BUMP MASSES AND RADII OF BL HERCULIS VARIABLES

Jørgen Otzen Petersen
Copenhagen University Observatory
Øster Voldgade 3, DK-1350 Copenhagen K, Denmark

ABSTRACT

Bump masses and radii are derived for 18 BL Her stars from the observed bump phase and the accurately known fundamental period. The mean mass $M/M_\odot = 0.60 \pm 0.09$ agrees precisely with predictions from standard stellar evolution theory and gives a new test of the theoretical models. The derived radius of V553 Centauri is in good agreement with the radius recently determined by an independent modified Baade-Wesselink method by Balona. Finally, a preliminary discussion of possible continuations of the BL Her bump progression is given.

1. INTRODUCTION

A characteristic feature in the light curves of many BL Herculis variables (≡ Population II Cepheids of periods from 1 to about 6 days) is the presence of a secondary bump (e.g. Payne-Gaposchkin, 1956; Stobie, 1973).

Secondary bumps have been found in several nonlinear pulsation calculations for models of classical Cepheids, starting with those of Christy (1966, 1968) and Stobie (1969a, 1969b). Using standard Cepheid models with homogeneous envelopes, phases of bumps in the observed light and velocity curves always give a bump mass near 60% of the evolution mass. The two-zone envelope models suggested by Cox et al. (1977, 1978) seem necessary to solve this problem, which has recently been reviewed by Cox (1980).

According to Christy and Stobie secondary bumps are associated with the echo phenomenon found in their pulsation models. Later, Simon and Schmidt (1976), proposed that bumps are due to a close resonance of the second overtone with the fundamental mode: $\Pi_2/\Pi_o \cong 0.50$. Variables with $\Pi_2/\Pi_o = 0.46 - 0.50$ show a bump during rising light, while period ratios 0.50 - 0.53 correspond to bumps on the falling part of the light curve. According to Cox (1980) theoretical pulsation models have shown

Space Science Reviews **27** (1980) 495–501. 0038–6308/80/0274–495 $01.05.

that the close resonance $\Pi_2/\Pi_0 \cong 0.50$ is always connected with secondary bumps. Hodson (1980) now also gets bumps as expected for BL Her models.

King et al. (1979) and Petersen (1980) showed that the observed properties of BL Her variables (including the bump progression) seem to be in good agreement with the predictions from standard stellar evolution models with homogeneous envelopes. In the present paper we derive masses and radii for 18 BL Her variables with reasonably secure bump features.

2. DETERMINATION OF BUMP MASSES AND RADII

In order to derive bump masses and radii we first calculate the second overtone to fundamental mode period ratio Π_2/Π_0 from the observed bump phase ϕ by the linear relation

$$\Pi_2/\Pi_0 = P_r + 0.04\ \phi/\phi_\ell \qquad (1)$$

P_r is the period ratio for bump phase 0.00 and ϕ_ℓ is the change in bump phase for a change of 0.04 in the period ratio. In rough agreement with Simon and Schmidt (1976) we adopt as standard values $P_r = 0.50$ and $\phi_\ell = 0.30$. Simon and Schmidt discussed the period ratio interval 0.46 - 0.53. Their analysis indicates that Eq. (1) is correct within ± 0.01. In the following we use Eq. (1) also somewhat outside the above mentioned interval, and in order to estimate possible uncertainties due to application of the approximate relation (1) we study the effects from relatively large changes in P_r and ϕ_ℓ.

Using the period ratio given by Eq. (1) and the precisely known fundamental period, mass and radius for each variable can be derived by methods that are well established from discussions of the double mode Cepheids of periods 2 - 7 days (e.g. Petersen, 1973). Here we use the fitting formulae for the pulsation parameters Q_0 and Q_2, constructed by Cox et al. (1972) for the extreme population I composition (X, Z) = (0.602, 0.044) Two equations with M and R as unknowns are solved as in Petersen (1973).

Searching the literature for well determined light curves of BL Her stars, we have been able to find 18 with a reasonably secure bump feature. Table 1 gives estimated bump phases and the derived bump masses and radii. In most cases it is possible to estimate bump phases within about ± 0.03. However, for the seven variables marked by a colon in coloumn 3 of Table 1, the bump phase is more uncertain due to uncommon features in the light curve or to insufficient observational material.

