ON THE ANGLES BETWEEN THE MAGNETIC AND ROTATION AXES

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Abstract

Several methods for evaluating the angle β between the magnetic moment and rotation axis of a pulsar are considered. Averaged values of this angle, estimated by different methods for 61 pulsars, decrease as the characteristic age increases. Many pulsars have small angles $\overline{\beta}$ ($\langle\overline{\beta}\rangle = 23°$) and large ages ($\langle\log\tau\rangle = 7.0$), so the alignment of the magnetic and rotation axes must take place during the evolution of pulsars.

1. The angle β between the magnetic and rotation axes is the most important parameter for examining pulsar models, classification schemes, and evolutionary tracks. A short review of the possible methods for calculating this angle is given below. Values of β are computed for a number of pulsars, and the observed dependence of β on characteristic age ($\tau = P/2\dot{P}$) is discussed.

The well known polar-cap model in which the generation of emission takes place in open field line region at distances $r \ll r_{\rm LC}$ ($r_{\rm LC} = cP/2\pi$ where r is the radius of the light cylinder) was applied in order to obtain all the results discussed below. In this model the following equations (Manchester and Taylor 1977) are used to determine β (see figure 1)

$$\cos\theta = \cos\beta\cos\zeta + \sin\beta\sin\zeta\cos\phi_{\rm p}, \qquad (1)$$

$$\tan\beta = \frac{\sin\beta\sin\phi}{\sin\zeta\cos\beta - \cos\zeta\sin\beta\cos\phi}, \qquad (2)$$

by adding the third equation connecting the angles β, ζ and θ.

2. At the instant when the line of sight passes through the center of the emission cone, eq.(1) reduces to

$$\sin\beta = \sin(\theta/2)/\sin(\phi_{\rm p}/2) \qquad (3)$$

By adding the relationship between θ and P

$$\theta = AP^{-a}, \qquad A = \text{constant} \qquad (4)$$

obtained from the diagram "observed profile width W_e (Kuz'min and Dagkesamanskaya 1983), W_{10} (Malov 1986) or W_{50} (Rankin 1990) *vs.* period P" (as a lower boundary of this diagram) we can calculate the angles β. This method can give good estimates only for pulsars with $\zeta = \beta$, *i.e.* for objects with core emission. For others it gives lower limits on β.

3. Taking polarization into account we can omit the assumption that the sight line passes through the center of the emission cone. It is then possible to use either the full position-angle traverse $\psi(\phi)$ of the integrated profile or the value of the derivative at the center of this profile

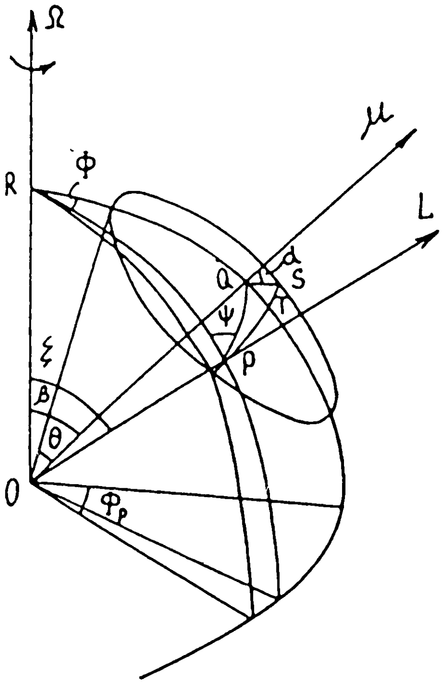

Figure 1 Pulsar geometry.

$$C = (d\psi/d\Phi)_{\max} = \sin\beta/\sin(\zeta - \beta) \qquad (5)$$

An expression relating θ and P [eq.(4)] can be used as the third equation (Kuz'min, Dagkesamanskaya, and Pugachev 1984, Malov 1986, Lyne and Manchester 1988). However, values of the angle β obtained by such a method are statistical estimates because a statistical relationship $\theta(P)$ is used.

4. Malov (1983) showed that the comparison of observed profiles with those calculated within the framework of the polar-cap model gave a third equation in the form

$$\zeta - \beta \approx \theta\sin\alpha, \qquad (6)$$

where the angle α was determined for the best agreement between the observed profiles and model profiles.

It is necessary to model the magnetosphere quite well (in particular the distribution of the emitting charges as a function of angle and energy) for this method to be effective.

