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On the Evolution of the Nova-like Variable AE Aquarii

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Abstract. It is shown here that the peculiar properties of AE Aqr can can be accounted for if the mass transfer from an evolved $0.7M_{\odot}$ secondary K4-5 star ($q_i \approx 0.8$, *i.e.* < 1) initiated when the orbital period of the binary was $P_{orb,i} \approx 8.5$ hours and the white dwarf period $P_{*,i} \approx 1$ hour. This resulted in a significant amount of orbital angular momentum being accreted by the white dwarf in an initial discless spin-up phase towards $P_{*} \approx 0.1 P_{orb,i}$. This destabilized the mass transfer, resulting in a run-away mass transfer from the secondary that lasted for approximately 10^4 years, with the orbital period evolving to $P_{orb} \approx 11$ hours until a critical mass ratio of $q_{crit} = 0.73$ had been reached. In this phase the mass transfer from the secondary occurred at a rapid rate of approximately $\dot{M}_2 \approx 10^{20}$ g s^{-1}, resulting in an accretion disc which spun-up the white dwarf to a period of approximately $P_{*} \approx 33$ s. For all $q \leq q_{crit} = 0.73$ the mass transfer proceeded on the thermal time scale of the secondary star, *i.e.* at a much slower rate, resulting in the binary converging ($\frac{\dot{a}}{a} < 0$) and forcing AE Aqr into the propeller phase. Applying stellar wind theory, this allow an estimate of the polar magnetic field of the secondary star, which is of the order of $B_{\circ} \approx (1600 - 2000)$ G. It has been shown here that the duration of mass transfer phase $q = q_{crit} \rightarrow 0.67$ (now) lasted for approximately $t_{\dot{M}_2} \sim 10^7$ years, similar to the spin-down time scale of the white dwarf, $t_{sd} = P_{*}/\dot{P}_{*} \approx 10^7$ years. The propeller ejection of matter in the current phase results in the dissipation of mhd power of $L_{mhd} \approx 10^{34}$ erg s^{-1}, probably channeled into mass ejection and non-thermal activity. This explains the non-thermal outbursts that are observed in radio wavelengths, and occasionally also in TeV energies, from AE Aqr.

1. Introduction

Since the nova-like variable AE Aqr resembled DQ Herculis in earlier optical and X-ray studies (eg. Patterson 1979), it was classified as a DQ Her type cataclysmic variable. The system consists of a 0.6 $M_{\odot}$, K4-5 secondary star, and a 0.9 $M_{\odot}$ white dwarf ($q = (M_2/M_1) = 0.67$) orbiting their common center of mass with a period of $P_{orb} = 9.88$ hours. Faint $P_{*} = 33.08$ s pulsations in the optical are associated with the spin period of the white dwarf (Patterson 1979). This implies that AE Aqr has the fastest rotating accreting white dwarf known to date. A long term analysis of the spin period of the white dwarf in AE Aqr (de Jager et al. 1994) shows that it is spinning down at a rate of $\dot{P} = 5.6\times$

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10^{-14} s s^{-1}, resulting in an inferred spin-down power of $P_{sd} \approx 10^{34}$ erg s^{-1}. The system shows rapid flaring in optical (Patterson 1979) with the optical intensity varying between $m_v \sim 12 - 10$ on a regular basis, initially believed to be the result of enhanced mass accretion onto the magnetic poles of the white dwarf. However, there is a peculiarly weak correlation between the amplitude of the 33 s oscillation and increased intensity during optical outbursts (Patterson 1979), casting doubt whether enhanced mass accretion onto the poles is the source of the optical outbursts. Circular polarization at the level of (0.05 ± 0.01) % (Cropper 1986) was reported in the optical light, which, if produced by cyclotron emission, may indicate a magnetic field in excess of $B_* \sim 10^6$ G (Chanmugam & Frank 1987). It has been shown recently (Patterson 1994; Eracleous & Horne 1996; Wynn, King & Horne 1997; Meintjes & de Jager 2000) that the fast rotating, highly magnetized white dwarf will act as a propeller, ejecting the inferred blob-like mass transfer flow of $\dot{M}_2 \sim (0.5 - 1.0) \times 10^{18}$ g s^{-1} (Eracleous & Horne 1996) from the system. Eracleous & Horne (1996) also showed that radiative cooling of ejected gas blobs is most probably the source of the optical outbursts in AE Aqr, explaining the weak correlation between optical flaring and the amplitude of the 33.08 s optical oscillations during flares.

