HIGH-FREQUENCY SOLAR OSCILLATIONS

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Abstract. This paper reviews the properties of the solar oscillation power spectrum at frequencies near and above the acoustic cut-off frequency for the atmosphere. This region of the spectrum contains over 50% of the total peak structure observed and is a source of information not only on the outer layers of the sun and the origin of solar oscillations, but also on the physics of the solar core and chromosphere.

1. Introduction

Solar acoustic waves with frequencies less than the acoustic cut-off frequency for the atmosphere, $\omega_{ac} \simeq 0.033$ rad s^{-1}, are trapped in resonant cavities beneath the photosphere. The boundaries for each cavity are defined by the positions where the outward propagating wave is reflected by the sharp transition in the density gradient near the solar surface, and where refraction of the inward propagating wave — due to the rapid rise in sound speed in the solar interior — renders the wave propagation horizontal. The resulting modes of oscillation, called p-modes, are governed by a dispersion relation and are manifest as discrete peaks in the $k - \omega$ power spectrum. As the acoustic wave frequency approaches and passes through ω_{ac}, the cavities develop "leaky" upper boundaries that become increasingly transparent. Eventually, for $\omega \gg \omega_{ac}$, the waves are no longer trapped and are able to propagate through the chromosphere to the base of the corona. Despite the traveling wave nature of the high-frequency waves, the oscillation power spectrum shows peak structure that extends well beyond ω_{ac} (see Figure 1). Three explanations have been put forward for the preservation of "mode-like" structure at high-frequencies: (1) The analysis procedure of viewing the sun in terms of spherical harmonic functions results in a "geometrical" interference of direct and indirect traveling waves emitted from a sub-photospheric source (Kumar *et al.*, 1990; Kumar and Lu, 1991). (2) Wave reflection from the chromosphere-corona transition region results in an "extended" acoustic cavity (Balmforth and Gough, 1990). (3) Chromospheric magnetism modifies the acoustic cut-off frequency such that waves with $\omega > \omega_{ac}$ are reflected at the photosphere (Jain and Roberts, 1996). Although all three mechanisms may be contributing to the observed signal, the ability of Kumar's model to reproduce the general characteristics of the high-frequency spectrum,

F.-L. Deubner et al. (eds.), New Eyes to See Inside the Sun and Stars, 415–422.
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https://doi.org/10.1017/S0074180900239090 Published online by Cambridge University Press

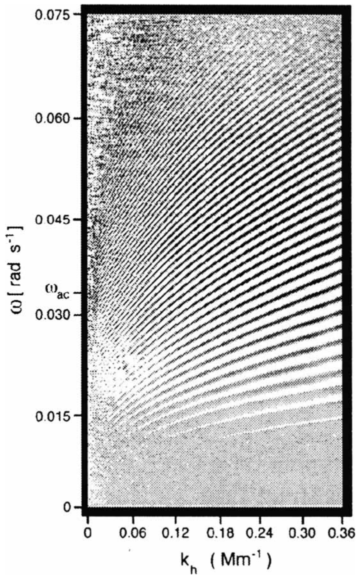

Figure 1. The oscillation power spectrum of 19 days of full-disk, Ca II K-line intensity observations acquired at South Pole over the period December 12, 1994 to January 1, 1995.

suggests that geometric interference accounts for the majority of the high-frequency structure (known as high-frequency interference peaks: HIPs). Unlike the p-modes, which have been extensively studied owing to their diagnostic capability for the solar interior, the HIPs have not been well studied. This is probably due, in part, to their intrinsically large widths (Duvall *et al.*, 1991) which prevents accurate frequency measurement (when compared to the p-modes). In this article I hope to demonstrate that the $k - \omega$ spectrum near and above ω_{ac} should receive more attention.

