Problems of Solar Convection

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ABSTRACT. Kinetic energy of convection is transported inwards in the main body of convection zone. The temperature gradient becomes super-radiative at the top of the overshooting zone. These two effects make the solar equilibrium model much less sensitive to the assumed mixing length in Xiong's eddy diffusion theory. Observed brightening of downflow at high level in the surface region over intergranular lanes seems to be consistent with the overshooting model. Momentum transport by convective motion is shown to be crucial in pushing down magnetic flux tubes against buoyancy. Also, subadiabatic layers are possibly formed temporarily in the middle of the convection zone, exciting oscillations and generating chaotic motions.

1. Hydrodynamics of Solar Convection

Solar convection is a three dimensional turbulent motion on which rotation, circulation, and oscillations are superposed and in which magnetic fields are embedded. Equations governing the motion are well known, but the complete treatment of the equations are still very difficult. In numerical studies (e.g. Nordlund, 1978; Glatzmaier, 1987), we have to choose a finite mesh size below which eddy diffusivities should be assumed. Different approximations with proper size of the resolution are appropriate for different problems.

The local mixinglength theory (Vitense, 1953) assumes the representative eddy size to be of the order of the scale height and neglects structures below the mixing length. It is the most convenient theory for constructing equilibrium model. Justification of the theory, however, was given much later by Antia, Chitre, and Pandey (1981) and Narashimha and Antia (1982). They showed that eddies of larger horizontal size essentially occupy deeper levels, if eddy diffusivities are taken into account. However, sensitivity of the equilibrium model on the assumed mixing length makes the local mixinglength theory unsatisfactory. Xiong's (1979) nonlocal mixinglength theory (better be called as the eddy diffusion theory) is advisable for constructing solar convection zone model (Unno, Kondo and Xiong, 1985). Xiong's theory differs from Vitense's theory in that it includes eddy diffusive transport of kinetic and thermal energies and temperature-velocity correlations and in that it can treat

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time-dependent problems. The model is insensitive to the assumed mixinglength, and it agrees well with observations. The hydrodynamic structure of the models has been studied by Unno and Kondo (1989). There are two reasons for the insensitivity of the model to the assumed mixing length. One is the structure of the overshooting layer in which the upper part shows larger temperature gradient than the radiative one (super-radiative) because the convective energy flux is negative (downward). The other reason is that the kinetic energy transport is directed downward in the main body of convective zone. Both of these effects require larger efficiency of convective transport (large Nusselt number) and therefore larger mixing length. Since the structure of the model saturates to the adiabatic one with the increase of mixing length, the largest possible length of the order of the distance to edge of the convection zone should be chosen. The characteristics of overshooting convection and the role of kinetic energy transport will be discussed in the following sections.

2. Structure and Dynamics of Overshooting Convection

Suemoto, Hiei, and Nakagomi (1987; 1989) observed brightness variations in continuum in the CaII K line wavelength region in its outer wing($|\Delta\lambda| > 3\mathring{A}$) and inner wing ($3\mathring{A} > |\Delta\lambda| > 0.5\mathring{A}$) for granules and intergranule space. They found that excess brightness of granules (hot layer) does not extend beyond $3\mathring{A} = |\Delta\lambda|$ or $\tau = 0.3$ and there are hot regions at high levels ($\tau < 0.1$) in intergranular lanes. The convective flux is downward for $\tau < 0.3$ as expected from Xiong's theory. The hydrodynamics of high levels of intergranular lanes seems to resemble that of plate-tectonics in which high density plates sink liberating gravitational energy through stable stratification. A similar situation was discussed by Ribes and Unno (1986) to interpret facular brightness by the downdraft along the magnetic flux tube. There are also some other problems associated with the convective overshooting. The layer is important not only for providing boundary conditions to hydrodynamics of deeper layers but also for exciting large scale motions. Kondo and Unno (1982; 1983) studied the penetrative convection for idealized two layer models composed of convectively unstable and stable layers (represented by subscripts 1 and 2, respectively, in Figure 1) with different constant viscosity and conductivity in each layer. The critical Rayleigh number defined by the unstable layer is greatly reduced from the usual value of the order of 10^3 for rigid and free boundaries of fixed temperature, if the viscosity of stable layer is very small. The penetration depth into weakly stable layer (small β_2) is large for eddies with large horizontal size, reducing viscous dissipation. With significant thermal conductivity, temperature fluctuations are smoothed out in up and downdrafts, giving rise to overstability of long period. No overstability of supergranulation has been reported, perhaps because upper layer is very stable. The physics of penetrative convection may better be applied to the main body of convective zone below the hydrogen ionization zone, as will be discussed later.