Understanding the evolution of AE Aqr may put this enigmatic source in context with respect to other intermediate polars. Therefore, a model is proposed here which accounts for its unique properties for the secular evolution of this system.

2. Secular evolution of AE Aqr

It is proposed (also Meintjes 2002) that mass transfer was initiated when the $M_{2,i} \approx 0.7\ M_\odot$ secondary K4-5 star evolved into contact with its Roche surface, filling it when the binary period was of the order of $P_{orb,i} \approx 8.5$ hours. It is also proposed that the mass ratio of the system, when mass transfer initiated, was of the order of $q_i \approx (M_{2,i}/M_{1,i}) \approx 0.8 (< 1)$ (*i.e.* $M_{1,i} \approx 0.87 M_\odot$). The initial rotation period of the white dwarf was also probably of the order of $P_{*,i} = 1$ hour (Wynn, King & Horne 1997; Meintjes 2002).

Evolved K-type stars with mass $M < 1 M_\odot$ most probably have convective envelopes (King 1988, Campbell 1997), resulting in an expansion of the star upon mass loss (King 1988). Hence, the initiation of mass transfer in AE Aqr resulted in an expansion of the secondary star, accompanied by high orbital angular momentum drainage from the system as the slowly rotating white dwarf accreted (discless) nearly all the orbital angular momentum lost by the secondary across the L_1 region while spinning up to $P_* \rightarrow 0.1 P_{orb,i}$ (King & Lasota 1991). This resulted in the secondary star flooding its Roche surface, leading to run-away mass transfer to the white dwarf. The instantaneous mass transfer across L_1 in this run-away phase is obtained by calculating the mass flux through $L_1 (\rho_{L_1} c_s)$ that crosses an area $(\pi (c_s/\Omega_{orb})^2)$, resulting in (Frank, King & Raine 1992)

$$-\dot{M}_2 \approx \frac{1}{4\pi} \rho_{L_1} c_s^3 P_{orb}^2 . \quad (1)$$

If it is assumed that the density of the gas flow through L_1 is of the same order of magnitude than the average photospheric density ($\rho_{L_1} \approx \rho_{phot} \sim 10^{-6}$ g cm^{-3}),

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one can show that the mass transfer could have reached values of

$$-\dot{M}_2 \approx 10^{20} \left(\frac{\rho}{\rho_{L_1}}\right) \left(\frac{T_*}{4000\,\mathrm{K}}\right)^{3/2} \left(\frac{P_{orb,i}}{8.5\,\mathrm{hr}}\right)^2 \mathrm{g\ s^{-1}} \quad (2)$$

which is approximately $\sim 0.2\dot{M}_{Edd}$. This mass transfer rate is several orders of magnitude larger than the angular momentum losses through magnetic braking, orbital angular momentum accretion by the white dwarf, and possibly mass loss from the system (Meintjes 2002), *i.e.* $\dot{M}_2/M_2 >> \dot{J}_{orb}/J_{orb}$. This results in an $\dot{a}/a > 0$ evolution in an attempt to conserve the orbital angular momentum as more matter is transferred closer to the center of mass of the system. The run-away mass tranfer lasted until a critical mass ratio, $q_{crit} = 0.73$ (Meintjes 2002) was reached. Meintjes (2002) showed that the high mass transfer phase from an initial q_i $(= 0.8) \rightarrow q_{crit}$ $(= 0.73)$ lasted for approximately $t_{\dot{M}_2} \leq 2.8 \times 10^4$ years. The evolution of the system in the high mass transfer phase is conservative since the high mass transfer is sufficient to develop an accretion disc in the system. By inverting the discless accretion argument (Warner 1995) during the initiation of the high mass transfer phase, an accretion disc will develop if the magnetic moment of the white dwarf in AE Aqr does not exceed

