14

Modeling the Dynamics of Socioenvironmental Transitions

"*Models are opinions embedded in mathematics.*"
(O'Neill 2016: Weapons of Math Destruction)

Introduction

In Chapter 11 I presented the qualities and limitations of information processing in various kinds of societal configurations under, respectively, universal control, partial control, and no control, and used a very simple percolation model to summarize the overall evolution of societal systems as a spreading activation net. In the second part of that chapter I discussed various aspects of heterarchical systems and the ways in which hierarchical and distributed information processing networks interact. It concluded with an argument to the effect that in such heterarchical systems, diversification of activities contributes substantially to the stability of the system.

This chapter is devoted to the dynamics and processes that occur between rural and urban contexts, engendering the transitions between these system states. The increasing connectivity that involves more and more people in the spreading activation net has major consequences for the structure of the information-processing network involved, and we need to look at them. That argument will be based on a complex systems model applied to the dynamics of information processing. Although this chapter is therefore based on a rather technical construct formulated in mathematical terms, I will initially present the argument as far as possible in non-technical terms. To demonstrate the potential and the relevance of the modeling approach, for readers who might be interested in some of the details, I will restate important elements of the argument in mathematical terms in Appendix B. Those who are not interested in this aspect can follow the overall reasoning of the book without interruption.

Second-Order Dynamics

To begin, we can gain a glimpse of the complex dynamics involved in the emergence of urbanism by identifying the long-term change in change dynamics (what I have called the second-order change dynamic) occurring in that process. This can be done by looking at the rhythms of the various processes that are involved. Whatever the societal form of organization, the human and environmental dynamics in it are interlocked in mutually interacting ways.

In rural situations, the environmental dynamic is the more complex and multilayered of the two, and is thus the slower one to change. The human dynamic, on the other hand, consists of relatively few superimposed rhythms and can change relatively quickly because people can learn. As a result, a faster human dynamic is essentially locked onto a slower environmental (natural) dynamic: humans adapt to the environment, and because the environment is slow to change the combined socioenvironmental system is rather stable.

In urban situations the two kinds of dynamic reverse their rhythms: the societal dynamic becomes more and more complex, and therefore more and more difficult and slow to change, whereas the environmental dynamic, in so far as it directly relates to the societal system, is simplified because humans have locally reduced the environmental complexity and diversity of their environment. The environment can now be adapted according to the needs of the society. But as the more rapid dynamic has now become the dominant one, the socioenvironmental system as a whole has become less stable. As Naveh and Lieberman (1984) put it, "the environment has become disturbance-dependent [on society]."

The above reversal is the fundamental one that has brought our societies to their current, unsustainable, situation, and it draws our attention to the fact that the temporal dimensions of the rhythms constituting socioenvironmental interaction are crucial in the coevolutionary transitions we are discussing here. I will come back to these later in this chapter in the form of models that show how these temporal differences affect urban–rural interaction.

Mobile and Early Sedentary Societies

Looking now at the first major organizational transition of society, that from mobile gatherer-hunter-fisher societies to sedentary ones (whether based on stable, naturally available resources such as salmon in the Pacific Northwest of the USA, or based on cultivation such as early farming communities in the Near East, East Asia, and the Valley of Mexico), from the perspective we are developing here we must emphasize a difference that has of course been noted but in my opinion not sufficiently emphasized: the change in the way resources are used. Mobile gatherer-hunter-fisher societies collected what nature had to offer – they had a multi-resource subsistence strategy in which they were wholly dependent on the rhythms of nature, and their only way to adapt to challenges was to move to other places with different natural rhythms. They harvested, but did not in any way invest in, their environment. Over the lifespan of individual gatherer-hunter (mobile) groups, once they had mastered sufficient knowledge of the dynamics of their environment they dealt effectively with change at daily and seasonal temporal scales by moving around from resource to resource. But they probably experienced very variable foraging success, and thus at that scale they experienced high levels of uncertainty, but hardly any risks because they had not substantially invested in the environment.

Sedentary societies, on the other hand, developed a reciprocal, interactive, relationship with their environment in which they invested in the latter by clearing spaces, working the soil, sowing, and waiting to harvest. In the process, they reduced the range of resources exploited by focusing much effort on one or more specific ones. They tried to some – very limited – extent to control some aspects of their environment, and their investment carried some risk with it. This was clearly a dynamic in which humans engaged with their environment, but remained essentially beholden to many of the vagaries of the latter, in the form of climate, soil, vegetation, etc. Herding societies also developed an interactive relationship with their environment, managing the natural dynamics of herd reproduction yet (as far as we know) not investing in a particular place, instead following the environmental rhythms of herds and their resources.

Though the information processing in all these cases was essentially under universal control (hunter-gatherer-fisher societies, early agricultural village societies, and herding societies were and are mostly egalitarian), the transition was the beginning of a shift from societies dominated by natural and slow (environmental) rhythms to environments that are being modified by more rapid human societal rhythms. Initially, because human groups were small and their technologies relatively unsophisticated, the human impact on these natural rhythms was limited, and the complex environmental dynamics ensured long-term overall stability of this mode of social organization and information processing.

But once human dynamic rhythms were introduced into the system alongside environmental ones, because people could adapt more quickly the former rhythms grew in importance in step with the growth of the population involved and the consequent growth in complexity and technological capability of societal systems. Ultimately, they took over so much of the Earth system that we now speak of the Anthropocene as the period in the Earth's history in which humans control (most of) the overall socioenvironmental dynamic on Earth. In the following sections, I will roughly outline how that process followed its course, ultimately leading to the rapid expansion of urban societies that we have seen over the last 150 years.