5. If the position angle is measured throughout the whole period, the value of β can be calculated by fitting the relationship $\psi(\phi)$ [eq.(2)] to the observations (Narayan and Vivekanand 1982). Such values were obtained by Malov (1986) for PSR 1937+21 and for a number of other pulsars by Lyne and Manchester (1988). In addition, some model calculations (as in ¶4) for these pulsars have been made by Malov (1986, 1990). A comparison of the results is given in table 1.

Table 1 Comparison of the angles obtained by Malov with those obtained by Lyne and Manchester.

PSR	β°_{M}	β°_{LM}
0531+21	85	-
0823+26	85	80
0826−34	<10	10
0950+08	7	10
1055−52	9	75
1702−19	85	80
1822−09	7	-
1929+10	8	15
1937+21	79	-

All these values are in a good agreement with each other with the exception of PSR 1055−52. Lyne and Manchester (1988) give a second evaluation of the angle β for this pulsar ($\beta = 17.9^\circ$) calculated on the basis of the position-angle derivative $(d\Psi/d\Phi)_{\max}$ in the main pulse. We obtained several values of β by different methods (Malov 1990) and the mean value was 22.6°. Therefore, we believe that PSR 1055−52 is a nearly aligned rotator. The existence of intense X-ray emission from this pulsar is an additional argument for such a conclusion (Malov 1989).

The data in table 1 show that interpulses occur both in orthogonal rotators and in aligned ones. Such a possibility was first discussed by Hankins and Cordes (1981), and the first calculations showing the existence of the two groups of pulsars with interpulses were made by Malov (1983).

6. If the observed radio emission is generated at the levels where $r \le 0.1r_{\mathrm{LC}}$ then

$$(\zeta - \beta) \le \theta = \sqrt{(r/r_{\mathrm{LC}})} \le 18^\circ \qquad (7)$$

and

$$C \ge 3.24 \sin\beta \qquad (8)$$

This relationship allows us to obtain an upper limit on the angle β in pulsars with small values of C (small β). Comparing these limits with lower ones (item 2) we can calculate more precise values of β for a number of pulsars (Malov 1990).

Table 2 Comparison of the values of β with age

PSR	$\overline{\beta}^\circ$	$\log\tau$	PSR	$\overline{\beta}^\circ$	$\log\tau$
0031−07	14	7.56	1540−06	33	7.10
0203−40	31	6.92	1552−23	22	7.08
0447−12	16	7.83	1556−44	30	6.60
0525+21	26	6.17	1557−50	47	5.78
0540+23	27	5.40	1600−49	20	6.71
0740−26	39	5.20	1609−47	18	6.98
0756−15	42	6.83	1718−02	9	7.94
0809+74	13	8.09	1737+13	37	6.94
0818−41	4	8.51	1745−12	24	6.71
0820+02	14	8.14	1820−31	40	6.19
0835−41	52	6.53	1821+05	21	7.72
0901−63	13	7.99	1831−04	8	7.31
0906−17	20	6.98	1842+14	31	6.50
0940−55	18	5.67	1845−01	25	6.30
0943+10	23	6.69	1922+20	20	6.26
0957−47	7	8.11	1924+16	25	5.71
1112+50	26	7.02	1924+14	16	7.97
1143−60	21	6.38	1933+16	50	5.98
1222−63	18	6.85	1941−17	28	7.13
1237+25	49	7.36	1942−00	20	7.97
1240−64	37	6.14	1944+17	5	8.46
1309−53	12	7.89	1946+35	29	6.21
1353−62	18	6.36	1953+50	36	6.78
1417−54	18	7.80	2003−08	11	8.36
1436−63	24	6.81	2016+28	36	7.77
1449−64	30	6.01	2045−16	36	6.45
1451−68	21	7.63	2048−72	16	7.44
1503−66	23	6.69	2123−67	10	7.36
1504−43	23	6.45	2319+60	18	6.71
1510−48	23	6.89			