$$\mu_{32} \leq 3 \left(\frac{P_{orb,i}}{8.5\,\mathrm{h}}\right)^{7/6} \left(\frac{M_1}{0.9\,M_\odot}\right)^{5/6} \left(\frac{-\dot{M}_{2,i}}{10^{20}\,\mathrm{g\ s^{-1}}}\right)^{1/2} \mathrm{G\ cm^3}. \quad (3)$$

However, the magnetic moment of the white dwarf in AE Aqr is probably of the order of ($\mu_{32} \sim 1 - 2$), in agreement with the magnetic moments of the other DQ Her stars, satisfying the condition above. This allows an independent constraint on the surface magnetic field strength of the white dwarf ($B_* \sim \mu/R_*^3$). By adopting the white dwarf mass-radius relation, we get for $R_* \sim 5 \times 10^8\,(M_*/0.9\,M_\odot)^{-0.8}$ cm (see also Eracleous & Horne 1996) a white dwarf surface field of

$$B_* \leq 2.4 \times 10^6 \left(\frac{\mu_{32}}{3}\right) \left(\frac{R}{R_*}\right)^{-3} \mathrm{G} \quad (4)$$

which is reconcilable with an inferred magnetic field strength of $B_* \sim 10^6$ G, deduced from circular polarization measurements (Chanmugam & Frank 1987). An independent estimate of the maximum initial mass transfer and subsequent accretion rate ($\dot{M}_{1,i}$) during the the run-away phase, can be obtained in the slow-rotator limit (Wang 1987). For

$$x_{o,i} \sim \left(\frac{P_{o,i}}{P_{*,i}}\right)^{2/3} \leq 0.01 \left(\frac{P_{o,i}}{P_{eq}}\right)^{2/3} \left(\frac{P_{*,i}}{3600\,\mathrm{s}}\right)^{-2/3} \quad (5)$$

where $P_{o,i} > P_{eq}$ (Wang 1987), and the initial rotation period of the white dwarf, $P_{*,i} \geq 1$ hour (Wynn, King & Horne 1997), this results in

$$\dot{M}_{1,i} \leq 2 \times 10^{20} \left(\frac{M_1}{0.9\,M_\odot}\right)^{\frac{-35}{24}} \left(\frac{\gamma}{\xi}\right)^{\frac{5}{8}} \alpha^{-9/32} \left(\frac{\eta}{0.2}\right)^{\frac{15}{8}} \left(\frac{P_{*,i}}{1\,\mathrm{h}}\right)^{\frac{-211}{96}} \mathrm{g\ s^{-1}} \quad (6)$$

which is still below the Eddington value, *i.e.* $\dot{M}_{1,i} \sim -\dot{M}_{2,i} \sim 0.4\,\dot{M}_{Edd}$. It can be shown that the white dwarf can be spun-up to a period of ~ 32.9 s (de Jager et al. 1994) in a timescale

$$t_{s-u} \approx 1.4 \times 10^4 \left(\frac{M_1}{0.9\,M_\odot}\right)^{1/3} \left(\frac{P_*}{32.9\,\mathrm{s}}\right)^{-4/3} \left(\frac{\dot{M}_1}{\dot{M}_{1,i}}\right)^{-1} \left(\frac{f(x_\circ)_{av}}{0.5}\right)^{-1} \mathrm{yr} \quad (7)$$

which pins the duration of the high mass transfer phase down, for $-\dot{M}_{2,i} \sim (0.2-0.4)\dot{M}_{Edd}$, to somewhere between $t_{\dot{M}} \sim (1.4-2.8) \times 10^4$ yr.