The Emergence of Hierarchies

How did hierarchies emerge in such societies? An example that I observed in Wiobo village in the Eastern Highlands of Papua New Guinea in 1990 can serve as an illustration. This is a highly isolated area, one of the last areas of Papua New Guinea to be opened up to western observation, this taking place in the 1950s. The society is a horticultural one, in which subsistence is provided locally by exploiting small gardens in which food is grown. When a dwelling for a new couple was being collectively built, a large part of the village came together around a meal prepared in a Polynesian oven. Suddenly an argument broke out between several males, concerning responsibility for a particular task in the village: keeping the landing strip alongside it in a serviceable state (cutting the grass, etc.). After a while, in which different contenders offered different solutions to the challenge, a consensus emerged that one person's suggestion was the best one, and he was elected to be what one could call the keeper of the landing strip.

From an information processing perspective, what was happening cut two ways. On the one hand this process selected a particular channel that favored a specific set of signals over many others referring to the same topic, relegating the others to the status of noise, and on the other hand the group created a degree of vertical integration by according one person control over a specific part of the information flow in the society, and thereby according that person a degree of responsibility and prestige, as well as the capacity to mobilize others for the task concerned. Both aspects of this action clearly rendered the fulfilling of this specific task more efficient by aligning the information processing of the people involved in it.

By thus "electing" candidates who offered what were considered to be the best solutions to challenges faced by the group, a group could create a number of domain-specific (short) hierarchies that improved the group's information processing substantively. Ultimately, of course, coordination between a growing number of such hierarchies, and thus between a number of job holders, would be necessary and would in all probability lead to a kind of coordinator function for which another individual was chosen. It is important to note that in the early stages of this development, these responsibilities were assigned ad hominem, were not heritable, and could also be revoked during a person's tenure.

The First Bifurcation

The next transition is one that sees the expansion of these small, sedentary (or herding) groups. They are still dependent on locally available energy and resources, and their information processing networks are hierarchical within the community. These hierarchies may at this point become more stable, giving rise to so-called great men and big men positions (Godelier 1982; van der Leeuw 1986) that ultimately may even become heritable. As the groups grow, the partial control of the different functional hierarchical information-processing networks creates inhomogeneities in the information pool. Those in control of a hierarchy process more information than others, which makes them leaders, but also leads to misunderstandings and potentially to conflicts. One way to deal with this is for the group to institute occasional or periodic group meetings to reduce communication distances between all members, and thus serve to rehomogenize the information pool and readjust it to changing circumstances, whether caused in the environment by human exploitation or by externally triggered fluctuations in the social or natural environment. One would expect these resets to occur more frequently as maladaptations between the state of the environment and the state of environmental information processing grow.

From a dynamic model perspective on information processing, one could characterize such systems as oscillating around a fixed-point attractor. Stability based on a fully shared information pool dominates.

But the societal system is subject to an oscillation between an accelerating/structuring phase and a decelerating/destructuring phase. In the former, the system is more deterministic, in the latter more stochastic. In more tangible everyday terms, people alternate between strengthening their system around a core set of ideas, customs, and institutions, and the opposite, widening the range of ideas and behaviors.

As contacts intensify, non-hierarchical distributed connections within groups are strengthened by family relationships maintained through networks of marriages. Owing to the combination of hierarchical and distributed information processing networks, information spreads very quickly, correcting imbalances in the information pool. But these societies are still slow to adapt as they are heavily constrained by slow environmental dynamic rhythms and have very few decision-makers (of limited diversity).

The Second Bifurcation

As societies grow in size, the hierarchical aspect of information processing also grows in depth and size, involving more and more people. As we saw at the end of Chapter 11, it also becomes more and more specific by losing a number of its branches as it focuses more sharply on tasks at hand, and thus becomes less adaptive. The distributed information processing network in the society, being more adaptive, gains in importance. We can thus imagine that at some point there could emerge a second bifurcation between hierarchical and distributed communication modes, in which they are separated spatially. This could for example occur when in some locations a faster adaptation of the socioenvironmental system is required than in others because the system is more dependent on the human dynamic than on the environmental one, whereas in other locations it is the reverse. Poorer environments, or environments that are more likely to be handicapped by certain environmental dynamics (climate, water, erosion) might trigger such more rapid adaptations, and favor distributed information processing.

Initially, this bifurcation might simply be enacted by certain people in a settlement who begin to specialize in communicating with others, for example in terms of exchanges or even trade, while others continue to be focused on immediate subsistence activities and to be linked to a hierarchical information-processing system. This would be one way to look at the prestige goods economy (e.g., Frankenstein & Rowlands 1978), which is in some places contemporaneous with emergent proto-urban centers and locally generates a settlement size hierarchy. Physically, this requires a point of connection between the distributed communications network and the hierarchical one. Because it is the point of introduction of new ideas and values, it quickly becomes the apex of the local hierarchy.

Over time, as the community of people linked into a distributed communication network grows, this may lead to the emergence of specialized periodic trading centers, such as the early medieval Northern European trading emporia, examples being Hedeby and Dorestat. These were located in geographical locations that were particularly suitable for communication, such as along rivers (at fords or branching points), along the coast, or at points where other conditions favor them.