2. A Brief Overview of Kumar's HIP Model

Consider an acoustic noise source that is located just beneath the base of the photosphere and is emitting waves with frequency $\omega \gg \omega_{ac}$. When the solar surface is viewed in terms of spherical harmonics, the observed velocity signal of waves with spherical harmonic degree ℓ is given by

$$v(t) = A(\omega, \ell)\mathrm{e}^{i\omega t}\left[1 + D\mathrm{e}^{-i2\theta^*}\right] , \tag{1}$$

where the first term represents a wave of amplitude $A(\omega, \ell)$ emitted upwards from the source (i.e. towards the solar surface), and the second term represents a downward emitted wave that returns to the source level, with amplitude $DA(\omega, \ell)$ and phase

$$2\theta^* = 2\int_{r_1(w)}^{r^*} k_r(\omega, r)dr - \pi/2 + \pi\delta \ , \tag{2}$$

after reflection at the lower turning point. Here k_r is the radial wavenumber, r^* and $r_1(w)$ are the radial locations of the excitation source and inner turning point,

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respectively, $w = \omega/(\ell + \frac{1}{2})$, the factor $\pi/2$ is the phase change that the inward wave undergoes on reflection at r_1 (Keller and Rubinow, 1960), and δ is a constant whose value depends on the type of radiation emitted by the source (0 for monopole and quadrupole radiation, 1 for dipole radiation). The power spectrum of $v(t)$,

$$P(\omega, \ell) = |\, A(\omega, \ell)\,|^2 \left[(1 - D)^2 + 4D\cos^2\theta^*\right] \ , \tag{3}$$

has maxima when $\theta^* = n\pi$. The frequencies of the HIPs thus depend on the properties of the solar interior and the oscillation source.
Since

$$\int_{r_1(w)}^{r^*} k_r(\omega, r)dr \simeq \omega F^*(w) \ , \tag{4}$$

where

$$F^*(w) \equiv \int_{r_1(w)}^{r^*} \left[1 - \frac{c^2}{r^2 w^2}\right]^{1/2} \frac{dr}{c} \tag{5}$$

(Jefferies *et al.*, 1994), and $\theta^* = n\pi$, then along with equation (2) we can see that

$$\omega F^*(w) = \pi \left(n + \frac{1}{4} - \frac{\delta}{2}\right) \ . \tag{6}$$

This is the dispersion relation for HIPs. The HIPs can be made to obey the dispersion relation for the trapped p-modes, (Jefferies *et al.*, 1994; Roxburgh and Vorontsov, 1995; Vorontsov *et al.*, 1997), i.e.

$$\omega F(w) = \pi[n + \alpha(\omega)] \ , \tag{7}$$

by defining the phase function

$$\alpha(\omega) \equiv \frac{\omega \Delta S}{\pi} + \frac{1}{4} - \frac{\delta}{2} \, . \tag{8}$$

Here

$$F(w) \equiv \int_{r_1(w)}^{R_\odot} \left[1 - \frac{c^2}{r^2 w^2}\right]^{1/2} \frac{dr}{c} \, , \tag{9}$$

$R_\odot$ is photospheric radius, c is the sound speed, n is the radial order, and

$$\Delta S \simeq \int_{r^*}^{R_\odot} \frac{dr}{c} \tag{10}$$

is the sound travel time between r^* and $R_\odot$. The phase function is, in general, sensitive to both the physics of the near-surface layers and the source properties. From equation (8) it can be seen that, in the limit of zero reflectivity,

$$\pi \frac{d\alpha(\omega)}{d\omega} = \Delta S \tag{11}$$

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provides a measure of the source location (with respect to $R_\odot$), while

$$\beta = -\omega^2 \frac{d}{d\omega}\left(\frac{\alpha}{\omega}\right) = \pm\frac{1}{4} \tag{12}$$

provides information on the source type ($+\frac{1}{4}$ for monopole or quadrupole radiation, and $-\frac{1}{4}$ for dipole radiation).