It is evident that the binary will expand ($\dot{a}/a > 0$) for $q \leq 1$, if the mass transfer rate from the secondary to the compact primary dominates over the rate at which angular momentum is drained from the binary. In the idealistic case of fully conservative mass transfer, *i.e.* when $\dot{J} = 0$, the rate at which the binary expands will obviously be a maximum. It is easy to show that for conservative mass transfer (in run-away phase), the binary orbital period evolution will follow (Verbunt 1993)

$$\frac{P_f}{P_i} = \left(\frac{M_{1,i}M_{2,i}}{M_{1,f}M_{2,f}}\right)^3 \quad (8)$$

which results in

$$P_{orb} \approx 11 \left(\frac{q_i}{0.8}\right)^3 \left(\frac{q_{crit}}{0.73}\right)^{-3} \left(\frac{P_{orb,i}}{8.5\,\mathrm{hr}}\right) \mathrm{hr}\,. \quad (9)$$

The violent mass transfer and accretion in this phase may have triggered several nova explosions, unless the mass accretion was above the critical value for stable nuclear burning on the surface of the white dwarf (Wynn 2002, personal communication). The critical mass accretion rate above which stable nuclear burning occurs is $\dot{M}_1 \approx 10^{19}(M_1/0.9\,M_\odot)^{3/2}$ g s^{-1} (Warner 1995). This means that stable nuclear burning could have occurred on the surface of the white dwarf in the high mass transfer, disc accreting phase. Therefore, during this phase AE Aqr could have turned into an ultrasoft X-ray source, like GQ Mus recently (Warner 1995).

For $q \leq q_{crit}$ the mass transfer is driven by magnetic braking of the secondary star. During this phase the Roche lobe of the secondary star may become detached from the secondary star's surface and further mass transfer in the system must be driven by angular momentum losses such as magnetic braking of the secondary star on the thermal timescale. It was shown (Meintjes 2002) that in this phase the mass transfer rate is such that $-\dot{M}_2 < 2 \times 10^{18}$ g s^{-1}, consistent with mass transfer inferred from observations. Meintjes (2002) showed that the spun-up ($P_* \sim 33$ s) magnetic white dwarf, experiencing a factor $\sim 100-1000$ decrease in the mass transfer rate when the binary evolves into the $q \leq q_{crit}$ phase, will act as a propeller. The propeller action spins down the white dwarf as matter is ejected from the binary. It has been shown (de Jager et al. 1994) that the spin-down power, inferred from the observed current spin-down rate of the white dwarf, is of the order $I\Omega\dot{\Omega} = 6 \times 10^{33}$ erg s^{-1}, which is several orders of magnitude higher than the accretion-induced UV luminosity $L_{UV} \sim 10^{31}$

erg s^{-1}, inferred from the *HST* data. This is in agreement with the propeller scenario. It can be shown that the power in mass ejection is

$$L_{mech} \approx \frac{1}{2}\dot{M}_{2,eject}v_{eject}^2 \tag{10}$$

and for $-\dot{M}_{2,eject} \geq 4 \times 10^{17}$ g s^{-1} (Eracleous & Horne 1996) and $v_{eject} = v_{esc}$ ~ 1550 km s^{-1} (Wynn, King & Horne 1997) this gives

$$L_{mech} \approx 5 \times 10^{33} \left(\frac{-\dot{M}_{2,eject}}{4 \times 10^{17}\,\mathrm{g\,s^{-1}}}\right)\left(\frac{v_{eject}}{v_{esc}}\right)^2 \mathrm{erg\,s^{-1}}\,. \tag{11}$$

This shows that a significant fraction of the spin down power goes into the supply of mechanical energy to the ejected matter, *i.e.* $I\Omega_*\dot{\Omega}_* \sim L_{mech}$, which was also pointed out by Wynn, King & Horne (1997). This explains the discrepancy between the spin-down power and the radiative luminosities observed. Another attractive feature of the ejector mechanism is that radiative cooling of expelled gas blobs is sufficient to explain the thermal outbursts from AE Aqr (Eracleous & Horne 1996). It can also explain why the amplitude of the 33 s optical, UV and X-ray oscillations is not dependant on the increasing intensity during flares (Eracleous & Horne 1996). It was further pointed out (Meintjes & de Jager 2000) that the spin-down power also provides a huge reservoir of drained rotational kinetic energy to drive, through various magnetospheric processes, the non-thermal radio synchrotron and TeV gamma-ray emission observed from this system.