In modeling terms, this is an information-processing system in which more permanent and spatially wider-spread communication corridors based on distributed information processing emerge between spatially separated hierarchical islands, structured as stochastic information webs wherever structured and unstructured oscillations form a pattern of interferences (Chernikov et al. 1987). Qualitatively, these webs involve information brokers between different hierarchically organized villages, such as ambulant tradesmen and others who are independent from the village hierarchies. In pre-classical Greece, one could also interpret priests in liminally placed sanctuaries, such as Delphi, as examples of such brokers. Currently, one finds them in very many places in the developing world.

The Third Bifurcation

The third bifurcation could be called preurban smouldering – a situation in which, at a regional level, limited-term and more complex structuring occurs here and there, after a while petering out, then rekindling elsewhere. The existence of long-distance distributed processing corridors that are relatively stable over a period, and sufficiently frequently used to have a sufficient channel capacity (bandwidth) to maintain the information flows involved, permit certain groups of hierarchically organized societies to integrate into a larger system. This has a locally destabilizing effect because the symbiotic, hierarchical systems' connectivity is enhanced through spatial extension (see White 2009). Dealing with this requires increased reliance on distributed information processing and energy obtained from elsewhere, and has probably led to instabilities in these systems, as I argue in Appendix B by constructing a set of dynamic models of these interactions.

Such a fluid and essentially discontinuous process of structuring and restructuring is imperfectly captured by any single spatial, all-encompassing, geometric structure as an explanation of societal organization. For example, under the type of dynamic evolution postulated here, territoriality and the societal boundedness of societies must have been subject to constant redefinition; a political tug of war between competing, adjacent polities for control and supremacy in exchange relations, both within the transportation and communication network itself and outside it. Under such circumstances, preeminent societal control by any single social group is unlikely for other than short periods.

Such essentially unstable systems were not confined to the European La Tène period, on which our models have been based. In Europe we see them again after the collapse of the Roman Empire, in the seventh to eleventh centuries CE. But I would surmise that we see them also in the Preclassic Maya (900 BCE–300 CE) area before the hegemony of Tikal and Caracol, in certain phases of Chinese history (such as the period of the warring states, 475–221 BCE), in the Uruk phase in the Near East (*c.* 4000–3100 BCE), and elsewhere.

An important aspect of the emergence of these long-distance distributed communications is that they infuse local hierarchical systems with new values (materials, objects, technologies, ideas). This enables them to extend the set of values of the community involved, and over time it enables the alignment of more and more people in different local systems into one value system.[1] I return to this aspect in Chapter 16.

The Fourth Bifurcation

In many parts of the world, the first real towns emerge as a network of small, more or less equivalent, city states in what has been called peer–polity interaction, invoking a kind of mutual bootstrapping (Renfrew & Cherry 1987, title). This phenomenon resembles in many ways that of convection and might be modeled as an example of Bénard-like convection (see Chapter 7; Nicolis & Prigogine 1977; Prigogine & Stengers 1984). The peer polity/convection cell model is essentially one of increasing information flow in a local circuit, which has a differentiating and structuring effect on the inhabitants of the cell itself: center–periphery, town–hinterland. The regional and supraregional exchange that takes place is initially effectively stochastic (down the line).

As these cells grow, the cores come to interact more closely and boundary phenomena take over: neighboring cores begin to exchange information on a regular basis, i.e., no longer in a stochastic manner but directionally. In this intermediate phase, long distance exchange becomes hybrid, i.e., between cells it moves stochastically, but once it hits the periphery of a unit, it cannot but go to its center. This entails a major reduction in stochasticity of communication as well as the beginnings of opening up the cells. Once the flows are directional, the cells can become dependent on them; the time delays in communication are drastically reduced, and this enables them to play to each other's needs.

As more and more individuals participate in the (now) heterarchical channels, long-distance communication becomes more and more directional, meets more and more needs, and eventually connects very large spaces to such a degree that the centers become dependent on their trade networks. Importantly, the way the individual centers developed is highly dependent on minimal differences in initial conditions and on the paths they took. Guérin-Pace (1993) sketches this highly variable dynamic at the regional level within a full-grown urban structure. The crucial variable in the transition seems to be the degree of long-distance complementarity.

Eventually, the growth of these large heterarchical systems threatens stability and increases sluggishness in adapting to change. Some degree of separation of interactive spheres may be a response (city states?) as well as internal hierarchization (for example in the early development of Greek city states, in which oscillations took place between tyranny and democracy). The towns eventually become permanent heterarchical systems.

Summary and Conclusion

In this chapter I have tried to outline a trajectory from early egalitarian societies to heterarchical urban ones. In doing so, I have used a conceptual model to link known observations about intermediate stages of this development by assuming several important bifurcation points (transitions, tipping points) between the different states of the information processing system. But I have not discussed the last stage of this evolution, which has led to the current challenging sustainability predicament. That is dealt with in Chapters 15–18. Altogether, it needs to be emphasized that this has no other purpose than to propose a different way to view social evolution from an a priori perspective rather than the existing a posteriori one. Whether such an approach will in the long run help us deal with a number of the issues involved remains to be seen.

Appendix B

Modeling Urban–Rural Interaction

I present this in conjunction with Chapter 14 as a voluntary exercise for the reader, which can be skipped if desired. It is based on a model that James McGlade and I designed to explore the essentially metastable dynamics of regional information-processing structures in the context of the European Iron Age (van der Leeuw & McGlade 1997),[2] but work that I have since done on the urban–rural dynamic in modern Epirus (van der Leeuw 2000) and Ancient Maya (van der Leeuw 2014) confirms that it is also relevant for the transformation of rural Epirus settlements and the emergence of the large Maya centers, and I think it might be interesting for other areas of the world where cities emerged. But that is subject to testing. To illustrate how such a model can help us conceive of such a complex trajectory, I will try and summarize its main elements.