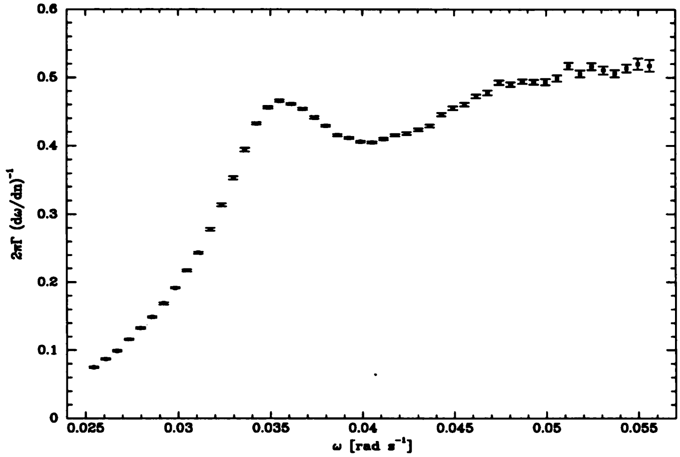

Figure 2. The quantity $2\pi\Gamma(\frac{\partial\omega}{\partial n})_\ell^{-1}$ as measured from the high-frequency region of the power spectrum shown in Figure 1, and averaged over the range $100 \leq \ell \leq 250$. The widths were measured using a single Lorentzian profile and then corrected, to first order, for the additional width introduced by the spatial leaks. Kumar's HIP model predicts a value of $\frac{1}{2}$ for $\omega \gg \omega_{ac}$ (see equation (13)).

Another property of the HIP model is that the frequency separation between two successive peaks, $\partial\omega/\partial n$, and the peak line width, Γ (FWHM), are determined entirely by the wave travel-time, $S^*(w)$, between the source and the inner turning point: i.e.

$$2\pi\Gamma = \frac{1}{2}\left(\frac{\partial\omega}{\partial n}\right) = \frac{\pi}{2}\left[\int_{r_1(w)}^{r^*} \frac{\partial k_r}{\partial\omega} dr\right]^{-1} = \frac{\pi}{2S^*(w)} \quad . \tag{13}$$

3. Determining the Properties of the Oscillation Source and the Near-Surface Layers

By varying the radial location and type of the acoustic source in a solar model, Kumar (1997) is able to produce a theoretical power spectrum which closely matches the observed power spectrum in Figure 1 when the source is located 140 +/- 60 km below the base of the photosphere and emits quadrupole radiation.

Based on the high-frequency behavior of $(d\alpha/d\omega)$ and β for the same data, Vorontsov *et al.* (1997) estimate the oscillation source to be located at the base of

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the photosphere (within a pressure scale height), and to emit both positive and negative parity radiation (i.e. monopole/quadrupole and dipole radiation respectively). The discrepancy with Kumar's result for the source type is not understood.

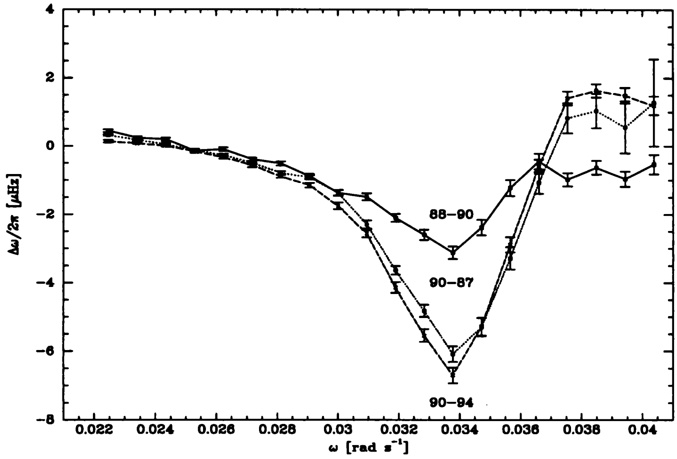

Figure 3. The frequency differences $[\omega(x) - \omega(y)]/2\pi$, measured for $100 \leq \ell \leq 250$ using a single Lorentzian profile: x and y are the years in which the observations were obtained. To first order, the spectral leakage is symmetric about each target peak. Thus the systematic error in ω incurred by fitting a multiplet with a single peak is small. The differences are plotted such that the mean solar activity level for the observations obtained in year x is higher than for year y. The data were all obtained from observations made at South Pole.

The observed frequency separation, $\frac{1}{2\pi}(\frac{\partial\omega}{\partial n})_\ell$ (Kumar *et al.*, 1994), wave travel time (Jefferies *et al.*, 1994), phase function (Duvall *et al.*, 1991; Vorontsov *et al.*, 1997), and line width (see Figure 2) all show a resonant-like behavior near ω_{ac}. This phenomenon has been interpreted as the manifestation of an acoustic resonance between the oscillation source and the effective level of partial reflection for the waves (Vorontsov *et al.*, 1997). The properties of this "source resonance" are sensitive to the acoustic reflectivity of the solar atmosphere, and to the location and parity of the excitation source. The observed frequency of the resonance for the data shown in Figure 2 is $\omega_r \simeq 0.035$ rad s^{-1}. For this value of ω_r to be consistent with the source being located near the base of the photosphere, the source needs to have composite parity. This agrees with the inference based on the behavior of β.

