A NEW METHOD OF LIGHT CURVE ANALYSIS: APPLICATION TO V444 CYGNI

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ABSTRACT-A new method of light curve analysis is introduced for systems involving an atmospheric eclipse.Occultation or partial eclipses may be treated, with or without the presence of transparency effects. Besides admitting an arbitrary degree of transparency to the eclipsing component, an arbitrary law of limb darkening may also be given the eclipsed star.

The method has successfully been applied to the study of narrow band continuum light curves of V444 Cygni obtained by Cherepashchuk and Khaliullin. Primary and secondary minima are examined separately.

1.INTRODUCTION

A novel approach to the determination of geometrical elements of eclipsing binary systems is here outlined. A more complete presentation will be given elsewhere. The method involves the solution of a system of non-linear, transcendental equations relating the morphology of eclipse minima and the geometrical elements. These include the relative radii, brightnesses, angle of incination, and coefficients of limb darkeming and transparency. The last parameter arises from the assumption that the eclipsing star can not be considered a wholly opaque disc.

The method is checked by using the solutions to the equations as parameters which generate a synthetic light curve. The model assumes spherical stars, circular orbits, spherically symmetrical, homogeneous extended atmospheres, no phase effects and symmetrical minima.

2. THE BASIC EQUATIONS

Let the total amount of light of the system be

M. J. Plavec, D. M. Popper and R. K. Ulrich (eds.), Close Binary Stars: Observations and Interpretation, 73–78.

$$\ell = L_1 + L_2 - \int_A J(r)\, dS \qquad (1)$$

where J is the brightness per unit area, dS the infinitessimal element of eclipsed area and A is the eclipsed area. If the brightness is uniform then $J(r)=L_1/\pi r_1^2$ where the subscripts refer to the eclipsed star. Following this we can say

$$1-\ell(r_1, r_2, \delta, J) = \alpha L_1$$

where

$$\alpha(r_1, r_2, \theta, i) = 1/\pi r_1^2 \int_A dS$$

and δ is the apparent separation of the two stars, θ the phase in degrees, and i the angle of inclination. Subsequently a plot of ℓ against $\sin^{2m}\theta$ may be represented by

$$A_{2m}(r_1, r_2, i, L_1) = \int_0^{\pi/2} \{1-\ell(r_1, r_2, \delta, J)\} \frac{d(\sin^{2m}\theta)}{d\theta}\,. \qquad (2)$$

The A2m's on the LHS may be empirically determined from the data and m equations of the form of eq.(2) may be inverted to determine r_1, r_2 and i.

Using a similar approach we develop a system of equations of the form

$$A_{2m} = A_{2m}(r_1, r_2, i, J, F)$$

where F is the transparency function involved by letting the eclipsing star vary in opacity across the disc much the same way the brightness is allowed to vary due to limb darkening. An expression of the form

$$A_{2m}(r_1, r_2, i, F, J) = \int_0^{r_2} F(s) \frac{\partial}{\partial s}\left(A_{2m}(r_1, s, i, J)\right) dS \qquad (3)$$

is derived and solved numerically to obtain the elements. Note that s in F(s) refers to the radius of the eclipsing star while dS represents an element of eclipsed area.

To solve equations of the form of eq.(3) the transparency function F(s) is adopted while further expressions for $\partial/\partial s\, A_{2m}(r_1, s, i, J)$ are derived. In the course of these derivations successive terms arise defined as

$$I_m = I_m(r_1, J) = \int_0^{r_1} \frac{J(r)}{r_1^{2m-2}} \frac{\partial}{\partial r}(\pi r^{2m})\, dr$$ where $I_m = L_1$ for a non-limb darkened star

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$$R_m = R_m(r_2, F) = \int_0^{r_2} \frac{F(s)}{r_2^{2m}} \frac{\partial}{\partial s}(s^{2m})\,ds$$

where $R_m = 1$ for an opaque star

$$Q = Q(r_1, r_2, J, F) = -\int_0^{\min(r_1, r_2)} F(s) J(r)\, 2\pi s\, ds\,.$$

Ultimately, expressions of the following form are determined.

$$A_2(r_1, r_2, i, J, F) = I_1 R_1 r_2^2 \csc^2 i + \cot^2 i\, Q - \psi \qquad (4)$$

for m=1

where the ψ term is treated as an error term. A similar system of equations may be derived using the Kernel $\cos^{2m}\theta$ which weights the data near mid-minima more heavily.

APPLICATION TO V444 CYGNI

Solutions have been obtained for narrow band continuum light curves of the Wolf-Rayet eclipsing binary V444 Cygni. The observations were obtained by Cherepashchuk and Khaliullin(1972). Primary and secondary eclipse were examined separately. For the primary eclipse (that of the 09 by the WN6) the adopted transparency function was

$$F(s) = \left(1 - V\left(s/r\right)^2\right) \qquad (5)$$

where v is the coefficient of transparency and $0 < v < 1$. The behavior of this function is shown in fig.1 below. The adopted law of transparency determines the coefficients R_m and here $R_m = 1 - v/(m+1)$, e.g. $R_1 = 1 - 0.5v$ for m=1. The loss