To begin with, we investigated the generic dynamics of urban impact on a rural environment, assuming that the rural environment is self-organizing, and that the dynamic can be described as a sigmoid growth process according to the following equation (Gallopin 1980, 240):

$$dR/dt = B(R-T)(K-R); R > 0 \qquad (14.1)$$

The important properties of such rural production can be summarized as (see Figure 14.1):

- Whenever R (the rural environment's production, represented by the black line in the figure) is above the upper asymptote K, for example at E, it tends to decrease and move toward that asymptote.

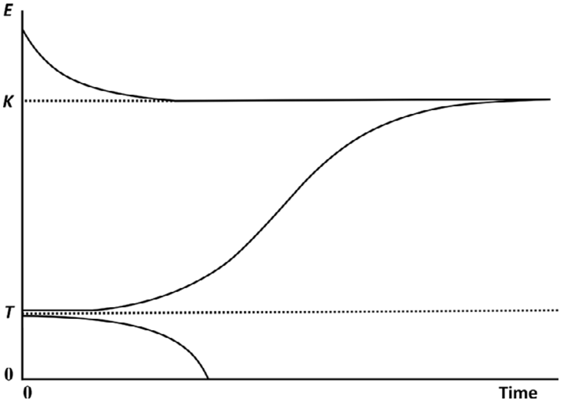

FIGURE 14.1 Generic time behavior of rural production according to the relationship $dR/dt = B(R - T)(K - R)\ ;\ R > 0\ (1)$, in which R = rural environment production (represented by the black line in the figure); T = lower threshold; K = upper asymptote; B = a positive growth function. (Source: van der Leeuw & McGlade 1997, by permission from Routledge)

- Whenever R is below K (but above the extinction threshold T), it converges on K in a sigmoid or logistic way.
- Whenever R is below T, it decreases to zero [R stops at zero because equation (14.1) is restricted to positive values of R].

Given this pattern of rural production, how will urban impact on the rural economy play out? This can be seen as a combination of urban development (U) and its rate of growth (dU/dt or U^*). The impact of urban growth on the rural environment can then be written as:

$$dR/dt(= U^*) = B(R - T)(K - R) + mU + ndU/dt; R > 0 \qquad (14.2)$$

in which both U and U^* are exogenous to the rural sector, and the relationship between U and U^* is not taken into account at this stage. The coefficients m and n indicate the sign and the strength of the unitary effect of urban development and its rate of growth on the rural environment, and can be considered as being composed of two factors, one accounting for negative, constraining effects, while the other accounts for positive, enhancing effects. Their sum gives the net effect: thus, $m = (y - g)$, $n = (e - v)$. One can then distinguish a number of effects of urbanism upon the rural sector, thus:

- m and/or $n = 0$: absence of a net effect of U and/or U^* upon R.
- m and/or $n < 0$: the net effect of urbanism (and/or its growth) is harmful, or exerts a negative effect on the rural environment.

- m and/or $n > 0$: The net effect of urban development (and/or its growth) is beneficial to the rural sector.

This equation's equilibrium values are not constant, but depend on the values of $\phi = mU + nU^*$. For different constant values of ϕ, the equilibria can be displaced (see Figure 14.2), where $\phi+$ and $\phi-$ indicate positive and negative values of ϕ respectively.

Additionally, when $\phi^+ > \beta TK$, the zero equilibrium becomes unstable ($dR/dt > 0$, and thus R begins to grow). For both ϕ^+ and ϕ^-, the upper equilibrium is stable, and the lower is unstable, if the equilibrium value of $R(R^*)$ is greater than $(K + T)/2$. For $\phi^- = -[(K - T)/2]^2\beta$, both equilibria collapse into one, such that R tends to move toward R^*, if $R > R^*$, but it tends to move away from R^* when $R < R^*$. Thus, for all practical purposes, this point is unstable. To the left of $\beta[(K + T)/2]^2$ there is no equilibrium, and the rate of change of R is always negative (R tends to go to zero for all values of ϕ^- lower than this).

The behavior of this system, particularly when the effect of urbanization (ϕ) can be assumed to change relatively slowly with respect to change in the rural sector (R), can be regarded as an example of Thom's (1989) fold catastrophe (see Zeeman 1979). This catastrophe exhibits three basic properties: bimodality (because of the double stable equilibria), discontinuity (catastrophic jump) and hysteresis (the path differs according to the direction of change).

Our analysis has, so far, assumed that the only effect of urban development on the rural environment was a negative one on rural subsistence production. It is represented in the model by the fact that the upper equilibrium value of the rural sector is significantly higher in the absence of urban development. But the influence of the urban sector need not necessarily be deleterious because the two sectors are largely symbiotic, particularly within the later prehistoric context: the rural production also has an impact on the urban one.

To examine this, we might look at the potential effects of urban development on T, the lower unstable threshold of the rural system. As we have noted earlier, T is a value of R such that if $R > T$, R tends to go to the upper stable equilibrium, and if $R < T$, R tends to go to zero. If T is very high, that means that rural production must be maintained at a high level, so as to avoid collapse (the highest possible value for T is when $T = K$, in which case the system collapses). $T = 0$ implies that rural subsistence production will tend to regenerate, even if R is pushed around zero values. Finally, increasing negative values of T affects the initial speed of growth of R when $R = 0$.