The data of Table 1 supplies a mean mass $M/M_\odot = 0.60 \pm 0.09$ in very good agreement with present stellar evolution theory. In post horizontal branch evolution stages all low mass stars are expected to have a mass $M/M_\odot$ = 0.5 - 0.7. The present bump masses yield a new test of the theoretical models.

https://doi.org/10.1017/S0252921100081975 Published online by Cambridge University Press

Table 1. Bump masses and radii of 18 BL Herculis variables of periods 1.1 - 3.2 days. Phase values refer to estimated phase differences between maximum light and the secondary bump: $\phi = \phi_{bump} - \phi_{max}$

Variable	Period Π_o (days)	Phase ϕ	References	Period ratio Π_2/Π_o	Data from bump $M_\phi/M_\odot$	$R_\phi/R_\odot$
WY CMa	1.144	0.20:	1	0.527	0.62	7.8
CE Her	1.209	0.20:	2	0.527	0.65	8.2
V527 Sgr	1.259	0.25	1, 3	0.533	0.73	8.9
BL Her	1.308	0.30	4	0.540	0.82	9.5
KZ Cen	1.520	0.00:	1	0.500	0.60	9.1
SW Tau	1.584	-0.15:	5	0.480	0.51	8.6
NW Lyr	1.601	-0.18	6	0.476	0.50	8.6
VZ Aql	1.668	-0.15	3	0.480	0.54	9.1
V839 Sgr	1.835	-0.15:	3	0.480	0.59	10.0
RT TrA	1.946	-0.35:	5	0.453	0.48	9.4
V553 Cen	2.061	-0.30	5	0.460	0.54	10.2
M13 6	2.113	-0.28	7	0.463	0.56	10.6
ωCen 61	2.274	-0.24:	7	0.468	0.64	11.7
XZ CMa	2.558	-0.32	1	0.457	0.64	12.4
BE Pup	2.871	-0.33	1	0.456	0.70	13.8
YZ CMa	3.157	-0.50	1	0.433	0.62	13.8
CM Pup	3.173	-0.62	1	0.417	0.54	12.9
BK Cen	3.174	-0.58	1	0.423	0.56	13.2

References to Table 1:

1. Payne - Gaposchkin (1956)
2. Nielsen (1940)
3. Kwee and Braun (1967)
4. Mitchell et al. (1964)
5. Dean et al. (1977)
6. Zessevich (1966)
7. Arp (1955)

3. UNCERTAINTIES IN BUMP MASSES AND RADII

Table 2 gives effects of relatively large changes in the assumed relation (1) between period ratio and bump phases. It is seen that a decrease in P_r (i.e. a decrease in the value of Π_2/Π_o used for derivation of bump properties) of 0.02 results in a decrease in the derived masses of ≅ 20%. From this information we now estimate uncertainties in the data of Table 1, taking into account the following three uncertainty sources: (i) observed bump phase, (ii) application of Eq.(1), and (iii) determination of mass and radius from Π_o and Π_2/Π_o.

https://doi.org/10.1017/S0252921100081975 Published online by Cambridge University Press

Table 2. Effects from changes in the parameters in the relation between period ratio and bump phase

Parameters in Eq. (1)		Mean mass	Bump radius R_ϕ (unit : $R_\odot$)	
P_r	ϕ_ℓ	(unit : $M_\odot$)	V553 Cen	SW Tau
0.50	0.3	0.60 ± 0.09	10.2	8.6
0.50	0.4	0.63 ± 0.07	10.7	8.8
0.48	0.3	0.49 ± 0.07	9.4	7.9
0.48	0.4	0.52 ± 0.06	9.8	8.1

As mentioned in Section 2 the observed bump phases can usually be estimated within ± 0.03. With $\phi_\ell \cong 0.3$ this corresponds to $\Delta\,(\Pi_2/\Pi_o) \cong \pm\, 0.004$, which gives $\sigma\,(M_\phi) \cong 4\%$. The discussion by Simon and Schmidt (1976) indicates that Eq. (1) is correct within ± 0.01 or better, which gives for contribution (ii) $\sigma\,(M_\phi) \lessapprox 10\%$.