Based on the solar-cycle variations in ω (Libbrecht and Woodard, 1991; Ronan *et al.*, 1994) and $\alpha(\omega)$ (Vorontsov *et al.*, 1997), Vorontsov *et al.* argue that the acoustic reflectivity increases with increasing solar activity. As the reflectivity of the atmosphere increases, the acoustic waves are repelled more and penetrate the atmosphere to a lesser extent: i.e. the effective acoustic cavity is smaller. Hence both the cavity eigenfrequencies and the source resonance frequency increase. Oscillations with frequencies well below ω_r, i.e. the p-modes, therefore show a positive difference for ω(high-activity) $-$ ω(low-activity) as is well known. Near ω_r, however, the frequency differences have the opposite sense (see Figure 3). They are also much larger than those observed for the p-modes. Although the observed frequency differences can be

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qualitatively explained by simultaneous magnetic field and temperature changes in the sun's chromosphere (Jain and Roberts, 1996), the temperature changes required are quite large. In addition, a large part of the frequency differences near ω_r may be caused by fitting symmetric profiles to asymmetric spectral peaks (Vorontsov *et al.*, 1997). Using a simple model for wave interference, it can be shown (Vorontsov *et al.*, 1997) that the spectral peaks in the immediate vicinity of ω_r are asymmetric with the amount of asymmetry decreasing as $\mid \omega - \omega_r \mid$ increases. Increasing the atmospheric reflectivity affects the line profiles in two ways. First, ω_r moves to higher frequencies thus moving the most asymmetrical peaks to higher frequencies. Second, the magnitude of the asymmetry increases. Under these circumstances, the systematic frequency offset produced by fitting a symmetric profile to an asymmetric spectral line can be much larger than (and in the opposite sense to) the change in the cavity eigenfrequency. The large asymmetries in the spectral profiles near the source resonance frequency are also responsible for some of the resonant-like behavior in the measured linewidths [c.f. Figure 2 with Figure 3 of Roxburgh and Vorontsov (1995)]. In summary, waves with $\omega \simeq \omega_{ac}$ appear to have an increased sensitivity to changes in the near surface layers of the sun (Guenther, 1991). However, careful modeling of the spectrum in this region is required before any inferences can be made about changing chromospheric temperatures, magnetic fields, etc.

4. Probing the Solar Core

HIP-like structure with a spacing of 4.4 10^{-4}rad s^{-1}, has been detected in the unimaged velocity observations from the GOLF experiment (García *et al.*, 1997). At first sight, this is a surprising result (Fossat *et al.*, 1992) as the GOLF experiment is only sensitive to partial waves with $\ell \leq 5$. (Acoustic waves with ray paths that pass close to the solar center can be considered as the superposition of low-ℓ partial waves.) Based on Kumar's HIP model, the expected velocity signal is given by

$$v(t) = \sum_{\ell=0}^{5} a_\ell A(\omega,\ell) \mathrm{e}^{i\omega t} \left[1 + D\mathrm{e}^{-i2\theta_\ell^*}\right] \tag{14}$$

where a_ℓ is the sensitivity of the observation to a given ℓ-value. Now for small ℓ, equation (7) reduces to

$$F(w) \simeq \int_0^{R_\odot} \frac{dr}{c} - \frac{\pi}{2}\,\frac{\ell + 1/2}{\omega} \tag{15}$$

where the first term corresponds to the acoustic radius of the sun ($\sim$ 3518 seconds (Gough, 1990)), and the second term is independent of the solar model. Assuming that the excitation source is located close to $R_\odot$, then $F^*(w) \simeq F(w)$ and using equations (2), (4) and (15) we have

$$\theta_\ell^* \simeq \omega \int_0^{R_\odot} \frac{dr}{c} + \frac{\pi}{2}(\delta - \ell - 1) = \theta_0^* - \frac{\pi\ell}{2}\ . \tag{16}$$