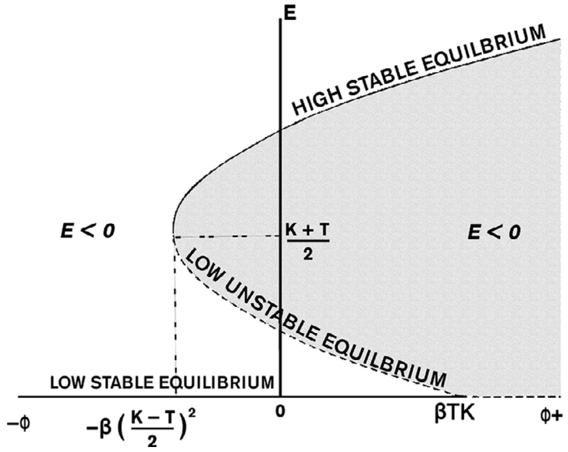

FIGURE 14.2 The rural environment equilibria represented as a function of $f = mU + nU^*$. For an explanation, see text. (Source: van der Leeuw & McGlade 1997, by permission from Routledge)

It is reasonable to infer for the model that different rural settlement systems might be characterized by different values of T. Those rural systems with an ability to recover rapidly from perturbation, such as relatively fast-growing settlements in the initial stages of colonization, should have a low value of T. By contrast, systems with a high T, such as systems whose persistence depends on management (agriculture), are probably characterized by a high degree of complexity and are relatively fragile and prone to collapse.

Another way in which urbanism can affect the persistence of the rural sector is by changing K, the upper stable equilibrium. When K is at its maximum, the rural production system is maintained at a peak of sustainability with a high rate of growth that is able to sustain major interaction with towns. Measures that are likely to modify K affect the maximum capacity of the rural environment, such as genetic improvement of crops (which increases K) or agricultural soil degradation through overuse, decreasing K.

Finally, urban development can alter the rural environmental system by affecting β, the parameter controlling rural production. Increasing β induces faster growth (or collapse) at all levels of urban development. The systems with higher ϕ can support greater levels of extraction or exploitation.

We have thus far treated urban development as a single parameter ϕ; however, remembering that $\phi = (y-g)U + (e-v)U^*$, the distinction between urban development, U, and its rate of growth, U^*, becomes important with respect to the viability of responses to rural overexploitation or potential collapse.

In summary, this model shows that sudden shifts from either center-based or rural-dominated landscapes to mixed settlement structures are a function of t. In this approach, long-distance trade is a factor in engendering unstable morphogenetic transitions. Let us now look at those dynamics in more detail.

Modeling Rural–Urban Interaction in a Regional System

Our assumption here is that slow gradual improvements in connectivity (e.g., the transportation network) have a discontinuous effect (Mees 1975). To model that, we start out with open, small, rural settlements in a localized landscape. The population of that area is initially assumed to be constant and divided between rural (p_r) and agglomerated (p_a). U_r and U_a are considered the utility levels of respectively the rural and the agglomerated populations, and t is the long-distance transportation cost of traded goods, and a proxy for communication. The level of t determines the size of the flow of trade goods and communication: if it is above a certain threshold all trade and communication is absent, but below that level, the lower the cost the larger the volume of trade. The demographic dynamics are represented by a utility maximizing migration function, given by:

$$dp_r/dt = p_r p_a (U_r - U_a) = -dp_a/dt \qquad (14.3)$$

The behavior of the system is illustrated in Figure 14.3. In the absence of trade and communication (t is high), E_m is the only stable equilibrium, representing a certain mix of rural and agglomerated population (Figure 14.3a). As the cost of trade and communication decreases, the system's dynamic can assume two forms (Figure 14.3b and c), depending on the overall population density, the average productivity of the region, and the agglomerated-rural area productivity difference.

With high population density and high productivity difference in favor of agglomerations, E_a is the stable equilibrium and the population is completely devoted to the products created in the agglomerations for long-distance trade. If the productivity difference is in favor of rural productivity, E_r is the equilibrium and rural products will be traded. We must conclude that this model relates any potential sudden shifts from