7. If the magnetic flux remains constant during the formation of a neutron star, its magnetic moment can reach $\sqrt{2} \times 10^{30}\,\mathrm{G\,cm^3}$ and have an arbitrary orientation. In addition to this magnetic field, superfluid protons inside the star can generate another field of the same order of magnitude parallel to the rotation axis (Sedrakian and Movsissian 1986). The total field of the neutron star is then the vector sum of these two contributions. If the value of this total field is maintained but its orientation changes, we can estimate the angle β from observational data. In fact, for the case of magnetodipolar losses of rotation energy, we have

$$-I\Omega\dot{\Omega} = 2H_0^2 R^6 \Omega^4 \sin^2\beta/(3c^3), \qquad (9)$$

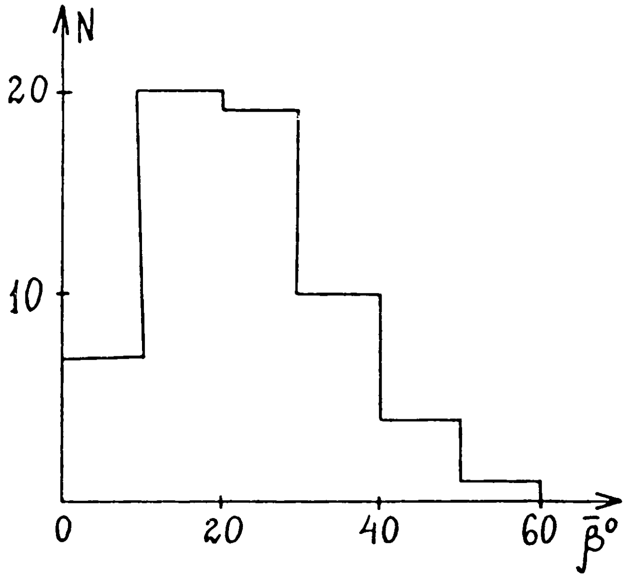

Figure 2 The distribution of angles $\bar{\beta}$ for 61 pulsars.

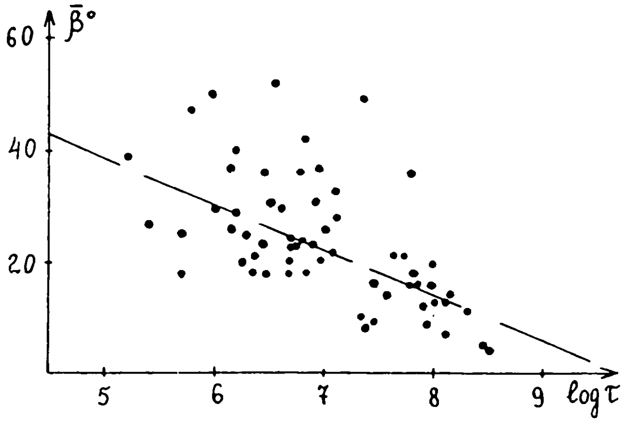

Figure 3 The dependence of the angle $\bar{\beta}$ on $\log\tau$.

or

$$P\dot{P} = 16\pi^2 H_0^2 R^6 \sin^2\beta/(3c^3 I) \qquad (10)$$

Let us assume that $H_0^2 R^6 = 2\times 10^{60}\,\mathrm{G}^2\,\mathrm{cm}^6$, $I = 10^{45}\,\mathrm{g\,cm}^2$, and $R = 10^6$ cm; eq.(10) then reduces to

$$\sin\beta = \sqrt{(P\dot{P}_{-15}/2)} \qquad (11)$$

For a number of pulsars $\sqrt{(P\dot{P}_{-15}/2)} > 1$. This means either that their magnetic moments exceed $\sqrt{2}\times 10^{30}\,\mathrm{G\,cm}^3$ or that $I_{45} < 1$.

It is interesting that estimates calculated from eq.(11) correlate with values of β obtained by the independent method (Malov 1986) (¶3). The correlation coefficient is equal to 0.5. Hence these two methods for calculating β support each other.

8. Analyses of the angle β obtained by different methods are in good agreement with each other for many pulsars. We calculated mean values $\bar{\beta}$ (table 2) on the basis of results obtained by Rankin (1990), Lyne and Manchester (1988) and Malov (1990). We believe that these values represent the most probable values of β at present. Table 2 does not contain any pulsars with interpulses. The best values for those pulsars are given in table 1.

The distribution of angles $\bar{\beta}$ for 61 pulsars is shown in figure 2. It is necessary to point out that there are many small angles in this distribution. The mean value of $\bar{\beta}$ is 23°. Since the average characteristic age is 10^7 years ($\overline{\log\tau} = 6.99$) this means that alignment of the magnetic and rotation axes must take place in the course of their evolution as radio pulsars. The dependence of the angle $\bar{\beta}$ on $\log\tau$ is presented in figure 3.

Other authors have also come to the conclusion that alignment of the axes occurs as pulsars evolve, for example Kuz'min and Dagkesamanskaya (1983) and Lyne and Manchester (1988).