If we now assume that the low-ℓ modes are excited to equal amplitudes, then

$$v(t) \simeq A' \mathrm{e}^{i\omega t} \left[1 + \eta D\mathrm{e}^{-i2\theta_0^*}\right] \tag{17}$$

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with $A' = A\sum_\ell a_\ell$ and

$$\eta = \frac{\sum_\ell a_\ell(\ell = \text{even}) - \sum_\ell a_\ell(\ell = \text{odd})}{\sum_\ell a_\ell} \quad . \tag{18}$$

Now for unimaged, full-disk observations, the numerator in equation(18) is much smaller than the denominator (Christensen-Dalsgaard and Gough, 1979) so that $\eta D \ll D$. Thus we can see from the power spectrum,

$$P(\omega) \simeq \mid A' \mid^2 \left[(1-\eta D)^2 + 4\eta D\cos^2\theta_0^*\right] \quad , \tag{19}$$

that the amplitude of any HIP structure will be extremely small. Moreover, the expected frequency separation between adjacent HIPs will be

$$\frac{d\omega}{dn} = \pi \left[\int_0^{R_\odot} \frac{dr}{c}\right]^{-1} \simeq 8.8\ 10^{-4}\text{rad s}^{-1} \quad , \tag{20}$$

and not the observed value of 4.4 10^{-4}rad s^{-1}.

To explain the observations requires that the high-frequency waves undergo reflection from a layer above the source (García *et al.*, 1997b). When there is wave reflection, the observed velocity signal is given by

$$v(t) = \mathrm{e}^{i\omega t}\sum_{\ell=0}^{5} a_\ell A(\omega,\ell)\left[\frac{1+D\mathrm{e}^{-i2\theta_\ell^*}}{1-R\mathrm{e}^{-i2\theta_\ell}}\right] \quad . \tag{21}$$

If R is small, so that $(1-R\mathrm{e}^{-i2\theta_\ell})^{-1} \simeq 1 + R\mathrm{e}^{-i2\theta_\ell}$, and if $\eta \simeq 0$, then the power spectrum is given by

$$P(\omega) \simeq \mid A' \mid^2 \left[(1-RD)^2 + 4RD\cos^2(\theta+\theta^*)\right] \tag{22}$$

and it is easy to see that the expected peak separation is close to 4.4 10^{-4}rad s^{-1}. However, high-frequency acoustic waves are not expected to be significantly reflected at the solar surface (Kumar, 1997), so where does the reflection come from? Recent time-distance measurements of the high-frequency waves (Jefferies *et al.*, 1997) suggest that a small amount of wave reflection ($\sim$ 6%) may be occurring in the sun's chromosphere. If the frequency dependence of the reflectivity for this atmospheric "boundary" is weak, then it may provide the reflection necessary to explain the GOLF spectrum at high ω. The detection of high-frequency structure in the GOLF spectrum is exciting as it represents waves which pass very close to the solar core and therefore provide information about the rotation and sound speed in this region. Unfortunately, the precision with which the peak frequencies can be measured is limited owing to the broad nature of the profiles. Despite this, the data are important as they cover a range of w which is otherwise poorly sampled.

5. Determining the Horizontal Structure of the Chromosphere

Since high-frequency acoustic waves are essentially vertically propagating in the solar atmosphere, it should be possible to directly measure the wave travel-time across the

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atmosphere. This could be accomplished by observing the acoustic wavefield simultaneously at two heights in the atmosphere (e.g. using photospheric and chromospheric lines) and using the cross-correlation techniques that have been developed for time-distance analyses (Duvall *et al.*, 1993). The wave travel-times could then be inverted to produce maps of the horizontal structure of the chromosphere, averaged over the vertical extent between the two observing heights, similar to the tomographic maps recently produced for the solar interior (Duvall *et al.*, 1996).

Acknowledgements
I would like to thank Y. Osaki, H. Shibahashi and S. Vorontsov for many enlightening conversations about the high-frequency oscillations, and for the "5-star" hospitality extended to me during my visits to the Universities of Tokyo and London. This material is based upon work supported by the National Science Foundation under grant numbers INT-9417091 and OPP-9219515.

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