Contribution (iii) from the application of the fitting formulae of Cox et al. (1972) is more difficult to evaluate. The formulae of Cox et al. are based upon models of extreme Population I composition (Z = 0.044) for classical Cepheids. BL Her stars have considerably higher L/M and R/M ratios, and although some BL Her variables seem to be "super metal rich" (Pop. I), others definitely belong to Pop. II. Stellingwerf (1975) compared data for Pop. II models (Z = 0.002) of mass $M/M_\odot \cong 0.6$ with the Cox et al. fitting formulae, and some of Stellingwerf's models have L/M and R/M ratios close to those of BL Her stars. He found that the main changes from Pop. I to Pop. II compositions are reductions in log Q_o by 0.014 and in log Q_2 by 0.0062. Thus Pop. II models have Π_2/Π_o about 0.009 (1.8%) larger than corresponding Pop. I models, and somewhat smaller masses are derived. Furthermore, King et al. (1979) found that Π_2/Π_o is not a unique function of log Π_o for models with same mass situated across the instability strip. This position effect corresponds to an additional uncertainty in Π_2/Π_o for a fixed Π_o, which we estimate to about $\Delta(\Pi_2/\Pi_o) \cong \pm\, 0.015$ or $\sigma\,(M_\phi) \cong 15\%$. Contribution (iii) is, therefore, about $\sigma\,(M_\phi) \cong 20\%$.

We conclude that the total uncertainty in a derived mass value in Table 1 is at most $\sigma\,(M_\phi) \cong 35\%$, and if the three contributions are uncorrelated not larger than 25%. From the pulsation equation we then find that the corresponding uncertainty in bump radii is 8 - 12%.

4. DISCUSSION

Table 2 gives radii of two BL Her stars with independent radius

https://doi.org/10.1017/S0252921100081975 Published online by Cambridge University Press

determinations by a modified Baade-Wesselink method. Balona (1977) gives for V553 Cen $R/R_\odot$ = 10.0 ± 0.2 (mean internal error), which is in perfect agreement with our best value $R_\phi/R_\odot$ = 10.2.

For SW Tau Stobie and Balona (1979) find $R/R_\odot$ = 12.8 ± 0.5, about 40% higher than our value $R_\phi/R_\odot$ = 8 - 9. Even taking uncertainties 10 - 15% in both methods into account these results disagree. A reasonable explanation for the discrepancy could be that our identification of a bump at $\phi \cong -0.15$ is wrong. SW Tau has a unique light curve among the BL Her stars (Stobie and Balona, 1979; Payne-Gaposchkin, 1956), which impedes the estimate of bump phase. However, also for assumed bump phases 0.00 to 0.15 too small radii are derived. If we take for granted that SW Tau is a Type II Cepheid of $M/M_\odot \cong 0.6$ oscillating with fundamental mode period 1.584 days, it is very difficult to explain a radius as large as $R/R_\odot \cong 13$.

An interesting speculation is that SW Tau is oscillating in the first overtone. With an assumed mass $M/M_\odot$ = 0.6 and Π_1 = 1.584 days we find $R/R_\odot \cong 11.8$ in reasonable agreement with the Wesselink radius. An argument for this interpretation is the striking similarity between the light curves of SW Tau and several RRc variables e.g. YZ Cap, BB Hyi, and AU Vir (Lub, 1977), which are known to be first overtone pulsators.

In Table 1 we have included five variables with periods 2.558 to 3.174 days, showing bumps that seem to continue the progression outside the interval in bump phase and period ratio discussed by Simon and Schmidt (1976). The fact that the masses derived for these variables are very reasonable suggests that the bump progression continues to at least $\phi \cong -0.60$ where $\Pi_2/\Pi_0 \cong 0.42$. Many W Vir stars with periods larger than 12 days show a more or less marked bump with phase ϕ = -0.6 to -0.8 (e.g. Payne-Gaposchkin, 1956). This also might indicate a continuation of the progression shown in Table 1. However, our method for determination of bump masses and radii cannot be extrapolated to W Vir stars.

Discussing bumps in RR Lyrae light curves, van Herk (1971) concluded that RRab variables usually show bumps at $\phi \cong 0.68$. This is confirmed by inspection of the very accurate light curves published by Lub (1977). The corresponding bump masses for RRab periods Π_0 = 0.4 - 0.7 days are $M_\phi/M_\odot$ = 0.5 - 0.7. These very reasonable masses indicate that the BL Her bump progression actually continues to at least $\phi \cong 0.7$. Data derived for individual RR Lyrae stars are, however rather uncertain for two reasons: The relationship between Π_2/Π_0 and ϕ for the RR Lyrae region has not yet been studied by nonlinear models, and there is no reason to believe that Eq.(1) can be used far from the resonance Π_2/Π_0 = 0.5. Furthermore, the application of the Cox et al. (1972) fitting formulae for derivation of masses and radii gives larger uncertainties at shorter periods.

We emphasize that the problem of the continuation of the bump progression outside the interval $-0.30 \lesssim \phi \lesssim 0.30$ should be studied more carefully before any conclusion is drawn. But if the continuation can be ascertained and bumps occur far from the resonance $\Pi_2/\Pi_0 = 0.5$, the resonance explanation for the existence of bumps seems in doubt. A bump progression through a large period interval can probably be easier understood in terms of the echo mechanism studied by Christy (1968) and Stobie (1969b).