Now, coming back to the problem of kinetic energy transport, we should estimate how large the effect should be. With the help of the gradient diffusion approximation, the kinetic energy flux, $F_c^{(K)}$, is estimated to be

$$F_c^{(K)} = \frac{1}{2}\overline{\rho v^2 v_z} = \frac{1}{2}\overline{\rho (v^2)' v_z} = -\frac{\rho\kappa_e}{2}\frac{\partial \overline{v^2}}{\partial z}, \tag{1}$$

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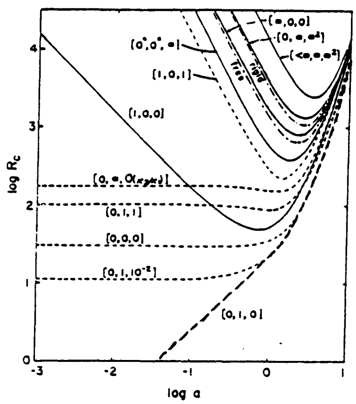

Fig. 1: The critical Rayleigh number R_c, ($R \equiv \alpha\beta_1 g z_1^4/\kappa_1\nu_1$), for various values of $[\nu_2/\nu_1,\ \kappa_2/\kappa_1,\ |\beta_2/\beta_1|]$, as functions of horizontal wavenumber a. Dashed curves represent cases of overstability, where Re$\lambda = 0$ and Im$\lambda \neq 0$, and solid curves the case of exchange of stability. Note low R_c values for $\nu_2 = 0$ and $\beta_2 = 0$ (Kondo and Unno, 1983).

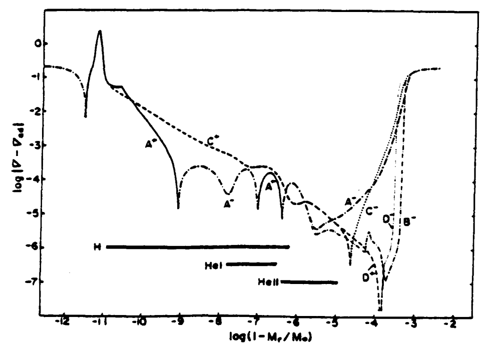

Fig. 2: $(\nabla - \nabla_{ad})$ in the solar convection zone. Case A and B (C and D) are for longer (shorter) mixing length but differ in the lower boundary conditions. A^- etc. show the internal subadiabatic layers (Unno and Kondo, 1989).

where the eddy conductivity is estimated to be

$$\kappa_e = 0.25(\overline{v^2})^{1/2} l_e, \tag{2}$$

(Nakano et. al, 1979), while the thermal energy flux, $F_c^{(th)}$, is given by

$$F_c^{(th)} = \overline{\rho W v_z} = \overline{\rho W' v_z} = \overline{c_p T' v_z} = \alpha^{-1}\gamma(\gamma-1)^{-1}\overline{\rho(v^2)^{3/2}}. \tag{3}$$

We obtain the ratio, $F_c^{(K)}/F_c^{(th)}$, to be (Unno and Kondo, 1989)

$$\frac{F_c^{(K)}}{F_c^{(th)}} = -0.02\alpha^2 \tag{4}$$

where α denotes the mixing length-scale height ratio. Numerical experiments of highly nonlinear compressible convection with a fixed flux boundary condition by Hurlbert et al. (1984), by Yamaguchi (1985), and by Chan and Sofia (1986) give the ratio to be negative and of some 4–8%. The value increases with increasing mixing length (Roxburg, 1987). In Unno and Kondo (1989), the ratio amounts up to some 20% in the lower part of the convection zone. The effective α value adopted may have been a little too large.

The inward flux of the kinetic energy in the result of continuity with faster speed in a narrower downdraft area and slower speed in a wider updraft area. In the gradient diffusion approximation, a faster downdraft is the result of larger velocity fluctuations in the parent region lying one mixing length above. In the same context, therefore, the confinement of magnetic tubes in the convection zone can be discussed by means of net momentum transport of anisotropic turbulence. The average vertical momentum transport per unit horizontal area $\overline{\rho v_z v_z}$ can be estimated in the gradient approximation as

$$\overline{\rho v_z v_z} = -\overline{\rho}\nu_e \frac{\partial(\overline{v^2})^{1/2}}{\partial z} = -\frac{\alpha}{5R_e}\overline{\rho v^2}, \quad (5)$$

where the effective Reynolds number R_e may be taken to be 10. A magnetic flux tube of the field strength B and the thickness d receives magnetic buoyancy balancing the above momentum transport and satisfies the following relation in equilibrium,

$$\frac{B^2}{8\pi} = \frac{1}{5R_e}\frac{\ell_e}{d}\overline{\rho v^2}. \quad (6)$$

If the field is stronger or the thickness is larger, the flux tube is lifted up. A thin weak flux tube will be pushed downward and finally find a rest position, flattened, if $Bd = const.$. Flat tubes will be piled up, forming a thick tube which will then expand or float to upper layers. At a depth of 1.5×10^{10}cm from the surface where $\rho = 7 \times 10^{-2}$gcm^{-3} and $v^2 = 6 \times 10^7$cm^{2}s^{-2}, the equilibrium field of $1.5 \times 10^3 G$ is given by equation (5) for a flux tube of $d = \ell_e$. Thus, the momentum transport by convection is an important factor in the dynamo mechanism.

