we will be surprised how rapidly a machine like the TI 92 and its inevitable successors becomes readily available. Assessment clearly raises some significant problems which have been discussed by Taylor [9] and others in recent issues of the *Gazette*. However, the biggest question is whether we can meet the challenge to our teaching methods. We may just ignore such developments in the hope that they will go away, in which case many students are likely to become machine dependent or be put off mathematics altogether because readily available technology is ignored. Alternatively, we can accept the challenge with all its pitfalls and possibilities and use the TI 92 as another useful tool for helping students develop understanding, appropriate skills and knowledge, confidence, clear thinking and an ability to use mathematics successfully.

To conclude, I return to square roots with a nice question which I have often used to show that a calculator will not do everything. More importantly, however, it shows the virtue of approaching some problems by looking at a simpler, but similar, problem first. So, how do we calculate

$$\sqrt{12345678987654321}?$$

Calculators will not usually handle this number of digits, but the answer is obvious once you see the similarity to

$$\sqrt{121} = 11 \quad \text{and} \quad \sqrt{12321} = 111.$$

To my annoyance I now find, as shown in Figure 8, that the TI 92 will work out square roots of such large numbers, so I am certainly in no doubt that there is still a strong case for doing some mathematics without having access to a calculator!

■ $\sqrt{12345678987654321}$	111111111

FIGURE 8

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Fry for President?

'When he grasped the completeness of my ignorance he went right back to the beginning and taught me something that I did not understand: the equals sign. I knew what 2 + 2 = 4 meant. I did not understand, however, even the rudimentary possibilities that flowed from that. The very thought of an equals sign approximating a pair of scales had never penetrated my skull. That you could do anything to an equation, so long as you did the same to each side, was a revelation to me.

There came a second revelation, even more beautiful than the first. Algebra, I suddenly saw, is what Shakespeare did. It is metonym and metaphor, substitution, transferral, analogy, allegory: it is poetry. Suddenly I could do simultaneous equations. And so it went on.'

Malcolm Perella saw this extract from Stephen Fry's autobiography *Moab is my washpot* (Hutchinson) in *The Sunday Times*, 21 September 1997.

Overheated

Barometers also hit 72F at Stansted Airport in Essex and RAF Lyneham in Wiltshire.

This howler from *Wales on Sunday,* 19 October 1997, led Mike Mudge to wonder if the recent tendency for examiners to provide the units in GCSE examinations is somehow to blame.