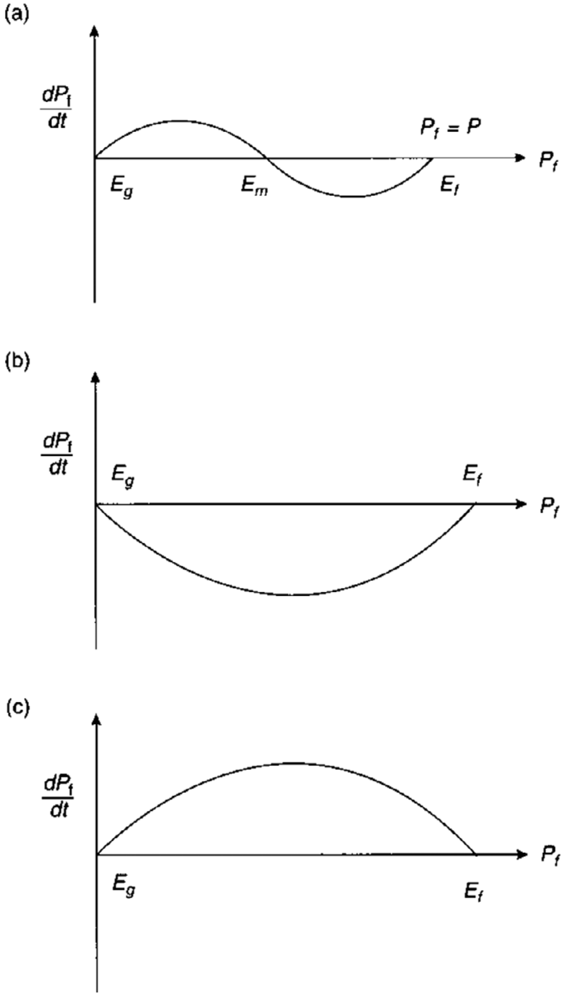

FIGURE 14.3 (a) Urban/rural equilibrium in the absence of long-distance trade; (b) center specializing in long-distance trade; (c) long-distance trade in a rural area. P_t is the urban population P_f the rural population; U_t is the urban and U_f the rural utility level; t is the long distance transportation cost. Above a certain threshold of t there is no trade (and very little, if any, information flow), but as t declines, trade and information flow increase. E is the equilibrium. (Source: van der Leeuw & McGlade 1997, by permission from Routledge)

either agglomeration dominated or rural-dominated landscapes to mixed settlements to the level of t.

With a high t, the only stable equilibrium is E_m, which is a mixture of urban–rural interaction. As t declines, interaction can take two forms as illustrated in Figure 14.3b and c, depending on: (1) the overall population density, (2) the average productivity, and (3) the productivity difference between urban and rural areas. In the case of Figure 14.3b, high population density and a high productivity differential has only E_g as a stable equilibrium. But when rural production for long-distance trade dominates, E_f is the stable equilibrium.

Concluding this qualitative analysis, we could state with some assurance that discontinuous and sudden shifts from either an urban-dominated or a rural-dominated regime to a mixed one are a function of the volume of trade, as it reflects the changes in the logistical networks enabling the connectivity.

If we now go a step further and look at how fluctuations in the rise of agglomerated central settlements are related to the interaction between faster and slower dynamics, we can do so by assuming (with Andersson 1986) a third order system of differential equations consisting of a fast one:

$$dY/dt = -T(Y^3/3 - rY - X) \tag{14.4}$$

and a slow one:

$$dX/dt = -T^{-1}Y \tag{14.5}$$

Here, r is a control parameter, and T an adjustment speed coefficient. Y is a center's production capacity, and X its access to a transportation and communication network. The model permits us to study discontinuous changes in centers' production as a function of access to the transportation/communication network. Figure 14.4 illustrates the relation between these two, where discontinuous changes in the value of urban productivity (Y) can be produced as a value of X (access to transportation and information). As the system's knowledge base (X) grows through access to information, it follows a trajectory in the L-zone of the system.

Assuming that abrupt changes in the dominance of individual centers are to be expected because of their unstable sociopolitical structures, these can be related to gradual changes in local resource accessibility. While the slow variable (productivity) is dominant, the fast variable can nevertheless flip the situation into another regime.

We might reasonably hypothesize that a form of network expansion through communication, trade, alliance, and even domination generated a slowly expanding system controlled from key locations occupied by the centers. In what follows, we see that network as a proxy for the information processing system. From Figure 14.4, we can see that as this

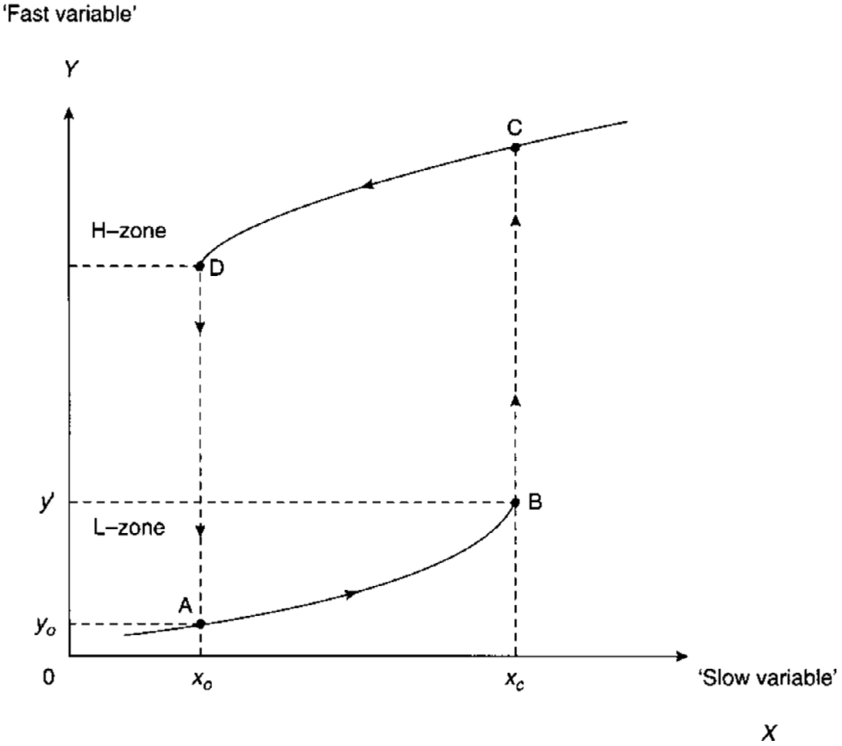

FIGURE 14.4 A generic cycle of fast and slow variables. *A* is the system's initial state; *X* is a slow variable symbolizing interactivity and knowledge; *Y* is a fast variable representing system evolution. In the *L*-zone, structures are stable; *B* is a threshold of change in urban productive activity (bifurcation point). In the *H*-zone, unstable structures emerge; at *D* the system returns to the initial state *A*. (Source: van der Leeuw & McGlade 1997, by permission from the publisher, Routledge)