3. The magnetic secondary star

Meintjes (2002) showed that stable mass transfer in the converging phase of AE Aqr's evolution must satisfy the condition (Wynn & King 1995)

$$\frac{-\dot{J}_{mb}}{J} \geq \left[1 - q(1-\alpha) - \frac{\alpha}{2}\left(\frac{M_2}{M}\right) - \eta\left(\frac{M}{M_1}\right)^{1/2}\left(\frac{R_{circ}}{a}\right)^{1/2}\right]\left(\frac{-\dot{M}_2}{M_2}\right) \tag{12}$$

where $\eta \approx 1$ and $\alpha \approx 1$ in the current propeller phase of AE Aqr (Wynn & King 1995). By adopting the Mestel & Spruit (1987) stellar wind model of magnetic braking in the fast rotator limit (Campbell 1997), *i.e.* $(\Omega_2/\Omega_\odot) = (P_\odot/P_2) >> 1$, assuming the secondary star's rotation period is tidally locked with the orbital period of the system, the orbital angular momentum loss via stellar wind (hence magnetic braking) can be derived from the basic stellar wind equations (Campbell 1997), for a field scaling with dynamo number n and the inverse Rossby number respectively (Campbell 1997):

$$\frac{\dot{J}_{mb}}{J_{orb}} \approx 5.8 \times 10^{-21}\left(\frac{\bar{r}}{R_2}\right)^{-5/3}\left(\frac{P_{orb}}{9.88\,\mathrm{hr}}\right)^{8/3}\left(\frac{M_1}{0.9\,M_\odot}\right)^{-1} \times$$
$$\left(\frac{M_2}{0.6\,M_\odot}\right)^{-4/3}\left(\frac{M}{1.5\,M_\odot}\right)^{1/3}\left(\frac{B_\circ}{B_\odot}\right)^{(4n+2)/3n} \tag{13}$$

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$$\frac{\dot{J}_{mb}}{J_{orb}} \approx 4.2 \times 10^{-21} \left(\frac{\bar{r}}{R_2}\right)^{-5/3} \left(\frac{P_{orb}}{9.88\,\mathrm{hr}}\right)^{20/9} \left(\frac{M_1}{0.9\,M_\odot}\right)^{-1} \times \left(\frac{M_2}{0.6\,M_\odot}\right)^{-4/9} \left(\frac{M}{1.5\,M_\odot}\right)^{1/3} \left(\frac{B_\circ}{B_\odot}\right)^{2}. \tag{14}$$

In both model equations, $\bar{r}/R_2$ represents the ratio of the wind dead zone to stellar radius of the secondary star (Mestel & Spruit 1987; Campbell 1997), and where $(B_\circ/B_\odot)$ is the ratio of the stellar to solar surface polar magnetic field strength. The condition securing stable mass transfer for AE Aqr in the current propeller phase ($\alpha \approx 1; \eta \approx 1$) (Meintjes 2002), is

$$\frac{\dot{J}_{mb}}{J} \geq 2.5 \times 10^{-16} \left(\frac{\dot{M}_2}{1 \times 10^{18}\,\mathrm{g\ s^{-1}}}\right) \left(\frac{M_2}{0.6\,M_\odot}\right)^{-1} \mathrm{s^{-1}}. \tag{15}$$