Acknowledgement

I wish to thank A.N. Cox for providing unpublished results.

REFERENCES

Arp, H.C.: 1955 Astron. J. 60, p 1
Balona, L.A.: 1977, Monthly Notices Roy. astr. Soc. 178, p 231
Christy, R.F.: 1966, Astrophys. J. 145, p 340
Christy, R.F.: 1968, Quart. J. Roy. astr. Soc. 9, p 13
Cox, A.N.: 1980, to be published in Ann. Rev. Astr. Astrophys.
Cox, A.N., Deupree, R.G., King, D.S., Hodson, S.W.: 1977, Astrophys. J. Letters 214, L127
Cox, A.N., Michaud, G., Hodson, S.W.: 1978, Astrophys. J. 222, p 621
Cox, J.P., King, D.S., Stellingwerf, R.F.: 1972, Astrophys. J. 171, p 93
Dean, J.F., Cousins, A.W.J., Bywater, R.A., Warren, P.R.: 1977, Mem. Roy. astr. Soc. 83, p 69
van Herk, G.: 1971, Highlights of Astronomy 2, Ed. C. de Jager, D. Reidel, Publ., Dordrecht-Holland, p 781
Hodson, S.W.: 1980, private communication by A.N. Cox
King, D.S., Cox, A.N., Hodson, S.W.: 1979, Bull. American Astron. Soc. 11, p 730
Kwee, K.K., Braun, L.B.: 1967, Bull. Astr. Inst. Netherlands Suppl. 2, p 77
Lub, J.: 1977, Astron. Astrophys. Suppl. 29, p 345
Mitchell, R.I., Iriarte, B., Steinmetz, D., Johnson, H.L.: 1964, Bull. Tonantzintla y Tacubaya Obs. 3, p 153
Nielsen, A.V.: 1940, Medd. Ole Rømer Obs., Århus No. 15
Payne-Gaposchkin, C.: 1956, Vistas in Astronomy 2, p 1142
Petersen, J.O.: 1973, Astron. Astrophys. 27, p 89
Petersen, J.O.: 1980, to be published in Astron. Astrophys.
Simon, N.R., Schmidt, E.G.: 1976, Astrophys. J. 205, p 162
Stellingwerf, R.F.: 1975, Astrophys. J. 195, p 441
Stobie, R.S.: 1969a, Monthly Notices Roy. astr. Soc. 144, p 485
Stobie, R.S.: 1969b, Monthly Notices Roy. astr. Soc. 144, p 511
Stobie, R.S.: 1973, Observatory 93, p 111
Stobie, R.S., Balona, L.A.: 1979, Monthly Notices Roy. astr. Soc. 189, p 641
Zessevich, V.: 1966, Sky and Telescope 32, p 201

DISCUSSION

J. COX: I wonder if the longer period bump Cepheids have a relation to your periods. Probably the period ratio falls outside the range 0.46 - 0.53.

PETERSEN: Some of the BL Her stars I have discussed simply fall outside this range.

SIMON: I don't think it is very easy, even for classical Cepheids, to talk about bumps because they are often in the eye of the beholder. For BL Her stars, which I had occasion to look at after reading the Carson, Stothers, and Vemury paper, the observations are terrible. Even for BL Her in the Mitchell, et. al. catalogue, there are lots of uncertainties. I would urge great caution in trying to interpret the so-called bumps. The best light curve I saw is Pel's DY Eri. It looks like, and Fourier analyzes like a population I Cepheid of 3-4 days. At first I didn't realize it was a population II Cepheid. My own feeling is that we need much better observations before we can proceed.

PETERSEN: I certainly agree with you that we should be very cautious, and that these problems should be studied much more carefully. I don't think that you are right that the bump observations are very bad. There are several cases where bumps are very clearly shown in the light curves. You can find at least, say, 10 cases that look, at least to me, very secure.

SIMON. In BL Her itself there is a maximum, then there is something that looks like a shoulder, and then down toward the bottom of the light curve there is a feature that some people, at least Stothers, calls a bump.

PETERSEN: BL Her is only one out of 18 that I have taken.

SIMON: I think they are very poor.

LUB: How did you find the bump? Was it with respect to the light maximum, or to the radial velocity curve?

PETERSEN: I defined the bump phase as the difference in phase between light maximum and the bump feature.

https://doi.org/10.1017/S0252921100081975 Published online by Cambridge University Press