network infrastructure and, consequently, the system's knowledge base (*X*) gradually grows, it follows the trajectory located in the *L* zone of the figure. The initial conditions of the system are given by *A*. As *X* changes, ultimately a point *B* is reached – a threshold beyond which the productive capacity of the urban system changes markedly. At this bifurcation point, the equilibrium loses its stability and a phase transition takes place. In this far from equilibrium phase (*H*-zone), the speed of change is determined by constraints on environment (natural resources), production techniques, and population (labor force). A prominent feature of this type of non-linear analysis is its cyclical nature. If, for example, the transportation/ communication network linking the centers is disrupted by other external competitive alliances or by warfare when the system is in the *H*-zone, unstable alliance structures are produced, and the system may follow the trajectory depicted on the *H*-zone until it converges on the initial state at *D*, and finally returns to the *L*-zone.

The role assumed by the critical points *B* and *D* needs to be further elaborated, so as to make the model dynamics easier to grasp. The essential underlying process is one of divergence, since a smooth but minor change in the transportation/communication network infrastructure can cause abrupt and unexpectedly large fluctuations in the equilibrium value of the production in certain locations. This (relatively sudden) phase transition takes place no matter how slowly the overall network capacity increases. By implication, the expansion of centers may simply be triggered by the addition of one small but important link in the network. Slight differences in transportation/communication conditions, for example as a result of changes in alliance structures, may eventuate major differences in the production capacity, if the centers' growth parameter finds itself at a critical point.

Modeling Instabilities in Inter-Regional Trade

This implies for example that the inherent instability in the communication and trading patterns of proto-urban centers in the phase in which they find themselves in the H-zone is largely a consequence of the interaction between slow and rapid dynamics. It has a major impact on the emergence of centers in a rural environment, as well as on the interaction between those centers and their rural environment once they have emerged. To conclude, I will briefly describe this, too, in modeling terms.

This model (based on McGlade 1990, 158) is not concerned with the fate of individual centers, but rather with a global regional dynamic. It assumes that there is initially limited interaction between centers in two regional systems involved in the limited export and import of specific high value goods, and denoted by X and Y. The dynamic system involved can be written in the following way:

$$dX/dt = F(X) - H(X) - X; \tag{14.6}$$

$$dY/dt = F(Y) + H(X) \tag{14.7}$$

where:

$$F(X) = rX(1 - X/N) + X^2 Y \tag{14.8}$$

$$F(Y) = -X^2 Y \tag{14.9}$$

$$H(X) = Q(K, L) - mK - C = \mathrm{Q_o K^m L^n} \qquad (14.10)$$

thus we have:

$$dX/dt = rX(1 - X/N) + X^2 Y - X[Q_o K^m L^n - mK - C] - X \qquad (14.11)$$

$$dY/dt = -X^2 Y + X[Q_o K^m L^n - mK - C] \qquad (14.12)$$

where r is the intrinsic rate of growth in commodity production; N is a production saturation level; Q is a measure of economic output; with Q_o as the initial value of Q; K is commodity stock; L is labor; m is the rate of commodity stock depreciation; C is consumption.

The production function *H(X)* is modeled as a nonlinear Cobb-Douglas function, with m as an exponential capital growth rate and n as an exponential accounting for the growth rate of labor. *H(X)* functions as an autocatalytic element in the system, effectively establishing the reaction-diffusion structure of the model.

Initially, region Y is seen as a major importer of prestige goods, with relatively little control of the trade routes. The $(+X^2 Y)$ term is essentially the status income accrued and exhibits strong self-reinforcing properties owing to the growing monopoly of the region in controlling trading transactions. The $(-X^2 Y)$ term represents constraints acting to prevent a total monopoly; it accounts for the loss of revenue as a result of the ability of the Y region to take part in alternative exchange systems. Additionally, the model assumes that the wealth of the X region – owing to its pre-eminence in trade – will grow as a logistic function over time, so long as the status quo is maintained, but will be reduced by any competing flow from region Y.

The steady state of this system, i.e. the state for which $dX/dt = dY/dt = 0$, corresponds to critical states X_o and $Y_o = F(X)/H(X) = (Q_o K^m L^n - mK - C)/rX\ (1 - X/N)$.

The critical transition point at which the system becomes unstable is given by:

$$H(X) > (1 + F(X\,)^2) = (Q_o K^m L^n - mK - C) > (1 + rX(i - X/N)^2 \qquad (14.13)$$

For example, when $F(X) = 1$, the critical point is unstable for $H(X) > 2$; as *H(X)* is increased, a Hopf bifurcation occurs, with the result that the system is attracted toward a limit cycle trajectory. Figures 13.5a–d show this behavior for increasing values of *H(X)*, since it is this function which controls the action/reaction nature of the trading system.

