Demonstrations of a Pair of Theorems in Geometry.

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I.

If, in any triangle ABC, the angle A > the angle B, and lines AD and BE be drawn so that the angles BAD and DAC are, respectively, greater than the angles ABE and EBC, then is BE > AD. (Fig. 6.)

Proof: Draw AD′ and AD″ making angles BAD′ and D″AC, respectively, equal to the angles ABE and EBC.

Then we can show that AD′ and AD″ are each less than BE.

First, by placing the triangle ABD′ on the triangle BAE so that AB and the angle BAD′ coincide respectively with BA and the angle ABE, we find, since angle A > angle B, AD′ < BE.

Second, by placing the triangle ACD″ on the triangle BCE so that the angles C in both triangles coincide, and the side CA falls along CB (AC < BC), the lines AD″ and BE will be parallel. Draw AF parallel to CE, and we find AD″ = FE < BE.

Now, of the three lines AD′, AD, and AD″, the one which is the most remote from the perpendicular, drawn from A upon BC, and consequently the greatest, must be one of the lines AD′ and AD″, which lie on opposite sides of AD.

Hence, AD, being less than either AD′ or AD″, which are each less than BE, must itself be less than BE.

Cor. If the angles ABE and EBC are, respectively, not greater than the angles BAD and DAC, and if AD = BE, it follows that the angle A = the angle B, and that these angles are divided in the same ratio by AD and BE.

Hence, if the angles A and B are bisected by AD and BE, and these lines are equal, the triangle ABC is isoceles.

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II.

If, in any triangle ABC, BC>AC, and lines AD and BE be drawn so that CD and DB are, respectively, greater than CE and EA, then is BE>AD. (Fig. 7.)

Proof: Draw AD″ and AD′ making BD′ and D″C, respectively equal to AE and EC.

Then we can show that AD″ and AD′ are each less than BE.

First, the triangles AD′B and BEA have two sides of the one respectively equal to two sides of the other, but the included angles A and B unequal (since BC>AC). As the angle A > the angle B, we find BE>AD′.

Second, the two triangles ACD″ and BCE have the angle C common, CD″=CE, and BC>AC. Therefore the angle CEB > the angle CD″A. Hence, the angle CAD″> the angle CBE. Also, the supplement of the angle CAD″> the angle CBE, since the supplement of the angle A> the angle B.

Now, place the triangle CAD″ on the triangle CBE so that the angles C in both triangles coincide, and CE coincides with CD″. Then, as each angle at A has been shown to be greater than the angle CBE, the oblique EB must be greater than EA.

Thus, EB is greater than both AD′ and AD″, one of which is, as shown in (I.), greater than AD. Therefore, EB>AD.

Cor. If CD and DB are, respectively, not less than CE and EA, and AD=BE, it follows that AC=BC, and these sides are divided in the same ratio by AD and BE.

Hence, if two medians of a triangle are equal, the triangle is isosceles.