In the fast rotator limit $(P_\odot/P_2) \approx 70$, applicable to AE Aqr, the ratio $(\bar{r}/R_2) \approx 3 - 5$ (Campbell 1997). For a linear $n = 1$ dynamo model and the inverse Rossby number law respectively, this results in estimates for the polar value of the secondary star's magnetic field which are

$$B_\circ \geq 1600 \left(\frac{B_\odot}{2.5\,\mathrm{G}}\right) \left(\frac{\dot{M}_2}{1 \times 10^{18}\,\mathrm{g\ s^{-1}}}\right)^{1/2} \left(\frac{P_{orb}}{9.88\,\mathrm{hr}}\right)^{-8/6} \mathrm{G} \tag{16}$$

and

$$B_\circ \geq 2000 \left(\frac{B_\odot}{2.5\,\mathrm{G}}\right) \left(\frac{\dot{M}_2}{1 \times 10^{18}\,\mathrm{g\ s^{-1}}}\right)^{1/2} \left(\frac{P_{orb}}{9.88\,\mathrm{hr}}\right)^{-20/18} \mathrm{G} \tag{17}$$

using $(M_2/0.6\,M_\odot)$, $(M_1/0.9\,M_\odot)$ and $(M/1.5\,M_\odot)$.

It can be shown that for an accretion disc to develop in AE Aqr (conservative mass transfer), the condition for accretion disc development in the current phase must satisfy

$$\frac{\dot{M}_2}{M_2} > \frac{\frac{\dot{J}_{orb}}{J_{orb}}}{\frac{4}{3} - q} \tag{18}$$

resulting in

$$\dot{M}_2 > 10^{18} \left(\frac{B_\circ}{2000\,G}\right)^{2} \left(\frac{P_{orb}}{9.88\,h}\right)^{20/9} \left(\frac{M_1}{0.9\,M_\odot}\right)^{-1} \left(\frac{M_2}{0.6 M_\odot}\right)^{-4/9} \mathrm{g\ s^{-1}} \tag{19}$$

which correlates with the fact that the current mass transfer of $\dot{M}_2 < 10^{18}$ g s^{-1} is propelled out of the system.

4. Discussion

It has been shown above that the wide binary and short rotation period of the white dwarf can be the result of an unique secular evolution process the system went through. It is proposed that mass transfer initiated from an evolved 0.7 $M_\odot$ K4-5 star to a heavier 0.87 $M_\odot$, slow rotating ($P_{*,i} \approx 1$ hour) white dwarf, when the initial orbital period was $P_{orb,i} \approx 8.5$ hours. High discless mass accretion rates in this phase destabilized the mass transfer, leading to a short, but violent, high mass transfer phase resulting in a mass transfer rate of $-\dot{M}_{2,i} \sim 0.2 - 0.4\dot{M}_{Edd}$, *i.e.* $\sim (1-2) \times 10^{20}$ g s^{-1}, lasting for a period of approximately $t_{\dot{M}} \sim (1-3)\times 10^4$ yr. The run-away mass transfer probably came to a halt when a critical mass ratio was reached, $q_{crit} \sim 0.73$ (evolved secondary). During this phase the orbital period evolved to approximately $P_{orb} \sim 11$ hrs as a result of the violent $q < 1$ mass transfer from the less massive secondary star to the heavier white dwarf. The high mass transfer rate resulted in an accretion disc spinning the white dwarf up to a period of ~ 33 s in this period. A substantial decrease in mass transfer occurred when $q \leq q_{crit}$, resulting in the system converging ($\dot{a}/a<1$) as mass transfer is driven by magnetic braking of the secondary on the thermal timescale. Mass loss from the system also occurs in this phase as a result of matter being ejected by the spun-up, fast rotating magnetosphere – the propeller-ejector effect. This allows an estimate of the secondary star's polar magnetic field, which lies between $\langle B \rangle \sim 1600 - 2000$ G. The duration of this low mass transfer phase is approximately $\sim 10^7$ yrs, which is in excellent agreement with the inferred spin down timescale of the white dwarf if it was spun up to a period of approximately 32.9 s during the high mass transfer phase.

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