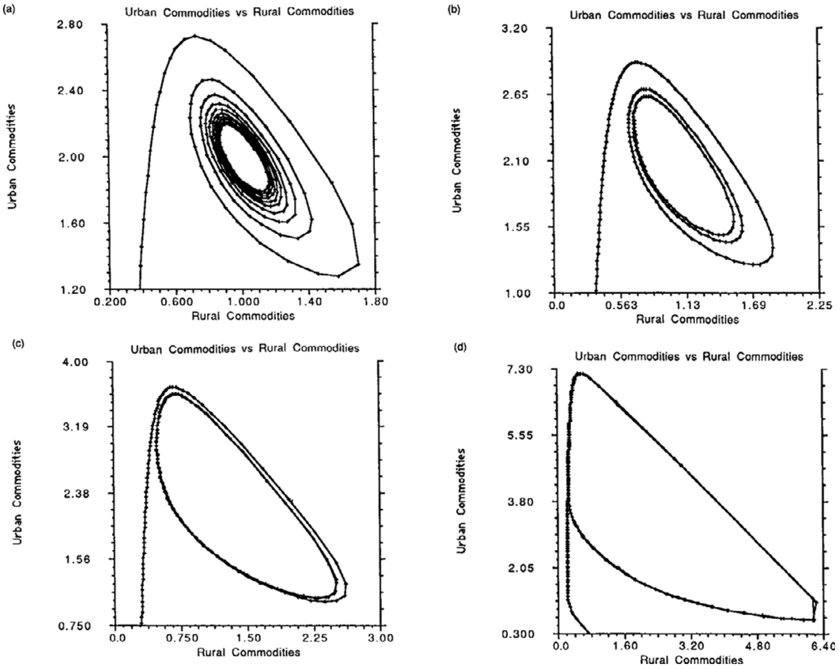

FIGURE 14.5 Simulation results of the center–rural environment interaction model (see text). (Source: van der Leeuw & McGlade 1997, by permission from the publisher, Routledge)

Instability is generated by purely endogenous factors, i.e., owing to the non-linearities in the system and their amplification by positive feedback mechanisms embedded in trade/exchange dynamics.

Trading systems are also subject to external fluctuations, for example owing to periodic increases in the volume of trade/exchange at particular times of the year. This we shall simulate by introducing a sinusoidal forcing term of amplitude a and frequency f. (see Tomita & Kai 1978). Thus, equation (13) becomes:

$$dX/dt = rX(1 - X/N) + X^2Y - X[Q_0K^mL^n - mK - C] - X + a\ \cos(ft) \tag{14.14}$$

Figures 14.6a–d show the results of such a perturbation, pushing the system progressively toward unstable orbits through a sequence of period-doubling bifurcations on the route to chaos.

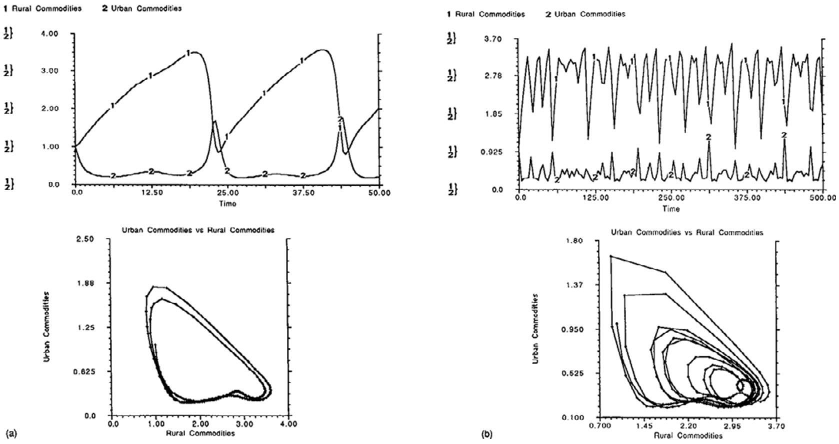

FIGURE 14.6 Another set of simulation results, showing how the system is slowly driven to chaotic behavior. (Source: van der Leeuw & McGlade 1997. Reproduced by permission from the publisher, Routledge)

Conclusion

These models are presented mainly to make the important point that many potential evolutionary pathways were open to such a system, given our stress on the twin concepts of nonlinearity and metastability, and their role in generating discontinuous evolution – particularly in logistical networks.

Modeling approaches such as these, based on open dissipative ideas can now be integrated into a new program of spatiotemporal dynamics that will demonstrate structural morphogenesis within urban and proto-urban settlement systems. More generally, what I have tried to demonstrate in this chapter is that archaeological approaches to questions of urban evolution have much to gain from an alignment with dynamical systems concepts – an alignment that is much more than metaphorical. Indeed, it is clear that the open, dissipative nature of urban/rural dynamics and their propensity to evolve through discontinuous transitions cannot be adequately understood by recourse to normative models; they require the combination of creative insight and experimental qualitative methods, which is the unique contribution of nonlinear dynamics. An interesting attempt to build further in this direction by modeling a number of the major transitions in societal systems is the recent volume edited by Sanders (2017), that looks at dynamic transitions from the Paleolithic to recent times in this manner.

Finally, I would like to stress again that none of these models pretends to represent reality, but they serve to help focus our minds on some of the issues concerned. Even if such a representation were theoretically possible (I do not think so), it is much too early to adopt one. For the moment, we are in that delightful phase of playing with hypotheses, and I have tried to show that there are wholly different games out there, which bring us some glimpses of insight that add to our understanding.

NOTES

1 This leads me to conclude that the size of the value space of a society must in some way or other be commensurate with the size of the overall group that is being aligned. This hypothesis has important implications for the present, and the difficulties encountered in and around globalization. I explore this in Chapter 17.

2 Not being a modeler myself, I owe a great debt of gratitude to James McGlade for his contributions to the original paper and this excerpt from it, reproduced by permission of Routledge.