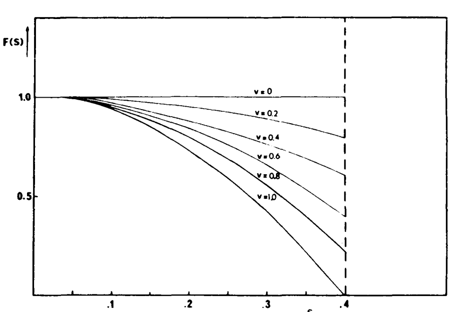

Fig.1 Law of transparency

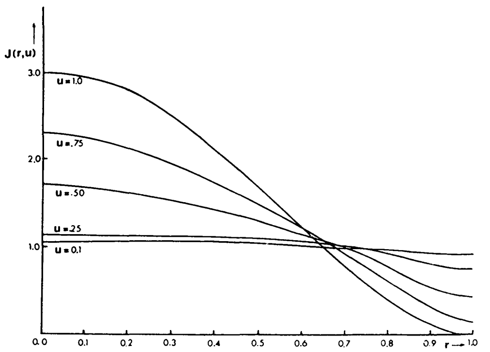

Fig.2 Law of limb-darkening

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of light for a concentric eclipse, Q, is similarly dependent on F(s) and is derived as

$$Q = L_1 (1 - 0.5v(r_1/r_2)^2) \quad (6)$$

while all I_m are equal to unity, the O star treated as non limb darkened.

For the secondary eclipse the O star is treated as opaque and the WR component is assigned a non-linear law of limb darkening(see Fig.2). This will determine the values of the coeffients I_m while all R_m=1. Due to lack of space solutions for the light curve taken at λ4789 are given below.In Fig. 3 the solid line represents the synthetic curve fit to the primary eclipse while the dotted lines represents the region in which all reasonable solution sets may lie. The parameters which generated this curve are the solutions to the primary eclipse listed in Table 1, along with solutions to the secondary eclipse. Although the solution for

	primary	secondary
WR	0.430+.045	0.134±.016
09	0.260±.026	0.260±.005
u	-	0.69±.04
v	0.39±.37	-
i	79.7° ± 1	80° adopted
L_1	0.252± .026	0.154±.098

Table 1.

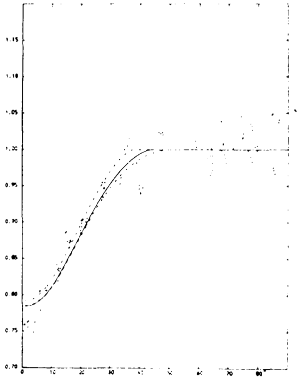

Fig.3

coefficients of transparency has an unacceptably large error it does indicate, as do the solutions for the other light curves, the presence of transparency effects. It is felt that, although mathematically sound, the method requires the use of highly accurate data before the application can be reliable to the degree required.

REFERENCE

Cherepashchuk,A.M. and Khaliullin,Kh.F.: 1972, Peremmenye Zvezdy,18, 321.

DISCUSSION FOLLOWING SMITH AND THEOKAS

Rucinski: The method by Cherepaschuk (applied by him most extensively to V444 Cyg) does not require any knowledge of shapes of functions describing the transparency or surface brightness versus radius; it is assumed that the functions are monotoneous, however. Could you comment how your method is related to the approach by Cherepaschuk?

Theokas: First the model does not require rectified data, simultaneous solution of the light curve minima or any precondition such as $r_1 > \cos\theta$ as does Cherepaschuk's method. Secondly the adoption of the law of transparency need not be completely arbitrary but other considerations may be taken into account such as those of Cherepaschuk you mention. This method in a sense thus may compliment his.

Mook: V444 Cyg has a phase-dependent linear polarization. Can your model predict this? (Your initial response that your model does not speak to conditions outside eclipse is not responsive to my question. You could write your transfer equations for polarized components and solve for the variation of polarization during the eclipse. That is, just as you solve for overall flux as a function of time--your synthesized light curve--you could also predict % polarization as a function of time and use that as an independent check.)

Theokas: Yes, in theory it would be possible to expand the model to examine polarization rather than flux as a function of time. Within the context of this model however, the mathematics would become considerably more complex diminishing its practicality.

Smak: What is the maximum photometric phase of eclipse, or, in other words, how close is it to totality?

Theokas: These are very nearly complete eclipses 70 - 80% or more. If they were total we would have the unusual occurrence of an occultation eclipse at primary and secondary.

van Paradijs: I hope that your model will allow one to determine the radial density distribution of stellar winds.

Theokas: Not directly. Some distributions may be ruled out by considering what laws of transparency will fit the data and which give poor approximations. The model is probably best applied to those systems where stellar winds are low as in ζ Aur.

Linnell: The semi-transparent layers are subject to substantial distortion by the companion, while your idealized model assumes spherical symmetry. How wide latitude of assumed model characteristics do you feel is possible which will produce a light curve in good agreement with the observations?

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Theokas: V444 Cygni perhaps represents the limit to that latitude as represented by the scatter in the light curve shown. This is due to variations in both the rate and amount of mass outflow so the assumption you mention is not well founded here and we should expect better results with less scatter. In other words an extended but quiescent atmosphere.

Al-Naimy: Why is there a great difference in the radius of the WR star?

Theokas: The fractional radius of the WR component is not the same for secondary and primary eclipse due to the presence of the highly extended atmosphere of the WR component. It behaves differently as a luminous object and does not take part in the secondary eclipse. This is just a confirmation of an old result.

Al-Naimy: How do you trust your results according to your assumption?

Theokas: I'm satisfied with the results for the fractional radii and brightnesses but do not trust the coefficients of transparency or limb darkening to tell us a great deal physically about the stars. They were very sensitive to changes in initial conditions. The accuracy of data is a problem here and perhaps satellite-based observations would overcome this.