3. Subadiabatic Layer in Convection Zone

With a large mixing length, subadiabatic layers appears in the middle of convection zone (Figure 2). It may be the result of too large mixing length assumed. However, such a stability reversal is seen in the Boussinesq convection for large Rayleigh numbers (Herring, 1963). Urata (1986) made numerical simulation of three-dimensional time-dependent Boussinesq convection with Rayleigh number of 2×10^4 and the horizontal aspect ratio of 2. Starting with the linear Benard cell pattern, the motion soon changes the mean temperature profile to be negative (subadabatic) in the central portion sandwiched with steep conductive temperature-gradient regions on both upper and lower edges. Then, the convective flux becomes oscillatory with the frequency of gravity wave mode standing in the stable stratification. The oscillation becomes much reduced after some time, Then, chaotic variations start to grow to large amplitudes. The chaotic variation are initiated with the growth of a new mode which shows large vertical vorticity without any initially imposed angular momentum to the system, a mode which was first found by Lopez and Murphy (1986). Coupling of three kinds of motion, Benard convection, gravity wave, and horizontal vorticity seems to be the cause of chaotic variations (Figure 3a and 3b). The subadiabatic layer in the middle of the solar convection zone, if it happens even temporarily, may be responsible for some of the observed oscillations or chaotic variations. Vertical vorticity mode should be important to convert toroidal field into poloidal field. It is interesting to note that vertical vorticity as well as sink and source patterns are visible in the SOUP flow vectors (Simon et al., 1988).

4. Nonlinear Magnetoconvection

Nonlinear magnetoconvection was studied and applied to solar convection problems by Rudraiah, Kumudini, and Unno (1985). It is found that turbulent magnetoconvection

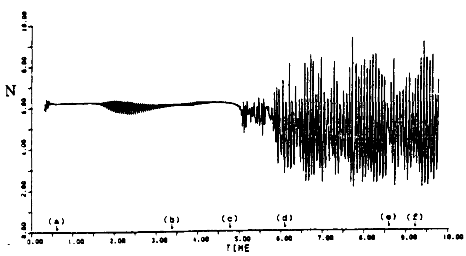

Fig. 3a: Time series of horizontally averaged total heat flux for $Pr = 1$ (Urata, 1987).

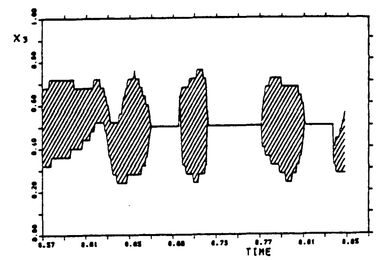

Fig. 3b: Variations of temporary stable (hatched) layer around the phase (e) in Fig. 3a (Urata, 1987).

is characterized by the effective Rayleigh number of about 4 times the critical Rayleigh number and by the effective Nusselt number of about 2. The amplification of the field strength is estimated to be of factor 5 in each cell ($\sim$ one scale height) so that e.g. 50G fields at the bottom of the convection zone could be amplified to a few thousand gauss until the magnetic flux tube reaches the surface layer. Also, darkness of sunspots can be interpreted by the nonlinear magnetoconvection with the effective Nusselt number of about 2. Another important effect of convection is that faster speed of down flow than upflow pushes down a magnetic tube very effectively, as discussed before. A flux tube can stay in the convection zone long enough for dynamo mechanism to work.

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DISCUSSION

WEISS: Recently there have been some impressive numerical simulations of three-dimensional compressible convection (Nordlund & Stein; Brummell, Cattaneo, Hurlburt & Toomre) which also show a downward kinetic energy flux associated with rapidly sinking plumes. Do you think it will be possible to calibrate mixing-length type theories by comparison with such models run on supercomputers?

UNNO: One way is to compare the kinetic energy transport to the total convective flux.

DALSGAARD: The principal quantity, as far as solar structure is concerned, obtained from mixing length theory is the entropy jump across the superadiabatic part of the convection zone. It is interesting that the detailed calculations give roughly the same value as obtained from mixing length theory, once this is calibrated to obtain the correct solar radius.

UNNO: However, use of the solar radius as a calibration is not entirely satisfactory from a theoretical point of view.

MOGILEVSKIJ: What is the difference in convection between active and quiet regions?

UNNO: The active-region magnetic field (except for sunspots) has practically no influence on large eddies like supergranule cells. Granules are smaller in size and elongated vertically by a magnetic field. For the strong field of a sunspot, heat transport is much reduced, if the temperature is fixed at the bottom of a convection cell.